314(2): Vector Format of the First Evans Identity.

From eq. (8) of Note 314(1):

$$F^{b}_{\mu a} \tilde{F}^{a\mu\nu} = 0 \quad -(1)$$

Now use:

$$F^{b}_{\mu a} = F^{b}_{\mu\nu} q^{\nu}_{a} \quad -(2)$$

So

$$q^{\nu}_{a} F^{b}_{\mu\nu} \tilde{F}^{a\mu\nu} = 0 \quad -(3)$$

A possible solution is:

$$F^{b}_{\mu\nu} \tilde{F}^{a\mu\nu} = 0 \quad -(4)$$

for all q^{ν}_{a}. So the identity is:

$$\boxed{F^{b}_{\mu\nu} \tilde{F}^{a\mu\nu} = 0} \quad -(5)$$

where:

$$F^{b}_{\mu\nu} = \begin{bmatrix} 0 & E_X & E_Y & E_Z \\ -E_X & 0 & -cB_Z & cB_Y \\ -E_Y & cB_Z & 0 & -cB_X \\ -E_Z & -cB_Y & cB_X & 0 \end{bmatrix} \quad -(6)$$

and

$$\tilde{F}^{a\mu\nu} = \begin{bmatrix} 0 & -cB_X & -cB_Y & -cB_Z \\ cB_X & 0 & E_Z & -E_Y \\ cB_Y & -E_Z & 0 & E_X \\ cB_Z & E_Y & -E_X & 0 \end{bmatrix} \quad -(7)$$

2) So eq. (5) is:

$$2\left(E_x^b B_x^a + E_y^b B_y^a + E_z^b B_z^a + B_z^b E_z^a + B_y^b E_y^a + B_x^b E_x^a\right) = 0 \quad -(8)$$

i.e.

$$\boxed{\underline{E}^b \cdot \underline{B}^a + \underline{B}^b \cdot \underline{E}^a = 0} \quad -(9)$$

For plane waves:

$$\underline{E}^{(1)} = \frac{E^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{i\phi} \quad -(10)$$

$$\underline{B}^{(1)} = \frac{B^{(0)}}{\sqrt{2}}\left(i\underline{i} + \underline{j}\right)e^{-i\phi} \quad -(11)$$

$$\underline{E}^{(2)} = \frac{E^{(0)}}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right)e^{-i\phi} \quad -(12)$$

$$\underline{B}^{(2)} = \frac{B^{(0)}}{\sqrt{2}}\left(-i\underline{i} + \underline{j}\right)e^{+i\phi} \quad -(13)$$

i.e.

$$a = (1), \quad b = (2) \quad -(14)$$

So

$$\underline{E}^{(1)} \cdot \underline{B}^{(2)} = \frac{E^{(0)}B^{(0)}}{2}\left(\underline{i} - i\underline{j}\right)\cdot\left(-i\underline{i} + \underline{j}\right)$$

$$= -E^{(0)}B^{(0)}\,i \quad -(15)$$

$$\underline{B}^{(1)} \cdot \underline{E}^{(2)} = \frac{E^{(0)} B^{(0)}}{2} \left(i\underline{i} + \underline{j} \right) \cdot \left(\underline{i} + i\underline{j} \right)$$

$$= E^{(0)} B^{(0)} i \qquad - (16)$$

So:

$$\underline{E}^{(1)} \cdot \underline{B}^{(2)} + \underline{E}^{(2)} \cdot \underline{B}^{(1)} = 0 \qquad - (17)$$

and eq. (9) is true for plane waves,
Q.E.D.

The first Evans identity of electromagnetism is true for all electric and magnetic fields.
