305(2): Analytical Solution of the Schroedinger Equation with a Gravitational Potential.

The Schroedinger equation is:

$$\left(-\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2}{4\pi\epsilon_0 r} - V_1\right)\psi = E\psi \quad -(1)$$

where

$$V_1 = -\frac{m_2 M G}{R} \quad -(2)$$

in the notation of the previous note.

Write:

$$\frac{e^2}{4\pi\epsilon_0 r_1} = \frac{e^2}{4\pi\epsilon_0 r} + V_1 \quad -(3).$$

so:

$$\boxed{\frac{1}{r_1} = \frac{1}{r} + \frac{4\pi\epsilon_0}{e^2}V_1} \quad -(4)$$

so the Schroedinger equation is:

$$\left(-\frac{\hbar^2}{2m}\nabla^2 - \frac{e^2}{4\pi\epsilon_0 r_1}\right)\psi = E\psi \quad -(5)$$

The solution of eq. (5) is the same as that of the usual Schroedinger equation with r replaced by r_1, i.e.

a)

$$\psi(r_1, \theta, \phi) = R(r_1)\,Y(\theta, \phi) \quad - (\cancel{1}6)$$

where :

$$R_{nl}(r_1) = \left(\frac{2Z_1}{na}\right)\left[\frac{(n-l-1)!}{2n[(n+l)!]^3}\right]\rho^{l} L_{n+l}^{2l+1}(\rho)\, e^{-\rho/2} \quad - (\cancel{1}7)$$

where :

$$\rho = \left(\frac{2Z_1}{na}\right) r_1 , \quad - (\cancel{1}8)$$

$$a = \frac{4\pi\epsilon_0 \hbar^2}{me^2} . \quad - (\cancel{1}9)$$

$$\frac{1}{r_1} = \frac{1}{r} + \frac{4\pi\epsilon_0}{e^2} V_1 \quad - (\overset{10}{\cancel{20}}) .$$

$$V_1 = \frac{m_2 M G}{R} \quad - (\overset{11}{\cancel{22}})$$

The solution of eq. (1) is eqs. $(\overset{6}{\cancel{16}})$ to $(\overset{11}{\cancel{22}})$.

The expectation values of energy are

$$E = \int \psi^* \hat{H} \psi \, d\tau \quad - (12)$$

and depend on gravitation. They must be worked out with r replaced by r_1