311(7): Checking the Inhomogeneous Field Equations

These are derived from the Cartan Evans identity:

$$D_\mu T^{a\mu\nu} = R^{a}{}_{\mu}{}^{\mu\nu} \quad -(1)$$

i.e.

$$\partial_\mu T^{a\mu\nu} = R^{a}{}_{\mu}{}^{\mu\nu} - \omega^{a}{}_{\mu b} T^{b\mu\nu} \quad -(2)$$

using for each a:

$$T^{\mu\nu} = \begin{bmatrix} 0 & -T^1(\text{orb}) & -T^2(\text{orb}) & -T^3(\text{orb}) \\ T^1(\text{orb}) & 0 & -T^3(\text{sp}) & T^2(\text{sp}) \\ T^2(\text{orb}) & T^3(\text{sp}) & 0 & -T^1(\text{sp}) \\ T^3(\text{orb}) & -T^2(\text{sp}) & T^1(\text{sp}) & 0 \end{bmatrix} \quad -(3)$$

The Coulomb law is derived from:

$$\nu = 0 \quad -(4)$$

i.e.

$$\partial_1 T^{a10} + \partial_2 T^{a20} + \partial_3 T^{a30} \quad -(5)$$

$$= R^{a}{}_{1}{}^{10} + R^{a}{}_{2}{}^{20} + R^{a}{}_{3}{}^{30} - \omega^{a}{}_{1b} T^{b10} - \omega^{a}{}_{2b} T^{b20} - \omega^{a}{}_{3b} T^{b30}$$

i.e.

$$\underline{\nabla} \cdot \underset{\sim}{T}^a(\text{orb}) = \underline{\omega}^a{}_b \cdot \underset{\sim}{T}^b(\text{orb}) - \underline{q}^b \cdot \underline{R}^a{}_b(\text{orb}) \quad -(6)$$

where:

$$\underline{T}^a(\text{orb}) = T^{a1}(\text{orb})\,\underline{i} + T^{a2}(\text{orb})\,\underline{j} + T^{a3}(\text{orb})\,\underline{k} \quad -(7)$$

and

$$\underline{R}^a{}_b(\text{orb}) = R^{a}{}_{b}{}^{1}(\text{orb})\,\underline{i} + R^{a}{}_{b}{}^{2}(\text{orb})\,\underline{j} + R^{a}{}_{b}{}^{3}(\text{orb})\,\underline{k} \quad -(8)$$

2) Now use:

$$\underline{E}^a = cA^{(0)} \underset{\sim}{T}^a (\text{vac}) \quad - (9)$$

and

$$\underline{A}^a = A^{(0)} \underline{q}^a \quad - (10)$$

to find that:

$$\underline{\nabla} \cdot \underline{E}^a = \underline{\omega}^a{}_b \cdot \underline{E}^b - c\underline{A}^b \cdot \underline{R}^a{}_b (\text{vac}) \quad - (11)$$

This is the same as UFT255 eq. (66) and Slide 25 of the Engineering Model, Q.E.D.

Now use:

$$\underline{E}^a{}_b = cW^{(0)} \underline{R}^a{}_b (\text{vac}) \quad - (11)$$

to find that:

$$\underline{\nabla} \cdot \underline{E}^a = \underline{\omega}^a{}_b \cdot \underline{E}^b - \frac{1}{W^{(0)}} \underline{A}^b \cdot \underline{E}^a{}_b \quad - (12)$$

$$= \underline{\omega}^a{}_b \cdot \underline{E}^b - \frac{1}{r^{(0)}} \underline{q}^b \cdot \underline{E}^a{}_b$$

Now remove indices using the methods developed in preceding notes and papers to find that:

$$\underline{\nabla} \cdot \underline{E} = 2\underline{E} \cdot \left(\frac{1}{r^{(0)}} \underline{q} - \underline{\omega} \right) \quad - (13)$$

3) This is the same as in notes 317(2) and 317(4), QED

Therefore:

$$\boxed{\begin{aligned} \underline{\nabla} \cdot \underline{E} &= 2\underline{E} \cdot \underline{\kappa} \\ \underline{\nabla} \cdot \underline{B} &= 2\underline{B} \cdot \underline{\kappa} \end{aligned}} \quad -(14)$$

where:

$$\underline{\kappa} = \frac{1}{r^{(0)}} \underline{q} - \underline{\omega} . \quad -(15)$$

In ECE2 there is exact symmetry between the Gauss law of magnetism and the Coulomb law.

