317(1): Background Theory for the Inhomogeneous Field Equations (Ref. UFT255)

The starting point is the Cartan Evans identity:

$$D_\mu \tilde{T}^a_{\ \nu\rho} + D_\rho \tilde{T}^a_{\ \mu\nu} + D_\nu \tilde{T}^a_{\ \rho\mu} := \tilde{R}^a_{\ \mu\nu\rho} + \tilde{R}^a_{\ \rho\mu\nu} + \tilde{R}^a_{\ \nu\rho\mu} \quad -(1)$$

which is equivalent to:

$$D_\mu T^{a\mu\nu} := R^{a\ \mu\nu}_{\ \mu} \quad -(2)$$

or:

$$\partial_\mu T^{a\mu\nu} = j^{a\nu} \quad -(3)$$

where:

$$j^{a\nu} = R^{a\ \mu\nu}_{\ \mu} - \omega^a_{\ \mu b} T^{b\mu\nu} \quad -(4)$$

The torsion tensor is defined by:

$$T^{\mu\nu} = \begin{bmatrix} 0 & -T^1(\text{orb}) & -T^2(\text{orb}) & -T^3(\text{orb}) \\ T^1(\text{orb}) & 0 & -T^3(\text{spin}) & T^2(\text{spin}) \\ T^2(\text{orb}) & T^3(\text{spin}) & 0 & -T^1(\text{spin}) \\ T^3(\text{orb}) & -T^2(\text{spin}) & T^1(\text{spin}) & 0 \end{bmatrix} \quad -(5)$$

The field equation (3) becomes:

$$\underline{\nabla} \cdot \underset{\sim}{T}^a(\text{orb}) = j^{a0} \quad -(6)$$

and

$$\underline{\nabla} \times \underset{\sim}{T}^a(\text{spin}) - \frac{1}{c}\frac{\partial \underset{\sim}{T}^a(\text{orb})}{\partial t} = \underline{j}^a \quad -(7)$$

where:

$$j^{a0} = \underline{\omega}^a{}_b \cdot \underline{T}^b(\text{orb}) - \underline{q}^b \cdot \underline{R}^a{}_b(\text{orb}) \quad -(8)$$

and:

$$\underline{j}^a = \omega^a{}_{0b} \underline{T}^b(\text{orb}) + \underline{\omega}^a{}_b \times \underline{T}^b(\text{spin}) - \left(q^b_0 \underline{R}^a{}_b(\text{orb}) + \underline{q}^b \times \underline{R}^a{}_b(\text{spin})\right) \quad -(9)$$

Now use:

$$\underline{E}^a = cA^{(0)} \underline{T}^a(\text{orb}) \ \text{V m}^{-1} \quad -(10)$$

where $A^{(0)}$ is in units of $\text{J s C}^{-1}\text{m}^{-1} = \text{tesla m}$.

Then the Coulomb Law is:

$$\underline{\nabla} \cdot \underline{E}^a = cA^{(0)} j^{a0} = \rho^a / \epsilon_0 \quad -(11)$$

where:

$$\rho^a = \epsilon_0 \left(\underline{\omega}^a{}_b \cdot \underline{E}^b - c\underline{A}^b \cdot \underline{R}^a{}_b(\text{orb}) \right) \quad -(12)$$

For a free field:

$$\rho^a = 0 \quad -(13)$$

so

$$\underline{\omega}^a{}_b \cdot \underline{E}^b = c\underline{A}^b \cdot \underline{R}^a{}_b(\text{orb}) \quad -(14)$$

The current is defined as:

$$\underline{j}^a = \omega^a{}_{0b} \underline{T}^b(\text{orb}) + \underline{\omega}^a{}_b \times \underline{T}^b(\text{spin}) - \left(q^b_0 \underline{R}^a{}_b(\text{orb}) + \underline{q}^b \times \underline{R}^a{}_b(\text{spin})\right) \quad -(15)$$

It follows that:

3)

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2}\frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a = A^{(0)} \underline{j}^a \quad -(16)$$

which is the Ampère Maxwell law, where:

$$\underline{J}^a = \frac{A^{(0)}}{\mu_0} \underline{j}^a \quad -(17)$$

in units of $C s^{-1} m^{-2}$, and where:

$$J^{\mu a} = (c\rho^a, \underline{J}^a), \quad -(18)$$

with:

$$\underline{J}^a = \epsilon_0 c \Big(\omega^a{}_{0b} \underline{E}^b - c A^b{}_0 \underline{R}^a{}_b(\text{orb}) + \underline{\omega}^a{}_b \times \underline{E}^b - c \underline{A}^b \times \underline{R}^a{}_b(\text{spin}) \Big) \quad -(19)$$

Now these equations must be translated into the ECE2 theory using the new hypotheses:

$$\underline{B}^a{}_b = W^{(0)} \underline{R}^a{}_b(\text{spin}) \quad -(20)$$

and

$$\underline{E}^a{}_b = c W^{(0)} \underline{R}^a{}_b(\text{orb}) \quad -(21)$$

and removal of tangent indices. This will be carried out in Note 317(2).