290 (2): Number of Photons per Unit Volume of the Refracted and Reflected Beams: Monochromatic Radiation

Using the derivation by Rayleigh and Jeans the number of radiation oscillators N in a volume V of radiation is:

$$\frac{N}{V} = \frac{1}{6\pi^2}\left(\frac{\omega}{c}\right)^3 \quad -(1)$$

the average energy of each oscillator in the Planck theory is:

$$\langle E \rangle = \left(\frac{x}{1-x}\right)\hbar\omega \quad -(2)$$

where

$$x = \exp\left(-\frac{\hbar\omega}{kT}\right) \quad -(3)$$

Here ω is the angular frequency of the oscillator, k is Boltzmann's constant, T the temperature and $\hbar$ the reduced Planck constant, c being the vacuum speed of light. For N oscillators the total energy per unit volume is:

$$\frac{E}{V} = \frac{N\langle E \rangle}{V} \quad -(4)$$

$$= \frac{\hbar}{6\pi^2 c^3}\omega^4\left(\frac{x}{1-x}\right)$$

i.e.

$$\boxed{\frac{E}{V} = \frac{\hbar\omega^4}{6\pi^2 c^3}\left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right)^{-1}} \quad -(5)$$

the intensity of the beam is:

$$I = c\left(\frac{E}{V}\right) \quad -(6)$$

a) Therefore:

$$I = \frac{\hbar\omega^4}{6\pi^2 c^2}\left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right)^{-1} \quad -(7)$$

in watts per square metre, or Joules second per square metre.

Eq. (5) is the equation for the total energy per cubic ($J m^{-3}$) of N oscillators. Each oscillator is defined by:

$$E = 0, \hbar\omega, \ldots, n\hbar\omega \quad -(8)$$

One photon is defined by:

$$E = \hbar\omega \quad -(9)$$

and the oscillator is made up of energy levels of photons. The total energy in eq. (5) is:

$$\frac{E}{V} = \hbar\omega\left[\frac{\omega^3}{6\pi^2 c^3}\left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right)^{-1}\right] \quad -(10)$$

Therefore the number of photons per unit volume is:

$$\boxed{\frac{N}{V} = \frac{E}{\hbar\omega V} = \frac{\omega^3}{6\pi^2 c^3}\left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right)^{-1}} \quad -(11)$$

Here N is the number of photons and

3)

V is the volume occupied by the photons – the radiation volume

The Evans / Morris effects show that the incident angular frequency of the beam is ω, the refracted angular frequency is ω_1 and the reflected angular frequency is ω_2. In terms of intensities:

$$\underline{I} = I_1 + I_2 \quad -(12)$$

and it is clear that the total intensity is conserved. In terms of number density of photons:

$$\frac{N}{V} = \left(\frac{N}{V}\right)_1 + \left(\frac{N}{V}\right)_2 \quad -(13)$$

i.e

$$\boxed{\omega^3\left(\frac{x}{1-x}\right) = \omega_1^3\left(\frac{x_1}{1-x_1}\right) + \omega_2^3\left(\frac{x_2}{1-x_2}\right)} \quad -(14)$$

Eq. (12) means that:

$$\boxed{\omega^4\left(\frac{x}{1-x}\right) = \omega_1^4\left(\frac{x_1}{1-x_1}\right) + \omega_2^4\left(\frac{x_2}{1-x_2}\right)} \quad -(15)$$

If the incident beam is made up of one photon of total energy:

$$E = \hbar\omega \quad -(16)$$

then from eq. (11) and (16):

4)

$$\frac{1}{V} = \frac{\omega^3}{6\pi^2 c^3}\left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right)^{-1} \quad -(17)$$

and the volume occupied by one photon is:

$$V = \frac{6\pi^2 c^3}{\omega^3}\left[\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right] \quad -(18)$$

From eq. (1) the volume occupied by one oscillator is:

$$V = 6\pi^2\left(\frac{c}{\omega}\right)^3 \quad -(19)$$

Eq. (19) is the Rayleigh Jeans volume of radiation. The Rayleigh Jeans theory is unphysical because the number density of oscillators becomes infinite as ω goes to infinity.

If there is only one photon, occupying one energy level

$$n = 1 \quad -(20)$$

of the oscillator theory of Planck, then eq. (4) becomes:

$$\frac{E}{V} = \frac{N}{V}\hbar\omega \quad -(21)$$

but:

$$N = 1 \quad -(22)$$

so

$$\frac{E}{V} = \frac{\hbar\omega}{V}. \quad -(23)$$

It follows that the intensity of a beam

made up of one photon is:

$$I = \frac{c\hbar\omega}{V} \quad -(24)$$

From eqs. (1) and (21), the energy per unit volume of a beam made up of one photon is:

$$\frac{E}{V} = \left(\frac{\hbar}{6\pi^2 c^3}\right)\omega^4 \quad -(25)$$

where

$$E = \hbar\omega \quad -(26)$$

so the volume occupied by one photon of energy (26) is:

$$\boxed{V = \frac{6\pi^2 c^3}{\omega^3}} \quad -(27)$$

This result is also obtained self consistently from eq. (1) when:

$$N = 1 \quad -(28)$$

At γ ray frequencies:

$$\omega \sim 10^{21} \text{ rad s}^{-1} \quad -(29)$$

so

$$V \sim 1.6 \times 10^{-37} \text{ m}^3 \quad -(30)$$

The volume of the electron is:

$$V = 9.4 \times 10^{-44} \text{ m}^3 \quad -(31)$$

So this is why the Compton theory works, the volume of one photon $\hbar\omega$ starts to approach the

) classical volume of the electron.

At the visible frequencies used by Evans and Morris:

$$\omega \sim 10^{15} \text{ to } 10^{16} \text{ rad s}^{-1} \quad -(32)$$

and the volume occupied by one photon is seventeen to eighteen orders of magnitude larger, i.e.:

$$V \sim 1.6 \times 10^{-20} \text{ to } 1.6 \times 10^{-21} \text{ m}^3 \quad -(33)$$

At microwave frequencies:

$$\omega \sim 10^{10} \text{ rad s}^{-1} \quad -(34)$$

So:

$$V \sim 1.6 \times 10^{-3} \text{ m}^3 \quad -(35)$$

which is many orders of magnitude larger than the classical volume of the electron, eq. (31).

Conclusions

The most complete theory of the Evans / Morris effects is the intensity theory, eqs. (12) or (15) because it accounts for the presence of many photons with the Planck distribution.