The Ampère Maxwell law is obtained from:

$$\nu = 1, 2, 3 \quad -(16)$$

in eq. (2). For:

$$\nu = 1 \quad -(17)$$

it follows that:

$$\begin{aligned} &\partial_0 T^{a01} + \partial_2 T^{a21} + \partial_3 T^{a31} \\ &= q^b{}_0 R^a{}_b{}^{01} + q^b{}_2 R^a{}_b{}^{21} + q^b{}_3 R^a{}_b{}^{31} \\ &\quad - \omega^a{}_{0b} T^{b01} - \omega^a{}_{2b} T^{b21} - \omega^a{}_{3b} T^{b31} \end{aligned} \quad -(18)$$

i.e.

$$\begin{aligned} &-\partial_0 T^1(\text{orb}) + \partial_2 T^3(\text{sp}) - \partial_3 T^2(\text{sp}) \\ &= -q^b{}_0 R^a{}_b{}^1(\text{orb}) + q^b{}_2 R^a{}_b{}^3(\text{sp}) - q^b{}_3 R^a{}_b{}^2(\text{sp}) \\ &\quad + \omega^a{}_{0b} T^{b1}(\text{orb}) - \omega^a{}_{2b} T^{b3}(\text{sp}) + \omega^a{}_{3b} T^{b2}(\text{sp}) \end{aligned} \quad -(19)$$

This means that:

$$-\frac{1}{c}\frac{\partial T_X(orb)}{\partial t} + \left(\underline{\nabla}\times\underline{T}(sp)\right)_X \qquad -(20)$$
$$= \omega^a{}_{0b}\, T^b{}_X(orb) + \left(\underline{\omega}^a{}_b \times \underline{T}^b(spin)\right)_X$$
$$- q^b{}_0\, R^a{}_{bX}(orb) - \left(\underline{q}^b \times \underline{R}^a{}_b(sp)\right)_X$$

There are similar results for the Y and Z components

so:

$$\underline{\nabla}\times\underline{T}^a(sp) - \frac{1}{c}\frac{\partial \underline{T}^a(orb)}{\partial t} \qquad -(21)$$
$$= \omega^a{}_{0b}\,\underline{T}^b(orb) - q^b{}_0\,\underline{R}^a{}_b(orb)$$
$$+ \underline{\omega}^a{}_b \times \underline{T}^b(sp) - \underline{q}^b \times \underline{R}^a{}_b(sp)$$

which is eq. (23) of UFT255, QED.

Now use:

$$\underline{B}^a = A^{(0)}\,\underline{T}^a(sp) \qquad -(22)$$
$$\underline{E}^a = cA^{(0)}\,\underline{T}^a(orb) \qquad -(23)$$
$$\underline{B}^a{}_b = W^{(0)}\,\underline{R}^a{}_b(sp) \qquad -(24)$$
$$\underline{E}^a{}_b = cW^{(0)}\,\underline{R}^a{}_b(orb) \qquad -(25)$$

To find that:

5)

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t}$$

$$= A^{(0)} \Big(\omega^a{}_{ob} \underline{T}^b(\text{orb}) - q^b{}_0 \underline{R}^a{}_b(\text{orb})$$

$$+ \underline{\omega}^a{}_b \times \underline{T}^b(\text{sp}) - \underline{q}^b \times \underline{R}^a{}_b(\text{sp}) \Big)$$

$$= \frac{\omega^a{}_{ob}}{c} \underline{E}^b - A^b{}_0 \underline{R}^a{}_b(\text{orb})$$

$$+ \underline{\omega}^a{}_b \times \underline{B}^b - \underline{A}^b \times \underline{R}^a{}_b(\text{sp}) \Big) \quad -(26)$$

This is the same as in the Engineering Model, slide 25, Q.E.D.

Now use eqs. (24) and (25) to find:

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2} \frac{\partial \underline{E}^a}{\partial t} \quad -(27)$$

$$= \frac{\omega^a{}_{ob}}{c} \underline{E}^b - \frac{A^b{}_0}{c W^{(0)}} \underline{E}^a{}_b + \underline{\omega}^a{}_b \times \underline{B}^b - \frac{\underline{A}^b \times \underline{B}^a{}_b}{W^{(0)}}$$

and remove indices to find that: $\quad -(28)$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = 2 \left[\left(\underline{q} - \frac{\underline{\omega}}{r^{(0)}} \right) \times \underline{B} - \frac{1}{c} \left(\omega_0 - \frac{q_0}{r^{(0)}} \right) \underline{E} \right)$$

which is Note 317(3), Eq. (13), Q.E.D.

6) Final Format of Field Equations.

$$\underline{\nabla} \cdot \underline{B} = 2\underline{B} \cdot \underline{\kappa} \quad -(29)$$

$$\frac{\partial \underline{B}}{\partial t} + \underline{\nabla} \times \underline{E} = 2\left(c\kappa_0 \underline{B} + \underline{\kappa} \times \underline{E} \right) \quad -(30)$$

$$\underline{\nabla} \cdot \underline{E} = 2\underline{E} \cdot \underline{\kappa} \quad -(31)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = 2\left(\underline{\kappa} \times \underline{B} - \frac{\kappa_0}{c} \underline{E} \right) \quad -(32)$$

where:

$$\kappa_0 = \omega_0 - \frac{q_0}{r^{(0)}} \quad -(33)$$

and

$$\underline{\kappa} = \underline{\omega} - \frac{\underline{q}}{r^{(0)}} \quad -(34)$$

There is a symmetry between the homogeneous and inhomogeneous field equations.