313(2): Final Version of the Proof of the Second Bianchi Identity

Consider firstly the following first Bianchi identities without torsion:

$$R^{\kappa}{}_{\tau\lambda\nu\rho} + R^{\kappa}{}_{\tau\rho\lambda\nu} + R^{\kappa}{}_{\tau\nu\rho\lambda} = 0 \quad -(1)$$

$$R^{\kappa}{}_{\tau\lambda\mu\nu} + R^{\kappa}{}_{\tau\nu\lambda\mu} + R^{\kappa}{}_{\tau\mu\nu\lambda} = 0 \quad -(2)$$

$$R^{\kappa}{}_{\tau\lambda\rho\mu} + R^{\kappa}{}_{\tau\mu\lambda\rho} + R^{\kappa}{}_{\tau\rho\mu\lambda} = 0 \quad -(3)$$

It follows that:

$$\begin{aligned} &D_\mu \left(R^{\kappa}{}_{\tau\lambda\nu\rho} + R^{\kappa}{}_{\tau\rho\lambda\nu} + R^{\kappa}{}_{\tau\nu\rho\lambda} \right) \\ &+ D_\rho \left(R^{\kappa}{}_{\tau\lambda\mu\nu} + R^{\kappa}{}_{\tau\nu\lambda\mu} + R^{\kappa}{}_{\tau\mu\nu\lambda} \right) = 0 \quad -(4) \\ &+ D_\nu \left(R^{\kappa}{}_{\tau\lambda\rho\mu} + R^{\kappa}{}_{\tau\mu\lambda\rho} + R^{\kappa}{}_{\tau\rho\mu\lambda} \right) \end{aligned}$$

Rearranging:

$$\begin{aligned} &D_\mu R^{\kappa}{}_{\tau\lambda\nu\rho} + D_\rho R^{\kappa}{}_{\tau\lambda\mu\nu} + D_\nu R^{\kappa}{}_{\tau\lambda\rho\mu} \\ &+ D_\mu \left(R^{\kappa}{}_{\tau\rho\lambda\nu} + R^{\kappa}{}_{\tau\nu\rho\lambda} \right) \quad -(5) \\ &+ D_\rho \left(R^{\kappa}{}_{\tau\nu\lambda\mu} + R^{\kappa}{}_{\tau\mu\nu\lambda} \right) = 0 \\ &+ D_\nu \left(R^{\kappa}{}_{\tau\mu\lambda\rho} + R^{\kappa}{}_{\tau\rho\mu\lambda} \right). \end{aligned}$$

Now assume that:

$$D_\mu R^{\kappa}{}_{\tau\lambda\nu\rho} + D_\rho R^{\kappa}{}_{\tau\lambda\mu\nu} + D_\nu R^{\kappa}{}_{\tau\lambda\rho\mu} = 0 \quad -(6)$$

2) and add eqs. (5) and (6) to obtain:

$$
\begin{aligned}
&D_\mu R^{\kappa}{}_{\lambda\nu\rho} + D_\rho R^{\kappa}{}_{\lambda\mu\nu} + D_\nu R^{\kappa}{}_{\lambda\rho\mu} \\
&+ D_\mu \left(R^{\kappa}{}_{\lambda\nu\rho} + R^{\kappa}{}_{\rho\lambda\nu} + R^{\kappa}{}_{\nu\rho\lambda} \right) \\
&+ D_\rho \left(R^{\kappa}{}_{\lambda\mu\nu} + R^{\kappa}{}_{\nu\lambda\mu} + R^{\kappa}{}_{\mu\nu\lambda} \right) = 0 \\
&+ D_\nu \left(R^{\kappa}{}_{\lambda\rho\mu} + R^{\kappa}{}_{\mu\lambda\rho} + R^{\kappa}{}_{\rho\mu\lambda} \right)
\end{aligned}
\quad -(7)
$$

Eqs. (6) and (7) are self consistent because Eq. (6) is the second of Eq (4), Q.E.D. Eq. (6) is the Bianchi identity without torsion. It can be expressed as

$$D^\mu G_{\mu\nu} = 0 \quad -(8)$$

where $G_{\mu\nu}$ is the Einstein tensor. The Einstein field equation of Nov. 1918 is obtained by comparing eq. (8) with the equation:

$$D^\mu T_{\mu\nu} = 0 \quad -(9)$$

where $T_{\mu\nu}$ is the energy momentum tensor. Eq. (9) can be derived from the Noether Theorem. It is then assumed that

$$G_{\mu\nu} = k T_{\mu\nu} \quad -(10)$$

where k is the Einstein constant.

