290(1): Relation between Wave Vectors from First Principles

From previous work the relation between wave vectors from first principles is given by:

$$\underline{\kappa} = \kappa(\underline{i}\sin\theta + \underline{j}\cos\theta) \quad -(1)$$

$$\underline{\kappa_1} = \kappa_1(\underline{i}\sin\theta_1 + \underline{j}\cos\theta_1) \quad -(2)$$

$$\underline{\kappa_2} = \kappa_2(\underline{i}\sin\theta_2 - \underline{j}\cos\theta_2) \quad -(3)$$

where θ, θ_1 and θ_2 are the incident, refracted and reflected angles. Snell's laws give:

$$\theta = \theta_2 \quad -(4)$$

and

$$n\sin\theta = n_1\sin\theta_1, \quad -(5)$$

where n and n_1 are the refractive indices of the incident and refracting media. If the phase velocities of the incident and refracting media are v and v_1, then:

$$n = \frac{c}{v},\ n_1 = \frac{c}{v_1},\ v = \frac{c}{n},\ v_1 = \frac{c}{n_1}. \quad -(6)$$

The three wave vector magnitudes are:

$$\kappa = \frac{n\omega}{c},\ \kappa_1 = \frac{n_1\omega_1}{c},\ \kappa_2 = \frac{n\omega_2}{c}. \quad -(7)$$

So:

$$\underline{\kappa} = \frac{n\omega}{c}(\underline{i}\sin\theta + \underline{j}\cos\theta) \quad -(8)$$

$$\underline{\kappa_1} = \frac{n_1\omega_1}{c}(\underline{i}\sin\theta_1 + \underline{j}\cos\theta_1) \quad -(9)$$

$$\underline{\kappa_2} = \frac{n\omega_2}{c}(\underline{i}\sin\theta - \underline{j}\cos\theta) \quad -(10)$$

If it is assumed that:

$$\underline{k} = \underline{k_1} + \underline{k_2} \quad -(11)$$

Then:

$$n\omega \sin\theta = n_1 \omega_1 \sin\theta_1 + n\omega_2 \sin\theta \quad -(12)$$

and

$$n\omega \cos\theta = n_1 \omega_1 \cos\theta_1 - n\omega_2 \cos\theta \quad -(13)$$

From eq. (5):

$$\sin\theta_1 = \frac{n}{n_1} \sin\theta \quad -(14)$$

and it follows from eq. (12) that:

$$\omega = \omega_1 + \omega_2 \quad -(15)$$

<u>Refraction</u>

The reflected frequency is eliminated using:

$$\omega_2 = \omega - \omega_1 \quad -(16)$$

From eq. (12):

$$n\omega \sin\theta = n_1 \omega_1 \sin\theta_1 + n(\omega - \omega_1) \sin\theta \quad -(17)$$

From eq. (13):

$$n\omega \cos\theta = n_1 \omega_1 \cos\theta_1 - n(\omega - \omega_1) \cos\theta \quad -(18)$$

From eq. (17):

$$\omega_1 (n_1 \sin\theta_1 - n \sin\theta) = 0 \quad -(19)$$

which leads back to Snell's law for all ω_1.

From eq. (18):

3)
$$\omega_1 \left(n_1 \cos\theta_1 + n\cos\theta \right) = \omega \left(n\cos\theta + n\cos\theta \right) \quad -(20)$$

So
$$\omega_1 = \left(\frac{2n\cos\theta}{n_1\cos\theta_1 + n\cos\theta} \right) \omega \quad -(21)$$

Reflection

The refracted frequency is eliminated using:
$$\omega_1 = \omega - \omega_2 \quad -(22)$$

From eqs. (12) and (22):
$$n\omega\sin\theta = n_1 \left(\omega - \omega_2 \right) \sin\theta_1 + n\omega_2\sin\theta \quad -(23)$$

so
$$\omega_2 \left(n\sin\theta - n_1\sin\theta_1 \right) = \omega \left(n\sin\theta + n_1\sin\theta_1 \right) \quad -(24)$$

and this implies the unphysical result:
$$\omega = ?\ 0 \quad -(25)$$

From eqs. (13) and (22)
$$n\omega\cos\theta = n_1 (\omega \overset{-\omega_2)}{\ } \cos\theta_1 - n\omega_2\cos\theta \quad -(26)$$

So:
$$-(27)$$
$$\omega \left(n\cos\theta - n_1\cos\theta_1 \right) = -\omega_2 \left(n_1\cos\theta_1 + n\cos\theta \right)$$

and
$$\omega_2 = \left(\frac{n_1\cos\theta_1 - n\cos\theta}{n_1\cos\theta_1 + n\cos\theta} \right) \omega \quad -(28)$$

+) Adding eqs. (21) and (28):

$$\omega_1 + \omega_2 = \left(\frac{n\cos\theta - n_1\cos\theta_1}{n_1\cos\theta_1 + n\cos\theta} \right) \omega \quad -(29)$$

which implies that:

$$n\cos\theta - n_1\cos\theta_1 = n_1\cos\theta_1 + n\cos\theta \quad -(30)$$

i.e.

$$2n_1\cos\theta_1 \overset{?}{=} 0 \quad -(31)$$

This is another unphysical result.

Conclusion

It becomes clear that:

$$\underline{\kappa} \neq \underline{\kappa_1} + \underline{\kappa_2} \quad -(32)$$

so conservation of energy and momentum must take place through another mechanism when considering reflection and refraction, because:

$$\hbar\underline{\kappa} \neq \hbar\underline{\kappa_1} + \hbar\underline{\kappa_2} \quad -(33)$$

and

$$\hbar\omega \neq \hbar\omega_1 + \hbar\omega_2 \quad -(34)$$

This is why the Planck oscillator and intensity methods are needed.