290(6) : Intensity Method for Calculating the Evans / Morris Shifts in Monochromatic Radiation

In radiation of angular frequency ω, the intensity in watts per square metre of N Planck oscillators in a volume of radiation V with:

$$\frac{N}{V} = \frac{1}{6\pi^2}\left(\frac{\omega}{c}\right)^3 \quad -(1)$$

is

$$I = \left(\frac{x}{1-x}\right)\frac{\hbar\omega^4}{6\pi^2c^2} \quad -(2)$$

where

$$x = \exp\left(-\frac{\hbar\omega}{kT}\right) \quad -(3)$$

The intensity of the refracted beam is:

$$I_1 = \left(\frac{x_1}{1-x_1}\right)\frac{\hbar\omega_1^4}{6\pi^2c^2} \quad -(4)$$

and the intensity of the reflected beam is:

$$I_2 = \left(\frac{x_2}{1-x_2}\right)\frac{\hbar\omega_2^4}{6\pi^2c^2} \quad -(5)$$

Experimentally:

$$I = I_1 + I_2 \quad -(6)$$

Therefore:

$$\left(\frac{x}{1-x}\right)\omega^4 = \left(\frac{x_1}{1-x_1}\right)\omega_1^4 + \left(\frac{x_2}{1-x_2}\right)\omega_2^4 \quad -(7)$$

Experiment also leads to Snell's laws:

$$\theta = \theta_2 \quad -(8)$$

$$n \sin\theta = n_1 \sin\theta_1 \quad -(9)$$

where θ, θ_1 and θ_2 are the angles of incidence, refraction and reflection respectively. Here n is the refractive index of the incident medium and n_1 is the refractive index of the refracted beam in the refracting medium. So:

$$n_1 = n \frac{\sin\theta}{\sin\theta_1} \quad -(10)$$

If the incident medium is air then:

$$n_1 = \frac{\sin\theta}{\sin\theta_1} \quad -(11)$$

because

$$n = 1. \quad -(12)$$

The refractive index is:

$$n_1^2 = \frac{1}{2}\left(\epsilon_1' + \left(\epsilon_1'^2 + \epsilon_1''^2\right)^{1/2}\right) -(13)$$

where ϵ_1' and ϵ_1'' are the relative dielectric permittivity and loss of the refracting medium.

) So:

$$\boxed{n^2\left(\frac{\sin\theta}{\sin\theta_1}\right)^2 = \frac{1}{2}\left(\epsilon_1' + \left(\epsilon_1'^2 + \epsilon_1''^2\right)^{1/2}\right)} \quad -(14)$$

This is a generally valid equation.

The refracted frequency ω_1 can be found by using a model for the dielectric permittivity and loss, for example the Debye theory:

$$\epsilon_1' = \epsilon_\infty + \frac{(\epsilon_0 - \epsilon_\infty)}{1+\omega_1^2\tau^2} \quad -(15)$$

and

$$\epsilon_1'' = (\epsilon_0 - \epsilon_\infty)\frac{\omega_1\tau}{1+\omega_1^2\tau^2} \quad -(16)$$

Therefore the Evans Morris frequency ω_1 can be found from eqs. (14) to (16).

Having found ω_1, the reflected frequency ω_2 can be found from eq. (7) in terms of the incident frequency ω.