3B(1): Proof of the Second Bianchi Identity and the Effect of Torsion.

Consider the Cartan identity:

$$D \wedge T := R \wedge q \quad -(1)$$

using the minimalist notation of previous papers. The second Bianchi identity with torsion can be proven from:

$$D_\mu (D \wedge T) := D_\mu (R \wedge q) \quad -(2)$$

Proof

First consider the method used erroneously in Einsteinian general relativity:

$$R \wedge q \overset{?}{=} 0 \quad -(3)$$

then reinstate torsion at the end of the proof. Eq. (3) is the first Bianchi "identity":

$$R^{\kappa}{}_{\lambda\nu\rho} + R^{\kappa}{}_{\rho\lambda\nu} + R^{\kappa}{}_{\nu\rho\lambda} = 0 \quad -(4)$$

It is clear that:

$$D_\mu \left(R^{\kappa}{}_{\lambda\nu\rho} + R^{\kappa}{}_{\rho\lambda\nu} + R^{\kappa}{}_{\nu\rho\lambda} \right) = 0 \quad -(5)$$

Similarly:

$$D_\rho \left(R^{\kappa}{}_{\lambda\nu\rho} + R^{\kappa}{}_{\rho\lambda\nu} + R^{\kappa}{}_{\nu\rho\lambda} \right) = 0 \quad -(6)$$

and

$$D_\nu \left(R^{\kappa}{}_{\lambda\nu\rho} + R^{\kappa}{}_{\rho\lambda\nu} + R^{\kappa}{}_{\nu\rho\lambda} \right) = 0 \quad -(7)$$

Now add eqs. (5) to (7):

)

$$
\begin{aligned}
&D_\mu \left(R^{\kappa}{}_{\lambda\nu\rho} + R^{\kappa}{}_{\rho\lambda\nu} + R^{\kappa}{}_{\nu\rho\lambda}\right)\\
&+ D_\rho \left(R^{\kappa}{}_{\lambda\nu\rho} + R^{\kappa}{}_{\rho\lambda\nu} + R^{\kappa}{}_{\nu\rho\lambda}\right) = 0\\
&+ D_\nu \left(R^{\kappa}{}_{\lambda\nu\rho} + R^{\kappa}{}_{\rho\lambda\nu} + R^{\kappa}{}_{\nu\rho\lambda}\right) \qquad -(8)
\end{aligned}
$$

Rearranging:

$$
\begin{aligned}
&D_\mu R^{\kappa}{}_{\lambda\nu\rho} + D_\rho R^{\kappa}{}_{\lambda\mu\nu} + D_\nu R^{\kappa}{}_{\lambda\rho\mu}\\
&+ D_\mu R^{\kappa}{}_{\rho\lambda\nu} + D_\rho R^{\kappa}{}_{\nu\lambda\mu} + D_\nu R^{\kappa}{}_{\mu\lambda\rho}\\
&+ D_\mu R^{\kappa}{}_{\nu\rho\lambda} + D_\rho R^{\kappa}{}_{\mu\nu\lambda} + D_\nu R^{\kappa}{}_{\rho\mu\lambda}\\
&= 0 \qquad -(9)
\end{aligned}
$$

Now add

$$
\begin{aligned}
&D_\mu R^{\kappa}{}_{\lambda\nu\rho} + D_\rho R^{\kappa}{}_{\lambda\mu\nu} + D_\nu R^{\kappa}{}_{\lambda\rho\mu}\\
&= 0 \qquad -(10)
\end{aligned}
$$

to eq. (9), and assume that this cyclic sum is zero. We obtain:

$$
\begin{aligned}
&D_\mu R^{\kappa}{}_{\lambda\nu\rho} + D_\rho R^{\kappa}{}_{\lambda\mu\nu} + D_\nu R^{\kappa}{}_{\lambda\rho\mu}\\
&+ D_\mu \left(R^{\kappa}{}_{\rho\lambda\nu} + R^{\kappa}{}_{\nu\rho\lambda} + R^{\kappa}{}_{\lambda\nu\rho}\right)\\
&+ D_\rho \left(R^{\kappa}{}_{\nu\lambda\mu} + R^{\kappa}{}_{\mu\nu\lambda} + R^{\kappa}{}_{\lambda\mu\nu}\right)\\
&+ D_\nu \left(R^{\kappa}{}_{\mu\lambda\rho} + R^{\kappa}{}_{\rho\mu\lambda} + R^{\kappa}{}_{\lambda\rho\mu}\right)\\
&= 0 \qquad -(11)
\end{aligned}
$$

3) Eq (11) simplifies to:

$$D_\mu R^\kappa{}_{\lambda\nu\rho} + D_\rho R^\kappa{}_{\lambda\mu\nu} + D_\nu R^\kappa{}_{\lambda\rho\mu} = 0 \quad -(12)$$

which is the second Bianchi identity, Q.E.D.

