313(3): New Second Bianchi Identities from the Jacobi Identity.

Consider the Jacobi identity of covariant derivatives:

$$[D_\rho, [D_\mu, D_\nu]] + [D_\mu, [D_\nu, D_\rho]] + [D_\nu, [D_\rho, D_\mu]] := 0 \quad -(1)$$

(L. H. Ryder, "Quantum Field Theory" p. 121 of the 1996 second edition, Cambridge University Press) and consider the commutator of covariant derivatives acting on a vector V^κ in any space of any dimension. In general:

$$[D_\mu, D_\nu] V^\kappa = R^\kappa{}_{\lambda\mu\nu} V^\lambda - T^\lambda{}_{\mu\nu} D_\lambda V^\kappa \quad -(2)$$

as in many previous UFT papers and S. M. Carroll chapter three.

The 1902 second Bianchi identity is obtained by omitting torsion erroneously.

For the sake of argument only it is shown how the 1902 second Bianchi identity is obtained by assuming that eq. (2) is:

2)

$$[D_\mu, D_\nu] V^\kappa = R^\kappa{}_{\lambda\mu\nu} V^\lambda \quad -(3)$$

Then:

$$[D_\rho, [D_\mu, D_\nu] V^\kappa] = [D_\rho, R^\kappa{}_{\mu\nu\lambda} V^\lambda] \quad -(4)$$

This commutator is worked out as follows:

$$[D_\rho, R^\kappa{}_{\mu\nu\lambda} V^\lambda] = (D_\rho R^\kappa{}_{\lambda\mu\nu}) V^\lambda + R^\kappa{}_{\lambda\mu\nu} D_\rho V^\lambda - R^\kappa{}_{\lambda\mu\nu} V^\lambda D_\rho \quad -(5)$$

$$= (D_\rho R^\kappa{}_{\lambda\mu\nu}) V^\lambda$$

Using this method in eq. (1) it follows that:

$$D_\rho R^\kappa{}_{\lambda\mu\nu} + D_\nu R^\kappa{}_{\lambda\rho\mu} + D_\mu R^\kappa{}_{\lambda\nu\rho} = 0 \quad -(6)$$

This is the original 1902 second Bianchi identity, QED.

Now use the correct eq. (2), with non zero torsion and antisymmetric connection.

Eq. (4) becomes:

3)

$$[D_\rho, [D_\mu, D_\nu]] = (D_\rho R^{\kappa}{}_{\lambda\mu\nu}) V^\lambda \quad -(7)$$
$$- [D_\rho, T^\lambda{}_{\mu\nu} D_\lambda V^\kappa]$$

The new commutator is:

$$[D_\rho, T^\lambda{}_{\mu\nu} D_\lambda V^\kappa] = D_\rho (T^\lambda{}_{\mu\nu} D_\lambda V^\kappa) \quad -(8)$$
$$- T^\lambda{}_{\mu\nu} D_\lambda V^\kappa D_\rho$$
$$= (D_\rho T^\lambda{}_{\mu\nu})(D_\lambda V^\kappa)$$

It follows that the 1902 Bianchi identity is changed to:

$$(D_\rho R^\kappa{}_{\lambda\mu\nu} + D_\nu R^\kappa{}_{\lambda\rho\mu} + D_\mu R^\kappa{}_{\lambda\nu\rho}) V^\lambda \quad -(9)$$
$$:= (D_\rho T^\lambda{}_{\mu\nu} + D_\nu T^\lambda{}_{\rho\mu} + D_\mu T^\lambda{}_{\nu\rho}) D_\lambda V^\kappa$$

<u>and</u> <u>eq. (6)</u> is <u>no longer</u> <u>true</u>.

