290(4): The Correct Derivation of the Stefan Boltzmann Law

The correct derivation of the intensity of radiation is:

$$I = c\frac{U}{V} = \frac{1}{\pi^2 c^2}\left[\int \langle E\rangle \omega^2 d\omega + \frac{2}{3}\int \langle E\rangle \omega (d\omega)^2 + \frac{1}{3}\int \langle E\rangle (d\omega)^3\right] \quad -(1)$$

Eq. (1) is:

$$I = \frac{1}{\pi^2 c^2}\left[\int_0^\infty \omega^2 f(\omega)\, d\omega + \int_0^\infty\int_0^\infty \omega f(\omega)\, d\omega\, d\omega + \int_0^\infty\int_0^\infty\int_0^\infty f(\omega)\, d\omega\, d\omega\, d\omega\right] \quad -(2)$$

where

$$f(\omega) = \frac{\hbar\omega}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1} \quad -(3)$$

The usual Stefan Boltzmann law is:

$$I = \frac{1}{\pi^2 c^2}\int_0^\infty \omega^2 f(\omega)\, d\omega = \left(\frac{\pi^2 k^4}{15 c^2 \hbar^3}\right) T^4 \quad -(4)$$

This is completely incorrect because it omits the second and third terms in eq. (2).

High Temperature or Low Frequency Limit

In this limit:

$$\hbar\omega \ll kT \quad -(5)$$

so:

$$\exp\left(\frac{\hbar\omega}{kT}\right) \sim 1 + \frac{\hbar\omega}{kT} \quad -(6)$$

2)

so

$$\langle E \rangle \to kT, \quad -(7)$$

For an infinite range of radiation as in blackbody radiation (eq. (2)), then in the limit (5):

$$I \to \frac{kT}{\pi^2 c^2}\left[\int_0^\infty \omega^2 d\omega + \int_0^\infty\int_0^\infty \omega\, d\omega\, d\omega + \int_0^\infty\int_0^\infty\int_0^\infty d\omega\, d\omega\, d\omega\right] \quad -(8)$$

The indefinite integrals are:

$$I \to \frac{kT}{\pi^2 c^2}\left[\int \omega^2 d\omega + \iint \omega\, d\omega\, d\omega + \iiint d\omega\, d\omega\, d\omega\right] \quad -(9)$$

$$= \frac{kT}{\pi^2 c^2}\left[\frac{1}{3} + \frac{1}{9} + \frac{1}{18}\right]\omega^3$$

$$= \frac{5}{9}\left(\frac{kT}{\pi^2 c^2}\right)\omega^3$$

The result in this same limit from the incorrect theory is:

$$I = \frac{1}{3}\left(\frac{kT}{\pi^2 c^2}\right)\omega^3 \quad -(10)$$

The usual result (4) is obviously incorrect because terms in $(d\omega)^2$ and $(d\omega)^3$ cannot be neglected. Claims to experimental precision of eq. (4) are obviously incorrect.