Eq. (12) is self consistent with eq. (10), and follows from eq. (11) because of the three first Bianchi identities: $-(13)$

$$R^\kappa{}_{\rho\lambda\nu} + R^\kappa{}_{\nu\rho\lambda} + R^\kappa{}_{\lambda\nu\rho} = 0 \quad -(14)$$

$$R^\kappa{}_{\nu\lambda\mu} + R^\kappa{}_{\mu\nu\lambda} + R^\kappa{}_{\lambda\mu\nu} = 0 \quad -(15)$$

$$R^\kappa{}_{\mu\lambda\rho} + R^\kappa{}_{\rho\mu\lambda} + R^\kappa{}_{\lambda\rho\mu} = 0$$

Therefore the second Bianchi identity is derived from:

$$D_\mu (R \wedge q) + D_\rho (R \wedge q) + D_\nu (R \wedge q) = 0 \quad -(16)$$

because

$$D_\mu (R \wedge q) = 0 \quad -(17)$$

$$D_\rho (R \wedge q) = 0 \quad -(18)$$

$$D_\nu (R \wedge q) = 0 \quad -(19)$$

When torsion is reinstated the correct

4)

second Bianchi identity is obtained from:

$$D_\mu(D \wedge T) := D_\mu(R \wedge q) \quad -(20)$$

$$D_\rho(D \wedge T) := D_\rho(R \wedge q) \quad -(21)$$

$$D_\nu(D \wedge T) := D_\nu(R \wedge q). \quad -(22)$$

These are direct consequences of <u>the Cartan identity</u>:

$$D \wedge T := R \wedge q \quad -(23)$$

Eq. (23) is ninety years old and is taught in all good universities. It is a rigorously correct identity, one side is identically equal to the other. This has been proven in great detail in previous UFT papers.

So the rigorously correct second Bianchi identity is derived from:

$$D_\mu(D \wedge T) + D_\rho(D \wedge T) + D_\nu(D \wedge T) :=$$

$$D_\mu(R \wedge q) + D_\rho(R \wedge q) + D_\nu(R \wedge q) \quad -(24)$$

In tensor notation, eq. (23) is:

$$D_\lambda T^\kappa{}_{\nu\rho} + D_\rho T^\kappa{}_{\mu\nu} + D_\nu T^\kappa{}_{\rho\mu} \quad -(25)$$

$$:= R^\kappa{}_{\lambda\nu\rho} + R^\kappa{}_{\rho\mu\nu} + R^\kappa{}_{\nu\rho\mu}$$

5) It follows that the left hand side of eq. (24) is:

$$D_\mu \left(D_\lambda T^\kappa{}_{\nu\rho} + D_\rho T^\kappa{}_{\mu\nu} + D_\nu T^\kappa{}_{\rho\mu} \right)$$
$$+ D_\rho \left(D_\lambda T^\kappa{}_{\nu\rho} + D_\rho T^\kappa{}_{\mu\nu} + D_\nu T^\kappa{}_{\rho\mu} \right)$$
$$+ D_\nu \left(D_\lambda T^\kappa{}_{\nu\rho} + D_\rho T^\kappa{}_{\mu\nu} + D_\nu T^\kappa{}_{\mu\nu} \right) \quad -(26)$$
$$= D_\mu \left(R^\kappa{}_{\lambda\nu\rho} + R^\kappa{}_{\rho\lambda\nu} + R^\kappa{}_{\nu\rho\lambda} \right)$$
$$+ D_\rho \left(R^\kappa{}_{\lambda\nu\rho} + R^\kappa{}_{\rho\lambda\nu} + R^\kappa{}_{\nu\rho\lambda} \right)$$
$$+ D_\nu \left(R^\kappa{}_{\lambda\nu\rho} + R^\kappa{}_{\rho\lambda\nu} + R^\kappa{}_{\nu\rho\lambda} \right).$$

This is the rigorously correct second Bianchi identity.