From the previous note and eq. (105) of 4FT255 the following result is obtained:

4)

$$\left(D_\rho R^\kappa{}_{\lambda\mu\nu} + D_\nu R^\kappa{}_{\lambda\rho\mu} + D_\mu R^\kappa{}_{\lambda\nu\rho}\right) V^\lambda$$

$$:= \left(D_\rho D_\lambda T^\kappa_{\mu\nu} + D_\nu D_\lambda T^\kappa_{\rho\mu} + D_\mu D_\lambda T^\kappa_{\nu\rho}\right) V^\lambda$$

$$:= \left(D_\rho T^\lambda_{\mu\nu} + D_\nu T^\lambda_{\rho\mu} + D_\mu T^\lambda_{\nu\rho}\right) D_\lambda V^\kappa \quad -(10)$$

There is also a completely new relation between the first and second Bianchi identities:

$$\left(R^\lambda{}_{\rho\mu\nu} + R^\lambda{}_{\nu\rho\mu} + R^\lambda{}_{\mu\nu\rho}\right) D_\lambda V^\kappa \quad -(11)$$

$$:= \left(D_\rho R^\kappa{}_{\lambda\mu\nu} + D_\nu R^\kappa{}_{\lambda\rho\mu} + D_\mu R^\kappa{}_{\lambda\nu\rho}\right) V^\lambda$$

Finally:

$$\left(D_\rho T^\lambda_{\mu\nu} + D_\nu T^\lambda_{\rho\mu} + D_\mu T^\lambda_{\nu\rho}\right) D_\lambda V^\kappa$$

$$:= \left(D_\rho D_\lambda T^\kappa_{\mu\nu} + D_\nu D_\lambda T^\kappa_{\rho\mu} + D_\mu D_\lambda T^\kappa_{\nu\rho}\right) V^\lambda \quad -(12)$$

The torsion tensor is defined by:

$$T^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu} \quad -(13)$$

5) It is clear from eqs. (2) and (13) that:

$$[D_\mu, D_\nu] V^\kappa = -(\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}) D_\lambda V^\kappa + R^\kappa{}_{\lambda\mu\nu} V^\lambda \quad -(14)$$

and it follows that the commutator and the connection have the same indices μ and ν and are both antisymmetric:

$$[D_\mu, D_\nu] = -[D_\nu, D_\mu] \quad -(15)$$

and

$$\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu} \quad -(16)$$

The Christoffel connection used by Ricci in 1902 was symmetric:

$$\Gamma^\lambda_{\mu\nu} = ?\ \Gamma^\lambda_{\nu\mu} \quad -(17)$$

and neither Ricci nor Einstein knew about eq. (14). The entire twentieth century general relativity of the Einstein type is incorrect because eq. (17) means:

$$[D_\mu, D_\nu] = 0 \quad -(18)$$

so curvature vanishes with torsion.

6)

In this note several new identities have been discovered by considering the Jacobi identity (1) and the correct eq. (2).

This is part of the great post Einstein paradigm shift - ECE theory

Einstein based his entire field equation development on eq. (6), and used the incorrect Christoffel connection (17). He expressed eq. (6) as:

$$D^\mu G_{\mu\nu} = 0 \quad -(19)$$

where $G_{\mu\nu}$, the Einstein tensor, can be derived if and only if the connection is symmetric. Here

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \quad -(20)$$

where $R_{\mu\nu}$ is the Ricci tensor and R the Ricci scalar. The metric is $g_{\mu\nu}$. This entire development is incorrect and is discarded entirely in ECE theory.

The latter is a far simpler theory

7) based directly on Cartan geometry:

$$T = D \wedge q \quad -(21)$$

$$R = D \wedge \omega \quad -(22)$$

$$D \wedge T := R \wedge q \quad -(23)$$

Eqs. (21) and (22) are the first and second Maurer Cartan structure equations. Effectively the structure equations are defined by eq. (2). It follows immediately that the connection in Cartan geometry is always antisymmetric.

The field equations of ECE are given by the Cartan identity (23) and the Evans identity:

$$D \wedge \tilde{T} := \tilde{R} \wedge q \quad -(24)$$

in four dimensions.