304(7): Solution for $V$ in terms of $\alpha$

The relevant equations are:

$$\frac{V}{c} = \frac{n'}{n'^2 + n''^2} \quad -(1)$$

where

$$n' = \frac{\omega \epsilon''}{\alpha c}, \quad n'' = \frac{\alpha c}{2\omega}. \quad -(2)$$

So:

$$V = \frac{\omega \epsilon''}{\alpha} \left[ \frac{1}{\left(\frac{\omega \epsilon''}{\alpha c}\right)^2 + \left(\frac{\alpha c}{2\omega}\right)^2} \right] \quad -(3)$$

$$= \frac{a \epsilon''}{b \epsilon''^2 + c'} \quad -(4)$$

where

$$a = \frac{\omega}{\alpha}, \quad b = \frac{\omega^2}{\alpha^2 c^2}, \quad c' = \left(\frac{\alpha c}{2\omega}\right)^2$$

So

$$V(b\epsilon''^2 + c') = a\epsilon'' \quad -(5)$$

i.e.

$$\epsilon''^2 - \left(\frac{a}{Vb}\right)\epsilon'' + \frac{c'}{b} = 0 \quad -(6)$$

$$\epsilon''^2 - \frac{\alpha}{\omega}\frac{c^2}{V}\epsilon'' + \frac{1}{4} = 0 \quad -(7)$$

Denote

$$A_1 = \frac{\alpha c^2}{\omega V} \quad -(8)$$

2) then:

$$\epsilon''^2 - A_1\epsilon'' + \frac{1}{4} = 0 \quad -(9)$$

and

$$\epsilon'' = \frac{1}{2}\left(A_1 \pm \left(A_1^2 - 1\right)^{1/2}\right) \quad -(10)$$

i.e

$$\epsilon'' = \frac{1}{2}\left(\frac{\alpha c^2}{\omega v} \pm \left(\left(\frac{\alpha c^2}{\omega v}\right)^2 - 1\right)^{1/2}\right) \quad -(11)$$

So

$$n' = \frac{\omega\epsilon''}{\alpha c}$$

$$n' = \frac{1}{2}\left(\frac{c}{v} \pm \frac{c}{v}\left(1 - \left(\frac{\omega v}{\alpha c^2}\right)^2\right)^{1/2}\right) \quad -(12)$$

and

$$n'' = \frac{\alpha c}{2\omega} \quad -(13)$$

Using eqs. (1), (12) and (13), the velocity $v$ can be expressed in terms of:

$$\alpha = \left(\frac{N}{V}\right)\frac{|\mu_{gi}|^2}{6\epsilon_0 v \hbar} \quad -(14)$$

$$:= \frac{d}{v}$$

where

$$d = \left(\frac{N}{V}\right)\frac{|\mu_{gi}|^2}{6\epsilon_0 \hbar} \quad -(15)$$

Therefore:

$$\frac{v}{c} = \frac{n'}{n'^2 + \frac{cd}{2v\omega}} \quad -(16)$$

and:
$$n' = \frac{1}{2}\frac{c}{v}\left(1 \pm \left(1 - \frac{\omega v^2}{dc^2}\right)^{1/2}\right) \quad -(17)$$

Therefore v may be found in terms of d. This is a general formula for any type of absorption.

Having found v, the photon mass is calculated with:

$$\hbar\omega = \gamma mc^2 \quad -(18)$$

so

$$m = \frac{\hbar\omega}{\gamma c^2} = \left(1 - \frac{v^2}{c^2}\right)^{1/2} \frac{\hbar\omega}{c^2} \quad -(19)$$

Therefore photon mass accompanies any absorption process.

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