3) Eq. (10) is the Einstein field equation of Nov. 1915, immediately rejected by Schwarzschild in Dec. 1915, by Schroedinger, Bauer, Verkov and many others. Finally, UFT88 refuted it completely by showing that it contains no torsion.

The correct identities (1) to (3) are the three Cartan identities:

$$D_\lambda T^\kappa{}_{\nu\rho} + D_\rho T^\kappa{}_{\lambda\nu} + D_\nu T^\kappa{}_{\rho\lambda} := R^\kappa{}_{\lambda\nu\rho} + R^\kappa{}_{\rho\lambda\nu} + R^\kappa{}_{\nu\rho\lambda} \quad -(11)$$

$$D_\lambda T^\kappa{}_{\mu\nu} + D_\nu T^\kappa{}_{\lambda\mu} + D_\mu T^\kappa{}_{\nu\lambda} := R^\kappa{}_{\lambda\mu\nu} + R^\kappa{}_{\nu\lambda\mu} + R^\kappa{}_{\mu\nu\lambda} \quad -(12)$$

$$D_\lambda T^\kappa{}_{\rho\mu} + D_\mu T^\kappa{}_{\lambda\rho} + D_\rho T^\kappa{}_{\mu\lambda} := R^\kappa{}_{\lambda\rho\mu} + R^\kappa{}_{\mu\lambda\rho} + R^\kappa{}_{\rho\mu\lambda} \quad -(13)$$

It follows that eq. (4) is replaced by:

$$\begin{aligned} &D_\mu\left(R^\kappa{}_{\lambda\nu\rho} + R^\kappa{}_{\rho\lambda\nu} + R^\kappa{}_{\nu\rho\lambda}\right) := D_\mu\left(D_\lambda T^\kappa{}_{\nu\rho} + D_\rho T^\kappa{}_{\lambda\nu} + D_\nu T^\kappa{}_{\rho\lambda}\right) \\ &+ D_\rho\left(R^\kappa{}_{\lambda\mu\nu} + R^\kappa{}_{\nu\lambda\mu} + R^\kappa{}_{\mu\nu\lambda}\right) + D_\rho\left(D_\lambda T^\kappa{}_{\mu\nu} + D_\nu T^\kappa{}_{\lambda\mu} + D_\mu T^\kappa{}_{\nu\lambda}\right) \\ &+ D_\nu\left(R^\kappa{}_{\lambda\rho\mu} + R^\kappa{}_{\mu\lambda\rho} + R^\kappa{}_{\rho\mu\lambda}\right) + D_\nu\left(D_\lambda T^\kappa{}_{\rho\mu} + D_\mu T^\kappa{}_{\lambda\rho} + D_\rho T^\kappa{}_{\mu\lambda}\right) \end{aligned} \quad -(14)$$

Rearranging, the following equation is obtained, it extends eq. (5) to include torsion:

4)

$$
\begin{aligned}
&D_\mu R^\kappa{}_{\lambda\nu\rho} + D_\rho R^\kappa{}_{\lambda\mu\nu} + D_\nu R^\kappa{}_{\lambda\rho\mu} := D_\mu D_\lambda T^\kappa{}_{\nu\rho} + D_\rho D_\lambda T^\kappa{}_{\mu\nu} + D_\nu D_\lambda T^\kappa{}_{\rho\mu} \\
&+ D_\mu \left(R^\kappa{}_{\rho\lambda\nu} + R^\kappa{}_{\nu\rho\lambda} \right) \qquad + D_\mu \left(D_\rho T^\kappa{}_{\lambda\nu} + D_\nu T^\kappa{}_{\rho\lambda} \right) \\
&+ D_\rho \left(R^\kappa{}_{\nu\lambda\mu} + R^\kappa{}_{\mu\nu\lambda} \right) \qquad + D_\rho \left(D_\nu T^\kappa{}_{\lambda\mu} + D_\mu T^\kappa{}_{\nu\lambda} \right) \\
&+ D_\nu \left(R^\kappa{}_{\mu\lambda\rho} + R^\kappa{}_{\rho\mu\lambda} \right) \qquad + D_\nu \left(D_\mu T^\kappa{}_{\lambda\rho} + D_\rho T^\kappa{}_{\mu\lambda} \right)
\end{aligned}
\qquad -(15)
$$

