308(1): Computation of Frequency Shifts for the Red Line of Atomic H

It would be very useful to compute the exact frequency shifts of the red line of atomic H from:

$$\omega = \omega_0 \exp\left(-\frac{AZ}{3}\right) \quad -(1)$$

where:

$$\bar{\nu}_0 = 15,241.4 \text{ cm}^{-1} \quad -(2)$$

and

$$A = \left(\frac{N}{V}\right) \frac{|\mu_{ij}|^2}{6\epsilon_0 c\hbar} \quad -(3)$$

A probe laser is tuned to $15,241.4 \text{ cm}^{-1}$ and is aligned according to eq. (1). The red line is at $n' = 2$ to $n = 3$, and for linear or unpolarized probe radiation the selection rules are:

$$\Delta l = \pm 1, \quad \Delta n = 0 \quad -(4)$$

So there are three transition dipole moments and the initial frequency ω_0 is split into ω_1, ω_2 and ω_3:

$$\omega_1 = \omega_0 \exp\left(-A(2s \to 3p) Z/3\right) \quad -(5)$$

$$\omega_2 = \omega_0 \exp\left(-A(2p \to 3s) Z/3\right) \quad -(6)$$

$$\omega_3 = \omega_0 \exp\left(-A(2p \to 3d) Z/3\right) \quad -(7)$$

The three transition dipole moments are:

$$\mu(2s \to 3p) = \int \psi_{2s}^* \mu_z \psi_{3p} \, d\tau \quad -(8)$$

$$\mu(2p \to 3s) = \int \psi_{2p}^{*} \mu_z \psi_{3s} \, d\tau \quad -(9)$$

$$\mu(2p \to 3d) = \int \psi^{*}_{2p} \mu_z \psi_{3d} \, d\tau \quad -(10)$$

where $$\mu_z = er\cos\theta \quad -(11)$$

Here:
$$\psi_{2s}^{*} = R_{20}^{*}(r) Y_{00}^{*}(\theta, \phi) \quad -(12)$$
$$\psi_{3p} = R_{31}(r) Y_{10}(\theta, \phi) \quad -(13)$$
$$\psi_{2p}^{*} = R_{21}^{*}(r) Y_{10}^{*}(\theta, \phi) \quad -(14)$$
$$\psi_{3s} = R_{30}(r) Y_{00}(\theta, \phi) \quad -(15)$$
$$\psi^{*}_{3d} = R_{32}(r) Y_{20}(\theta, \phi) \quad -(16)$$

and $$\int \psi^{*} \mu_z \psi \, d\tau = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \int_{r=0}^{\infty} \psi^{*} er^{3} \cos\theta \sin\theta \, \psi \, dr \, d\theta \, d\phi \quad -(17)$$

This computation was carried out in UFT 300 and UFT 304 and it would be very useful to find ω_1, ω_2 and ω_3 for various values of the concentration N/V and path length Z.

The experiment consists of chosing a sample cell of length Z filled with H gas

3) at concentration N/V, where N is the number of H atoms and V the volume of the sample cell. The radiation emerging from the cell at a distance Z, the length of the cell, should be split into the three frequencies ω_1, ω_2 and ω_3. These frequencies can be calculated precisely from the wavefunction of H, the concentration N/V, and the path length Z.

It is important to know the frequencies ω_1, ω_2 and ω_3 precisely from the theory. They depend on the concentration of the H gas and the square of the transition dipole moment. The frequencies can be converted into wavenumber using

$$1\ cm^{-1} = 29.97925\ GHz \quad -(18)$$

Different sites give slightly different values for the red line of H. One site gives:

