316(1): Removal of the Tangent Indices in ECE Theory.

Consider the ECE Hypothesis:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(1)$$

and

$$F^a_{\mu\nu} = A^{(0)} T^a_{\mu\nu} \quad -(2)$$

where q^a_μ is the Cartan tetrad and $T^a_{\mu\nu}$ the Cartan torsion. Here A^a_μ is the electromagnetic potential and $F^a_{\mu\nu}$ the electromagnetic field. The scaling factor $A^{(0)}$ is a scalar. So electromagnetism is based directly on Cartan geometry. There are similar equations for the gravitational and weak and strong nuclear forces.

The index a may be removed using:

$$A_\mu = A^a_\mu e_a \quad -(3)$$

and

$$F_{\mu\nu} = F^a_{\mu\nu} e_a \quad -(4)$$

where e_a is the unit vector in the Cartan tangent space. In the Cartesian basis:

$$e_a = (1, -1, -1, -1) \quad -(5)$$

and in the complex circular basis:

2)

$$e_{(a)} = \left(1, -\frac{1}{\sqrt{2}}(1-i), -\frac{1}{\sqrt{2}}(1+i), -1\right) \quad -(6)$$

Eqs. (3) and (4) follow from the fundamental definition of the Cartan tetrad:

$$V^a = q^a_\mu V^\mu \quad -(7)$$

and

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu \quad -(8)$$

So

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \omega^a_{\mu b} A^b_\nu - \omega^a_{\nu b} A^b_\mu \quad -(9)$$

Therefore

$$F_{\mu\nu} = F^a_{\mu\nu} e_a$$
$$= \partial_\mu A_\nu - \partial_\nu A_\mu + \omega_{\mu\nu} - \omega_{\nu\mu} \quad -(10)$$

where we have used:

$$\partial_\mu (A^a_\nu e_a) = \partial_\mu A_\nu \quad -(11)$$

and

$$\omega^a_{\mu b} A^b_\nu e_a = \omega_{\mu\nu} \quad -(12)$$

and so on. <u>So the definition of the field tensor is simplified to eq. (10).</u>

3) Similarly, define the electromagnetic field tensor by:

$$F^a{}_{b\mu\nu} := W^{(0)} R^a{}_{b\mu\nu} \quad -(13)$$

where $R^a{}_{b\mu\nu}$ is the curvature two-form of Cartan geometry:

$$R^a{}_{b\mu\nu} = q^a{}_\kappa q^\lambda{}_b R^\kappa{}_{\lambda\mu\nu} \quad -(14)$$

where $R^\kappa{}_{\lambda\mu\nu}$ is the Riemann curvature tensor. In eq. (13) $W^{(0)}$ is a scalar with the units of weber, (tesla m^2), or magnetic flux as in UFT 215.

It follows that:

$$\boxed{F_{\mu\nu} = e_a e^b F^a{}_{b\mu\nu}} \quad -(15)$$

in analogy to eq. (10). A direct analogy is obtained by defining:

$$F^a{}_{\lambda\mu\nu} = q^a{}_\kappa F^\kappa{}_{\lambda\mu\nu} \quad -(16)$$

so

$$F_{\lambda\mu\nu} = e_a F^a{}_{\lambda\mu\nu}. \quad -(17)$$

Now define

$$F_{b\mu\nu} = q^\lambda{}_b F_{\lambda\mu\nu} \quad -(18)$$

4) So
$$F_{\mu\nu} = e^b F_{b\mu\nu} \quad -(19)$$
i.e. from eqs. (17) and (18):
$$F_{\mu\nu} = e_a e^b F^a{}_{b\mu\nu} \quad -(20)$$
which is eq. (15) Q.E.D.

The curvature two-form is defined by:
$$R^a{}_{b\mu\nu} = \partial_\mu \omega^a{}_{\nu b} - \partial_\nu \omega^a{}_{\mu b} + \omega^a{}_{\mu c}\omega^c{}_{\nu b} - \omega^a{}_{\nu c}\omega^c{}_{\mu b} \quad -(21)$$

So: $\quad -(22)$
$$\begin{aligned} R_{\mu\nu} &= e^b e_a R^a{}_{b\mu\nu} \\ &= \partial_\mu \omega_\nu - \partial_\nu \omega_\mu + \omega_{\mu c}\omega^c{}_\nu - \omega_{\nu c}\omega^c{}_\mu \\ &:= \partial_\mu \omega_\nu - \partial_\nu \omega_\mu + \omega^2_{\mu\nu} - \omega^2_{\nu\mu} \end{aligned}$$
where
$$\omega_\nu = e_a e^b \omega^a{}_{\mu b} \quad -(23)$$
and
$$\omega^2_{\mu\nu} = \omega_{\mu c}\omega^c{}_\nu = e_a e^b \omega^a{}_{\mu c}\omega^c{}_{\nu b} \quad -(24)$$

Therefore:
$$\boxed{F_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + W^{(0)2}\left(\omega^2_{\mu\nu} - \omega^2_{\nu\mu}\right)} \quad -(25)$$

5) where:

$$\boxed{W_\nu = W^{(0)} \omega_\nu} \quad -(26)$$

is the magnetic flux potential.

Therefore the curvature based ECE hypotheses are:

$$W_\nu = W^{(0)} \omega_\nu \quad -(27)$$

and

$$F_{\mu\nu} = W^{(0)} R_{\mu\nu} \quad -(28)$$

Here $F_{\mu\nu}$ has the units of tesla, $W^{(0)}$ is in units of weber = m^2 tesla, and $R_{\mu\nu}$ has the units of inverse metres squared. The magnetic flux potential W_ν has the units of weber per metre.

As shown in UFT 315, eq. (28) follows directly from the Jacobi Cartan Evans (JCE) identity, the second Bianchi identity corrected for torsion.