317(2): The Coulomb Law in ECE2.

From the previous note:

$$\underline{\nabla} \cdot \underline{E}^a = \underline{\omega}^a{}_b \cdot \underline{E}^b - c\underline{A}^b \cdot \underline{R}^a{}_b(\text{orb}) \quad -(1)$$

where:

$$\underline{R}^a{}_b(\text{orb}) = \frac{1}{cW^{(0)}} \underline{E}^a{}_b \quad -(2)$$

so

$$\underline{\nabla} \cdot \underline{E}^a = \underline{\omega}^a{}_b \cdot \underline{E}^b - \frac{\underline{A}^b}{W^{(0)}} \cdot \underline{E}^a{}_b \quad -(3)$$

$$= \underline{\omega}^a{}_b \cdot \underline{E}^b - \frac{A^{(0)}}{W^{(0)}} \underline{q}^b \cdot \underline{E}^a{}_b$$

Removing the a index:

$$\boxed{\begin{aligned} \underline{\nabla} \cdot \underline{E} &= \underline{\omega}_b \cdot \underline{E}^b - \frac{A^{(0)}}{W^{(0)}} \underline{q}^b \cdot \underline{E}_b \\ &= 2\underline{E} \cdot \left(\frac{1}{r^{(0)}} \underline{q} - \underline{\omega} \right) \end{aligned}} \quad -(4)$$

where

$$\frac{A^{(0)}}{W^{(0)}} = \frac{1}{r^{(0)}} \quad -(5)$$

As in UFT255 use the simple example:

$$\underline{E} = E_r \underline{e}_r = \frac{-e}{4\pi\epsilon_0 r^2} \underline{e}_r \quad -(6)$$

2) So:

$$\frac{\partial E_r}{\partial r} = \frac{e}{2\pi\epsilon_0 r^3} \quad -(7)$$

if

$$E_r = -\frac{e}{4\pi\epsilon_0 r^2} \quad -(8)$$

Fm eq. (4):

$$\frac{\partial E_r}{\partial r} = 2E_r\left(\frac{1}{r^{(0)}}q_r - \omega_r\right) \quad -(9)$$

$$= \frac{e}{2\pi\epsilon_0 r^2}\left(\omega_r - \frac{1}{r^{(0)}}q_r\right)$$

Fm eqs. (7) and (9):

$$\boxed{\omega_r - \frac{q_r}{r^{(0)}} = \frac{1}{r}} \quad -(10)$$

As:

$$r \rightarrow \infty \quad -(11)$$

then

$$\omega_r \rightarrow \frac{q_r}{r^{(0)}} \quad -(12)$$

and this is the free space result, Q.E.D.