314(1): Vector Format of the First Evans Identity

Consider the first Evans identity of UFT 109:

$$T^{\lambda}_{\mu\nu} T^{\alpha}_{\rho\lambda} + T^{\lambda}_{\rho\mu} T^{\alpha}_{\nu\lambda} + T^{\lambda}_{\nu\rho} T^{\alpha}_{\mu\lambda} := 0 \quad -(1)$$

Replace the λ indices with a indices:

$$T^{a}_{\mu\nu} T^{\alpha}_{\rho a} + T^{a}_{\rho\mu} T^{\alpha}_{\nu a} + T^{a}_{\nu\rho} T^{\alpha}_{\mu a} := 0 \quad -(2)$$

Now use:

$$T^{\alpha}_{\rho\lambda} = T^{a}_{\rho\lambda} q^{\alpha}_{a} \quad -(3)$$

so

$$\left(T^{a}_{\mu\nu} T^{b}_{\rho a} + T^{a}_{\rho\mu} T^{b}_{\nu a} + T^{a}_{\nu\rho} T^{b}_{\mu a} \right) q^{\alpha}_{b} := 0 \quad -(4)$$

A possible solution is:

$$T^{a}_{\mu\nu} T^{b}_{\rho a} + T^{a}_{\rho\mu} T^{b}_{\nu a} + T^{a}_{\nu\rho} T^{b}_{\mu a} := 0 \quad -(5)$$

which is found by multiplying eq. (4) from the right by q^{b}_{α}.

Eq. (5) is:

$$T^{b}_{\rho a} \wedge T^{a}_{\mu\nu} := 0 \quad -(6)$$

Using the ECE hypothesis eq. (6) is transformed to

2)

$$F^b_{\rho a} \wedge F^a_{\mu\nu} := 0 \quad -(7)$$

where the wedge product is formed between the tensor valued one-form $F^b_{\rho a}$ and the vector valued two-form $F^a_{\mu\nu}$.

Now express eq. (5) as:

$$F^b_{\mu a} \tilde{F}^{a\mu\nu} := 0 \quad -(8)$$

where the Hodge dual field tensor is:

$$\tilde{F}^{a\mu\nu} = \begin{bmatrix} 0 & -cB^a_X & -cB^a_Y & -cB^a_Z \\ cB^a_X & 0 & E^a_Z & -E^a_Y \\ cB^a_Y & -E^a_Z & 0 & E^a_X \\ cB^a_Z & E^a_Y & -E^a_X & 0 \end{bmatrix} \quad -(9)$$

In direct analogy the homogeneous field equations of ECE are:

$$\partial_\mu \tilde{F}^{a\mu\nu} := 0 \quad -(10)$$

However there is a sign change between eqs. (8) and (10) because:

$$F^b_{\mu a} = \left(F^b_{0a}, -\underline{F}^b_{\ a}\right) \quad -(11)$$

and

3)

$$\partial_\mu = \left(\frac{1}{c} \frac{\partial}{\partial t}, \underline{\nabla} \right) \quad -(12)$$

The vector format of eq. (10) is:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad -(13)$$

and

$$\frac{\partial \underline{B}^a}{\partial t} + \underline{\nabla} \times \underline{E}^a = 0 \quad -(14)$$

So the vector format of Eq. (8) is

$$\boxed{\begin{aligned} \underset{\sim}{F}{}^b{}_a \cdot \underline{B}^a &= 0 \quad -(15) \\ \text{and} \quad c F^b{}_{oa} \underline{B}^a &= \underline{F}^b{}_a \times \underline{E}^a \quad -(16) \end{aligned}}$$

In the next note the field $F^b{}_{\mu a}$ will be discussed. It is a new field tensor in the format of a tensor valued one form of differential geometry.