309(1): Scattering Theory and Generalized Beer Lambert Law

In scattering theory the generalized Beer Lambert law is:

$$\frac{dI}{dx} = -Q I \quad -(1)$$

where I is a flux of particles and x the sample length.

Here:

$$Q = \frac{1}{\lambda} = n\sigma = \rho / \tau \quad -(2)$$

where λ is the mean free path, n the number of targets per unit volume, σ the area cross section, ρ the mass density of the target and τ the density / mean free path. From eq. (1):

$$\frac{I}{I_0} = \exp(-Q\Delta x) \quad -(3)$$

For example in conventional Rayleigh scattering:

$$\frac{I}{I_0} = \frac{8\pi^4 \alpha^2}{\lambda^4 R^2}(1+\cos^2\theta) \quad -(4)$$

where α is the polarizability and λ the wavelength, and R and θ are well defined geometrical parameters.

In conventional Compton scattering:

$$\lambda - \lambda_0 = \frac{h}{mc}(1-\cos\theta) \quad -(5)$$

where

$$\lambda f = \frac{\lambda\omega}{2\pi} = c, \quad \lambda = \frac{2\pi c}{\omega} \quad -(6)$$

So:

2)

$$\frac{1}{\omega} - \frac{1}{\omega_0} = \frac{\hbar}{mc^2}\left(1 - \cos\theta\right) \quad -(7)$$

i.e.

$$\frac{\omega}{\omega_0} = 1 - \frac{\hbar\omega}{mc^2}\left(1 - \cos\theta\right) \quad -(8)$$

The intensity of the incoming γ ray radiation of the Compton effect can be defined either as watts per square metre or joules per square metre. Both definitions are found in the literature. The latter definition uses:

$$I = \frac{cE}{V} \quad -(9)$$

where E is the energy of the electromagnetic radiation in a volume of radiation V.

Using the Planck theory the mean energy of an oscillator is:

$$\langle \hbar\omega \rangle = \frac{\hbar\omega}{e^y - 1} \quad -(10)$$

where

$$y = \frac{\hbar\omega}{kT}. \quad -(11)$$

Here k is the Boltzmann constant and T is the temperature.

In Compton scattering the incoming γ ray radiation is scattered from a stationary electron of rest energy mc^2. By conservation of energy:

3)

$$\hbar\omega + mc^2 = \hbar\omega' + \gamma mc^2 \quad -(12)$$

and by conservation of momentum:

$$\hbar\underline{\kappa} = \hbar\underline{\kappa}' + \gamma m\underline{v} \quad -(13)$$

Here γ is the Lorentz factor of the electron:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(14)$$

and m is the mass of the electron. The velocity of the electron after collision is $\underline{v}$. The initial wavenumber of the photon is $\underline{\kappa}$ and the final wavenumber is $\underline{\kappa}'$. For the electron the de Broglie / Einstein equations are used:

$$\hbar\omega = \gamma mc^2 \quad -(15)$$

$$\hbar\underline{\kappa} = \gamma m\underline{v} \quad -(16)$$

In this conventional theory the obsolete dogma of massless photon is used. The corrected theory with finite photon mass is given in UFT158 ff.

Therefore in this theory it is considered that

$$\langle \hbar\omega \rangle = \hbar\omega \quad -(17)$$

so from eq. (10):

$$e^{y} = 2 \quad -(18)$$

and

$$\hbar\omega = kT \log_e 2 \quad -(19)$$

4) If it is assumed that the volume V occupied by the energy $\hbar\omega$ is the same as the volume V occupied by $\hbar\omega'$ then:

$$\frac{I}{I_0} = \frac{\omega}{\omega_0} \quad -(20)$$

and from eqs. (8) and (20):

$$\frac{I}{I_0} = 1 - \frac{\hbar\omega}{mc^2}(1-\cos\theta) \quad -(21)$$

In this type of scattering it is appropriate to consider:

$$\frac{I}{I_0} = \exp(-\eta\sigma\Delta x) - (22)$$

as defined in eq. (2). The target is one electron of area cross section σ, so:

$$\eta = \frac{1}{V_e} \quad -(23)$$

where V_e is the volume occupied by the one electron. Therefore:

$$\frac{I}{I_0} = \exp\left(-\frac{\sigma\Delta x}{V_e}\right) := \exp(-\zeta) \quad -(24)$$

where

$$\zeta = \frac{\sigma\Delta x}{V_e} \quad -(25)$$

5) is a ratio of volumes ... So:

$$\frac{I}{I_0} = \frac{\omega}{\omega_0} = \exp\left(-\xi\right) \doteq 1 - \frac{\hbar\omega}{mc^2}\left(1-\cos\theta\right) \quad -(26)$$

If it is assumed that:

$$\xi << 1 \quad -(27)$$

then:

$$e^{-\xi} \sim 1 - \xi \quad -(28)$$

so

$$\xi = \frac{\hbar\omega}{mc^2}\left(1-\cos\theta\right) \quad -(29)$$

So Compton scattering is an Evans / Morris scattering with initial frequency ω_0 and final frequency ω. Evans / Morris scattering is the most general type of scattering.