The second Bianchi identity used by Einstein is eq. (12). It is seen that even if Einsteinian general relativity is corrected it is completely unworkable. It has been saved and then replaced by ECE theory Cartan identity eq. (1).

Eq. (26) can be rearranged to give the following expression:

6)
$$
\begin{aligned}
& D_\mu D_\lambda T^\kappa{}_{\nu\rho} + D_\rho D_\lambda T^\kappa{}_{\nu\rho} + D_\nu D_\lambda T^\kappa{}_{\nu\rho} \\
& + D_\mu \left(D_\rho T^\kappa{}_{\lambda\nu} + D_\nu T^\kappa{}_{\rho\lambda} + D_\lambda T^\kappa{}_{\nu\rho} \right) \\
& + D_\rho \left(D_\nu T^\kappa{}_{\lambda\mu} + D_\mu T^\kappa{}_{\nu\lambda} + D_\lambda T^\kappa{}_{\mu\nu} \right) \qquad -(27) \\
& + D_\nu \left(D_\mu T^\kappa{}_{\lambda\rho} + D_\rho T^\kappa{}_{\mu\lambda} + D_\lambda T^\kappa{}_{\rho\mu} \right) \\
:= \; & D_\mu R^\kappa{}_{\lambda\nu\rho} + D_\rho R^\kappa{}_{\lambda\mu\nu} + D_\nu R^\kappa{}_{\lambda\rho\mu} \\
& + D_\mu \left(R^\kappa{}_{\rho\lambda\nu} + R^\kappa{}_{\nu\rho\lambda} + R^\kappa{}_{\lambda\nu\rho} \right) \\
& + D_\rho \left(R^\kappa{}_{\nu\lambda\mu} + R^\kappa{}_{\mu\nu\lambda} + R^\kappa{}_{\lambda\mu\nu} \right) \\
& + D_\nu \left(R^\kappa{}_{\mu\lambda\rho} + R^\kappa{}_{\rho\mu\lambda} + R^\kappa{}_{\lambda\rho\mu} \right)
\end{aligned}
$$

Using cyclic permutation of eq. (25), eq. (27) simplifies to the Cartan identity,

$$
\boxed{
\begin{aligned}
& D_\mu D_\lambda T^\kappa{}_{\nu\rho} + D_\rho D_\lambda T^\kappa{}_{\nu\rho} + D_\nu D_\lambda T^\kappa{}_{\nu\rho} \\
& \quad := D_\mu R^\kappa{}_{\lambda\nu\rho} + D_\rho R^\kappa{}_{\lambda\mu\nu} + D_\nu R^\kappa{}_{\lambda\rho\mu}
\end{aligned}
}
\qquad -(28)
$$

which is eq. (105) of UFT255, Q.E.D.

7) The First and Second Evans Identities

Denote the Hodge duals of the torsion and curvature tensors by a tilde. It follows that the first Evans identity is:

$$D_\lambda \tilde{T}^{\kappa}{}_{\nu\rho} + D_\rho \tilde{T}^{\kappa}{}_{\nu\mu} + D_\nu \tilde{T}^{\kappa}{}_{\rho\mu} \quad -(29)$$

$$:= \tilde{R}^{\kappa}{}_{\lambda\nu\rho} + \tilde{R}^{\kappa}{}_{\rho\mu\nu} + \tilde{R}^{\kappa}{}_{\nu\rho\mu}$$

and the second Evans identity is:

$$D_\mu D_\lambda \tilde{T}^{\kappa}{}_{\nu\rho} + D_\rho D_\lambda \tilde{T}^{\kappa}{}_{\nu\rho} + D_\nu D_\lambda \tilde{T}^{\kappa}{}_{\nu\rho}$$

$$:= D_\mu \tilde{R}^{\kappa}{}_{\lambda\nu\rho} + D_\rho \tilde{R}^{\kappa}{}_{\lambda\mu\nu} + D_\nu \tilde{R}^{\kappa}{}_{\lambda\rho\mu}$$

$$-(30)$$

Eq. (29) in minimalist notation is:

$$D \wedge \tilde{T} := \tilde{R} \wedge q \quad -(31)$$

and is used for part of the ECE field equations as in UFT 303.

The Einsteinian approach is completely incorrect geometrically and if corrected for torsion becomes unworkable.