309(4): The Condition for Rayleigh Scattering

Consider eq. (27) of the previous note 309(3):

$$\exp\left(-\frac{\hbar(\omega_s-\omega_0)}{kT}\right) = \left(\frac{\mu \sin\phi\, \omega_0^2}{16\pi c^2 \epsilon_0 R E_0}\right)^2 \quad -(1)$$

In general

$$\omega_s \neq \omega_0 \quad -(2)$$

and this represents an Evans / Morris shift in light scattering. For Rayleigh scattering:

$$\omega_s = \omega_0 \quad -(3)$$

so:

$$\mu \sin\phi\, \omega_0^2 = 16\pi c^2 \epsilon_0 R E_0 \quad -(4)$$

where

$$\mu = \alpha E_0 \quad -(5)$$

So the condition for Rayleigh scattering is:

$$\boxed{\omega_0^2 = \omega_s^2 = \frac{16\pi c^2 \epsilon_0 R}{\alpha \sin\phi}} \quad -(6)$$

Units Check

$$\alpha = J^{-1} C^2 m^2$$
$$\epsilon_0 = J^{-1} C^2 m^{-1}$$

so:

$$\omega_0^2 = \frac{m^2 s^{-2} J^{-1} C^2 m^{-1} m}{J^{-1} C^2 m^2} = s^{-2}$$

Therefore in Rayleigh scattering:

$$\boxed{\Phi_0 = \Phi_s} \quad -(7)$$

2) The initial flux density is given by:

$$\Phi_0 = \frac{1}{2} c \epsilon_0 E_0^2$$

$$= \frac{1}{3} \frac{\hbar \omega_0^4}{c^3 \pi^2} \left(\frac{1}{e^{y_0} - 1} \right) \quad - (8)$$

where

$$y_0 = \frac{\hbar \omega_0}{kT} \quad - (9)$$

so

$$\boxed{E_0^2 = \frac{2}{3} \frac{\hbar \omega_0^4}{\epsilon_0 c^4 \pi^2} \left(\frac{1}{e^{y_0} - 1} \right)} \quad - (10)$$

In general however:

$$\omega_S \neq \omega_0 \quad - (11)$$

so:

$$\Phi_S \neq \Phi_0 \quad - (12)$$

and in general, the Evans / Morris frequency shift for Rayleigh scattering is:

$$\omega_0 - \omega_S = \frac{kT}{\hbar} \log_e \left(\frac{\mu \sin\phi \, \omega_0^2}{16 \pi c^2 \epsilon_0 R E_0} \right)^2 \quad - (13)$$

i.e.:

$$\boxed{\omega_0 - \omega_S = \frac{kT}{\hbar} \log_e \left(\left(\frac{\omega_0^2 \alpha \sin\phi}{16 \pi c^2 \epsilon_0 R} \right)^2 \right)} \quad - (14)$$

For Stokes scattering:

3)

$$\omega_0 - \omega_S = \omega \quad (15)$$

so the transition frequency is:

$$\omega = \frac{kT}{\hbar} \log_e \left(\left(\frac{\omega_0^2 \alpha \sin\phi}{16\pi c^2 \epsilon_0 R} \right)^2 \right) \quad (16)$$

This appears as a Stokes Raman line. In general however ω_S need not be the frequency of a Raman line. Therefore new types of shift can be looked for in this way.