Now assume that:

$$
\begin{aligned}
&D_\mu D_\lambda T^\kappa{}_{\nu\rho} + D_\rho D_\lambda T^\kappa{}_{\mu\nu} + D_\nu D_\lambda T^\kappa{}_{\rho\mu} \\
&\quad := D_\mu R^\kappa{}_{\lambda\nu\rho} + D_\rho R^\kappa{}_{\lambda\mu\nu} + D_\nu R^\kappa{}_{\lambda\rho\mu}
\end{aligned}
\qquad -(16)
$$

and add eqs. (15) and (16). We obtain:

$$
\begin{aligned}
&D_\mu D_\lambda T^\kappa{}_{\nu\rho} + D_\rho D_\lambda T^\kappa{}_{\mu\nu} + D_\nu D_\lambda T^\kappa{}_{\rho\mu} \\
&+ D_\mu \left(D_\rho T^\kappa{}_{\lambda\nu} + D_\nu T^\kappa{}_{\rho\lambda} + D_\lambda T^\kappa{}_{\nu\rho} \right) \\
&+ D_\rho \left(D_\nu T^\kappa{}_{\lambda\mu} + D_\mu T^\kappa{}_{\nu\lambda} + D_\lambda T^\kappa{}_{\mu\nu} \right) \\
&+ D_\nu \left(D_\mu T^\kappa{}_{\lambda\rho} + D_\rho T^\kappa{}_{\mu\lambda} + D_\lambda T^\kappa{}_{\rho\mu} \right) \\
&:= D_\mu R^\kappa{}_{\lambda\nu\rho} + D_\rho R^\kappa{}_{\lambda\mu\nu} + D_\nu R^\kappa{}_{\lambda\rho\mu} \\
&\quad + D_\mu \left(R^\kappa{}_{\rho\lambda\nu} + R^\kappa{}_{\nu\rho\lambda} + R^\kappa{}_{\lambda\nu\rho} \right) \\
&\quad + D_\rho \left(R^\kappa{}_{\nu\lambda\mu} + R^\kappa{}_{\mu\nu\lambda} + R^\kappa{}_{\lambda\mu\nu} \right) \\
&\quad + D_\nu \left(R^\kappa{}_{\mu\lambda\rho} + R^\kappa{}_{\rho\mu\lambda} + R^\kappa{}_{\lambda\rho\mu} \right)
\end{aligned}
\qquad -(17)
$$

5) From eqs. (11) to (13) it is seen that eqs. (16) and (17) are self consistent.

Eq. (16) is the correct second Bianchi identity with torsion, eq. (105) of UFT 255. It can never reduce to the Einstein field equation because it must contain an antisymmetric connection. Eq. (16) can be seen as a consequence of a generalized Noether Theorem in which there must appear a quantity akin to torsion.

It gives new ECE field equations of electrodynamics and gravitation.

1) Electrodynamics

$$F^{\kappa}{}_{\nu\rho} = A^{(0)} T^{\kappa}{}_{\nu\rho} \quad -(18)$$

and so on. Therefore:

$$D_\mu D_\lambda F^{\kappa}{}_{\nu\rho} + D_\rho D_\lambda F^{\kappa}{}_{\mu\nu} + D_\nu D_\lambda F^{\kappa}{}_{\rho\mu} \quad -(19)$$
$$= A^{(0)} \left(D_\mu R^{\kappa}{}_{\lambda\nu\rho} + D_\rho R^{\kappa}{}_{\lambda\mu\nu} + D_\nu R^{\kappa}{}_{\lambda\rho\mu} \right)$$

Using the tetrad postulate:

$$D_\mu D_\lambda F^{a}{}_{\nu\rho} + D_\rho D_\lambda F^{a}{}_{\mu\nu} + D_\nu D_\lambda F^{a}{}_{\rho\mu} \quad -(20)$$
$$= A^{(0)} \left(D_\mu R^{a}{}_{\lambda\nu\rho} + D_\rho R^{a}{}_{\lambda\mu\nu} + D_\nu R^{a}{}_{\lambda\rho\mu} \right)$$

2) Gravitation

$$\Phi^{\kappa}{}_{\nu\rho} = \Phi^{(0)} T^{\kappa}{}_{\nu\rho} \quad -(21)$$

So using the tetrad postulate:

$$D_\mu D_\lambda \Phi^{a}{}_{\nu\rho} + D_\rho D_\lambda \Phi^{a}{}_{\mu\nu} + D_\nu D_\lambda \Phi^{a}{}_{\rho\mu}$$
$$= \Phi^{(0)} \left(D_\mu R^{a}{}_{\lambda\nu\rho} + D_\rho R^{a}{}_{\lambda\mu\nu} + D_\nu R^{a}{}_{\lambda\rho\mu} \right)$$

It is seen that these are new cyclic field equations of electrodynamics and gravitation.