$$\lambda(\alpha H) = 656.281\ nm \quad -(19)$$

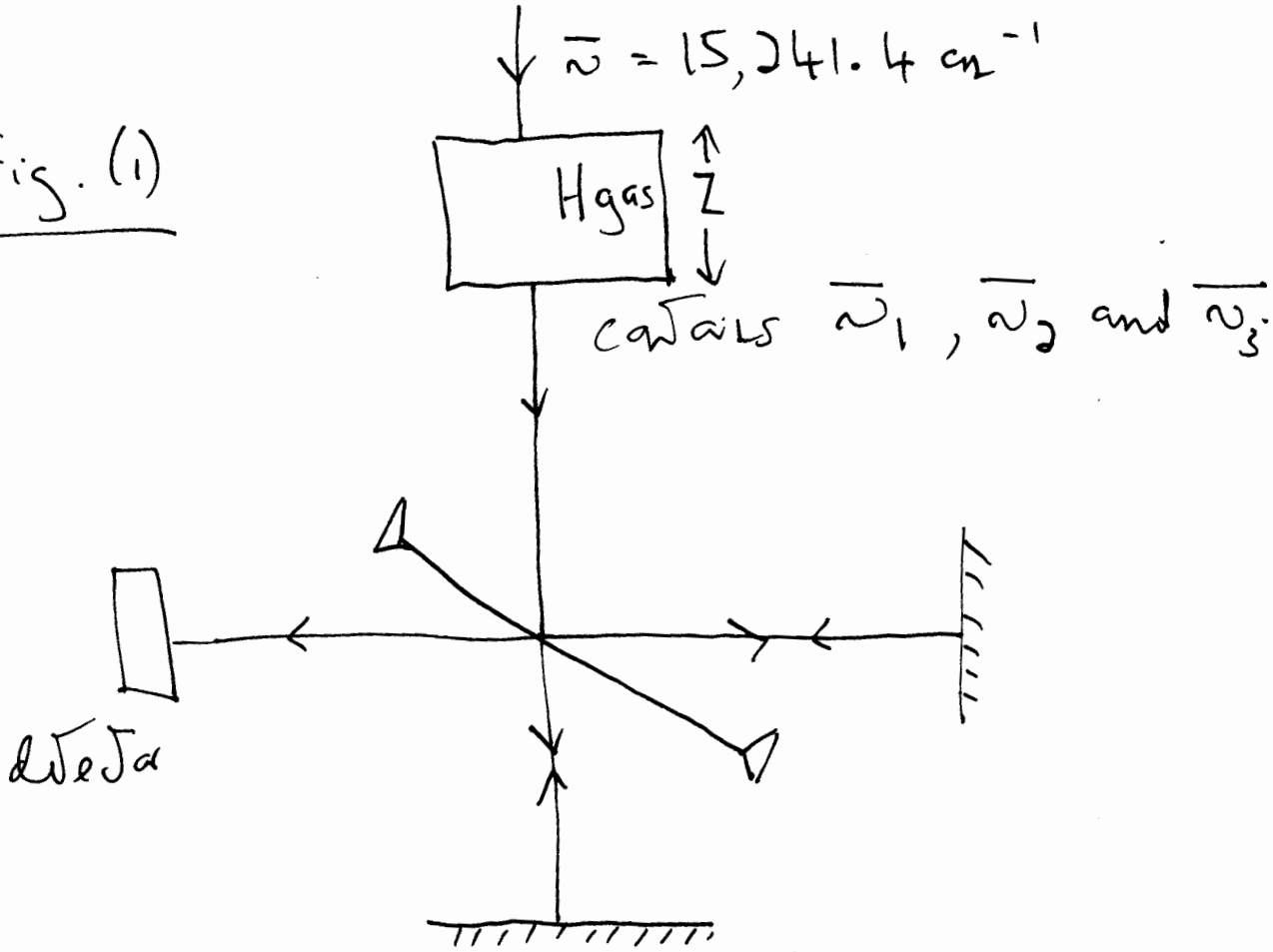
The wavenumber in cm^{-1} is:

$$\bar{\nu} = \frac{0.01}{\lambda}\ cm^{-1} \quad -(20)$$

$$= 15,237.3\ cm^{-1}$$

4) The quantity that is actually measured in the visible range is wavelength or wavenumber. Recall that three different wavenumbers should emerge from the sample. These three frequencies can be used as the input beam to a Michelson interferometer:

Fig. (1)



The interferogram is Fourier transformed to give a spectrum that should contain three peaks of $\bar{\nu}_1$, $\bar{\nu}_2$ and $\bar{\nu}_3$.