314(4): Development of the Second Evans Identity

The second Evans identity is:

$$\tilde{T}^{\lambda}_{\mu\nu}\tilde{T}^{\alpha}_{\rho\lambda} + \tilde{T}^{\lambda}_{\rho\mu}\tilde{T}^{\alpha}_{\nu\lambda} + \tilde{T}^{\lambda}_{\nu\rho}\tilde{T}^{\alpha}_{\mu\lambda} := 0 \quad -(1)$$

and is an exact identity in four dimensions. The tilde denotes Hodge dual. It can be developed as follows:

$$\tilde{T}^{a}_{\mu\nu}\tilde{T}^{\alpha}_{\rho a} + \tilde{T}^{a}_{\rho\mu}\tilde{T}^{\alpha}_{\nu a} + \tilde{T}^{a}_{\nu\rho}\tilde{T}^{\alpha}_{\mu a} := 0 \quad -(2)$$

i.e. $$\left(\tilde{T}^{a}_{\mu\nu}\tilde{T}^{b}_{\rho a} + \tilde{T}^{a}_{\rho\mu}\tilde{T}^{b}_{\nu a} + \tilde{T}^{a}_{\nu\rho}\tilde{T}^{b}_{\mu a}\right)q^{\alpha}_{b} := 0 \quad -(3)$$

so $$\tilde{T}^{a}_{\mu\nu}\tilde{T}^{b}_{\rho a} + \tilde{T}^{a}_{\rho\mu}\tilde{T}^{b}_{\nu a} + \tilde{T}^{a}_{\nu\rho}\tilde{T}^{b}_{\mu a} = 0 \quad -(4)$$

i.e. $$\boxed{\tilde{T}^{b}_{\mu a}T^{a\mu\nu} = 0} \quad -(5)$$

in which $$\tilde{T}^{b}_{\mu a} = q^{\nu}_{a}\tilde{T}^{b}_{\mu\nu} \quad -(6)$$

so $$\boxed{\tilde{T}^{b}_{\mu\nu}T^{a\mu\nu} = 0} \quad -(7)$$

2) Eqs. (5) and (7) give the field equations:

$$\tilde{F}^{b}_{\mu a} F^{a\mu\nu} = 0 \quad -(8)$$

and

$$\tilde{F}^{b}_{\mu\nu} F^{a\mu\nu} = 0 \quad -(9)$$

where for each polarization index: $\quad -(10)$

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -cB^3 & cB^2 \\ E^2 & cB^3 & 0 & -cB^1 \\ E^3 & -cB^2 & cB^1 & 0 \end{bmatrix} ; \quad \tilde{F}^{b}_{\mu\nu} = \begin{bmatrix} 0 & cB^1 & cB^2 & cB^3 \\ -cB^1 & 0 & E^3 & -E^2 \\ -cB^2 & -E^3 & 0 & E^1 \\ -cB^3 & E^2 & -E^1 & 0 \end{bmatrix}$$

Define:

$$\tilde{F}^{b}_{\mu a} = \left(\tilde{F}^{b}{}_{0a}, -\underline{\tilde{F}}^{b}{}_{a}\right) \quad -(11)$$

Eq. (8) gives:

$$\underline{\tilde{F}}^{a}{}_{b} \cdot \underline{E}^{b} = 0 \quad -(12)$$

and

$$\underline{\tilde{F}}^{a}{}_{b} \times \underline{B}^{b} + \frac{1}{c}\tilde{F}^{a}{}_{0b}\,\underline{E}^{b} = 0 \quad -(13)$$

and eq. (9) gives:

$$\underline{E}^{b} \cdot \underline{B}^{a} + \underline{B}^{b} \cdot \underline{E}^{a} = 0 \quad -(14)$$

The complete set of equations from the first and second Evans identities are, in four dimensions:

$$\underline{F}^b{}_a \cdot \underline{B}^a = 0 \qquad -(15)$$

$$cF^b{}_{0a}\,\underline{B}^a = \underline{F}^b{}_a \times \underline{E}^a \qquad -(16)$$

$$\underline{\tilde{F}}^b{}_a \cdot \underline{E}^b = 0 \qquad -(17)$$

$$\underline{\tilde{F}}^a{}_b \times \underline{B}^b + \frac{1}{c}\tilde{F}^a{}_{0b}\,\underline{E}^b = \underline{0} \qquad -(18)$$

and from self identities:

$$\underline{E}^b \cdot \underline{B}^a + \underline{B}^b \cdot \underline{E}^a = 0 \qquad -(19)$$

Eqs. (15) to (18) are identical in structure to the following set of free space equations in the Engineering Model, UFT 303:

$$\underline{\omega}^a{}_b \cdot \underline{B}^b = 0 \qquad -(20)$$

$$c\,\omega^a{}_{0b}\,\underline{B}^b = \underline{\omega}^a{}_b \times \underline{E}^b \qquad -(21)$$

$$\underline{\omega}^a{}_b \cdot \underline{E}^b = 0 \qquad -(22)$$

$$\underline{\omega}^a{}_b \times \underline{B}^b + \frac{1}{c}\,\omega^a{}_{0b}\,\underline{E}^b = \underline{0} \qquad -(23)$$

where:

$$\omega^a{}_{\mu b} = \left(\omega^a{}_{0b},\ -\underline{\omega}^a{}_b\right) \qquad -(24)$$