304(5): High Frequency Approximation and Balmer Series.

In the notation of note 304(4), the Evans Morris shifts are described by:

$$\left(\frac{\omega}{\omega_0}\right)^3 \left(\frac{e^{y_0}-1}{e^{y}-1}\right) = \exp\left(-A\ell\right) \quad -(1)$$

where $$A = \left(\frac{N}{V}\right) \frac{|\mu_{gi}|^2}{6\epsilon_0 v \hbar} \quad -(2)$$

In the visible range of the Balmer line:

$$\hbar\omega_0 \gg kT, \quad \hbar\omega \gg kT \quad -(3)$$

so $$\frac{e^{y_0}-1}{e^{y}-1} \to \exp\left(y_0 - y\right)$$

$$= \exp\left(\frac{\hbar(\omega_0-\omega)}{kT}\right) \quad -(4)$$

so:

$$\left(\frac{\omega}{\omega_0}\right)^3 = \exp\left(-\left(A\ell + \frac{kT}{\hbar(\omega_0-\omega)}\right)\right)$$

$$= \exp\left(-A\ell\right) \quad -(5)$$

to an excellent approximation. So:

$$\boxed{\omega = \omega_0 \exp\left(-\frac{A\ell}{3}\right)} \quad -(6)$$

2) The integrated power absorption coefficient A can be measured experimentally or computed from the wave function of the H atom.

Eq. (6) is easy to use and shows that any absorption is accompanied by an even Mössbauer frequency shift.

The Balmer series of atomic H is described by:

$n'=2$ to $n=$	at
3	656.3 nm are line
4	486.1
5	434.0
6	410.2
7	397.0
∞	365.0

The complete set of quantum numbers are:

$n'=2,\ \ell=1,\ m_\ell=-1,0,1\ ,\ \ell=0$

$n=3,\ \ell=2,\ m_\ell=-2,\dots,2$

$\ell=1\ \ m_\ell=-1,\dots 1\ ,\ \ell=0$

$n=4,\ \ell=3\ \ m_\ell=-3,\dots,3$

$2\ \ m_\ell=-2,\dots 2$

$1\ \ m_\ell=-1,\dots 1,\ \ell=0$

$n=5,\ \ell=4\ \ m_\ell=-4,\dots 4$

$3\ \ m_\ell=-3,\dots,3$

$2\ \ m_\ell=-2,\dots,2$

$1\ \ m_\ell=-1,\dots,1,\ \ell=0$

and so on.

) The selection rules are:

$$\Delta l = \pm 1 \quad -(7)$$

i.e. the Laporte selection rule, and

$$\Delta m_l = 0, \pm 1 \quad -(8)$$

depending on polarization. For linearly polarized light:

$$\Delta m_l = 0 \quad -(9)$$

For left circularly polarized light:

$$\Delta m_l = 1 \quad -(10)$$

For right circularly polarized light:

$$\Delta m_l = -1 \quad -(11)$$

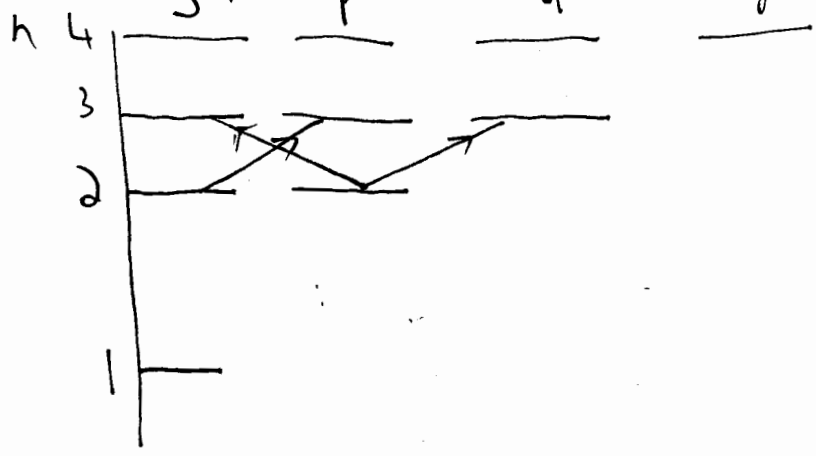
The energy levels of H are:

$$E_n = -\left(\frac{m e^4}{32\pi^2 \epsilon_0^2 \hbar^2}\right)\frac{1}{n^2}, \quad -(12)$$

$$n = 1, 2, \ldots \quad -(13)$$

and these values are independent of l and m_l.

Fig(1)



4) The first few levels are sketched in Fig. (1).
The 2s state is $n=2, \ell=0, m=0$.
The 2p state is $n=2, \ell=1, m=-1, 0, 1$.
The 3s state is $n=3, \ell=0, m=0$
The 3p state is $n=3, \ell=1, m=-1, 0, -1$
The 3d state is $n=3, \ell=2, m=-2, -1, 0, 1, 2$

The energy levels of the 2p and 2s states are the same, so it is a degenerate state. There are four states in all, so it is four fold degenerate. Similarly there are nine states in all of the $n=3$ level, which is nine fold degenerate.

Fig (1) shows the possible transitions between $n'=2$ and $n=3$ states at 656.3 nm:

a) $2s \rightarrow 3p$ $(\Delta\ell = 1, \Delta m = 0)$
b) $2p \rightarrow 3s$ $(\Delta\ell = -1, \Delta m = 0)$
c) $2p \rightarrow 3d$ $(\Delta\ell = 1, \Delta m = 0)$

The three transitions all occur at the same wavelength of 656.3 nm and appear as one line, first observed in the late nineteenth century.

In a relativistic treatment with the ECE femia equation, fine structure occurs. The above theory is non relativistic.