309(2): A New Type of Inelastic Light Scattering from Planck Theory and Rayleigh Scattering.

Consider the energy flux density Φ in the Rayleigh theory of the density of states. It is defined by:

$$\Phi = c \frac{U}{V} \quad -(1)$$

where U is the energy of the electromagnetic field and V is the volume occupied by the radiation. Here c is the speed of light in a vacuum. The number of oscillators in a unit volume of the Rayleigh theory is:

$$\frac{N}{V} = \frac{1}{6} \frac{\omega^3}{c^3 \pi^2} . \quad -(2)$$

For two states of polarization:

$$\frac{N}{V} = \frac{1}{3} \frac{\omega^3}{c^3 \pi^2} . \quad -(3)$$

Note that the $\underline{B}^{(3)}$ field adds another contribution to eq. (3). However for the sake of argument we use eq. (3). The above theory is for a monochromatic e/m field of one angular frequency ω. For a polychromatic e/m field:

$$\frac{1}{V} dN = \frac{1}{V} \left(N(\omega + d\omega) - N(\omega) \right) \quad -(4)$$

as in immediately preceding papers.

2) Each oscillator has a mean energy:

$$\langle \hbar\omega \rangle = \frac{\hbar\omega}{e^{y}-1} \quad -(5)$$

where

$$y = \frac{\hbar\omega}{kT} \quad -(6)$$

and where k is the Boltzmann constant at a temperature T. Here $\hbar$ is the reduced Planck constant.

Consider a process of any kind in which there is an initial energy flux density Φ_0 of the e/m field and a final flux density Φ. In this theory the e/m field is monochromatic. It follows that:

$$\frac{\Phi}{\Phi_0} = \left(\frac{\omega}{\omega_0}\right)^4 \left(\frac{e^{y_0}-1}{e^{y}-1}\right) \quad -(7)$$

and this is true regardless of the number of polarizations.

For visible frequencies:

$$y_0 \gg 1, \; y \gg 1 \quad -(8)$$

so

$$\frac{\Phi}{\Phi_0} = \left(\frac{\omega}{\omega_0}\right)^4 \exp\left(-(y-y_0)\right) \quad -(9)$$

where

$$y = \frac{\hbar\omega}{kT}, \quad y_0 = \frac{\hbar\omega_0}{kT} \quad -(10)$$

Therefore the process is described by the <u>generalized</u>

3)

Beer Lambert law :

$$\boxed{\frac{\Phi}{\Phi_0} = \left(\frac{\omega}{\omega_0}\right)^4 \exp\left(-\frac{\hbar(\omega-\omega_0)}{kT}\right)} \quad -(11)$$

This is the direct result of the fundamental Planck theory.

The usual theory of Rayleigh scattering starts from the energy flux density generated at a distance R from an oscillating electric dipole moment :

$$\Phi = \frac{\pi^2 c p^2 \sin^2\phi}{2\epsilon_0 \lambda^4 R^2} \quad -(12)$$

$$= \left(\frac{p^2 \sin^2\phi}{32\pi^2 c^3 \epsilon_0 R^2}\right)\omega^4$$

where $\lambda = 2\pi c / \omega$. — (13)

Here ϵ_0 is the vacuum permittivity, and ϕ a well defined angle of observation. The electric dipole moment p is induced by an incident electric field strength E_0 of the e/m field incident on an object such as a spherical particle, atom or molecule. The incident energy flux density is :

$$\Phi_0 = \frac{1}{2}\epsilon_0 c E_0^2 = c\frac{E_n}{V} \quad -(14)$$

4) where:

$$\frac{E_n}{V} = \frac{1}{2}\epsilon_0 E_0^2 \quad -(15)$$

It is assumed that the induced electric dipole moment is:

$$\underline{p} = \alpha \underline{E_0} \quad -(16)$$

where α is the polarizability. In general α is a tensor but for a spherical object or atom or molecule of certain symmetry it is a scalar.

So:

$$\Phi = \left(\frac{\alpha \sin\phi\, \omega^2}{4\pi\epsilon_0 c^2 R}\right)^2 \Phi_0 \quad -(17)$$

and

$$\frac{\Phi}{\Phi_0} = \left(\frac{\alpha \sin\phi\, \omega^2}{4\pi\epsilon_0 c^2 R}\right)^2 \quad -(18)$$

From eqs. (11) and (18):

$$\left(\frac{\omega}{\omega_0}\right)^4 \exp\left(\frac{\hbar(\omega_0-\omega)}{kT}\right) = \left(\frac{\alpha(\sin\phi)\omega^2}{4\pi\epsilon_0 c^2 R}\right)^2 \quad -(19)$$

i.e.

$$\exp\left(\frac{\hbar(\omega_0-\omega)}{kT}\right) = \left(\frac{\alpha(\sin\phi)\omega_0^2}{4\pi\epsilon_0 c^2 R}\right)^2 \quad -(20)$$

Therefore an entirely new result is obtained:

5)

$$\boxed{\omega_0 - \omega = \frac{kT}{\hbar} \log_e \left[\left(\frac{\alpha \omega_0^2 \sin\phi}{4\pi \epsilon_0 c^2 R} \right)^2 \right]} \quad -(21)$$

In the usual theory of Rayleigh scattering only eq. (18) is used, and the existence of the frequency shift $\omega_0 - \omega$ is missed completely.

Eq. (21) represents an inelastic Rayleigh scattering from spherical objects. This is not Raman scattering because the latter requires anisotropy of polarizability. Eq. (21) is a kind of Evans / Morris effect because there is the same type of frequency shift as in absorption theory.