290(8): Corrected Rayleigh Jeans Density of States

Consider eq. (4) of Note 290(3), in the same notation:

$$\frac{dN}{V} = \frac{\omega^2}{\pi^2 c^3}\, d\omega + \frac{2\omega}{3\pi^2 c^3}(d\omega)^2 + \frac{(d\omega)^3}{3\pi^2 c^3} \quad -(1)$$

$$= \frac{1}{V}\left(dN_1 + dN_2 + dN_3\right)$$

Then:

$$\frac{dN_1}{V} = \frac{\omega^2}{\pi^2 c^3}\, d\omega, \quad -(2)$$

$$\frac{dN_2}{V} = \frac{2\omega (d\omega)^2}{3\pi^2 c^3} \quad -(3)$$

$$\frac{dN_3}{V} = \frac{(d\omega)^3}{6\pi^2 c^3} \quad -(4)$$

It follows that:

$$\frac{dN_2}{dN_1} = \left(\frac{2\pi^2 c^3}{3\omega^3}\right)\frac{dN_1}{V} \quad -(5)$$

and

$$\frac{dN_3}{dN_2} = \left(\frac{\pi^2 c^3}{4\omega^3}\right)\frac{dN_1}{V} \quad -(6)$$

so

$$\frac{dN_3}{dN_1} = \frac{dN_2}{dN_1}\frac{dN_3}{dN_2} = \frac{1}{12}\frac{(d\omega)^2}{\omega^2} \quad -(7)$$

Therefore:

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$$\frac{dN}{dN_1} = 1 + \frac{2}{3\omega} d\omega + \frac{1}{12\omega^2}(d\omega)^2 \quad -(8)$$

Assume that:

$$\frac{dN}{dN_1} \sim 1 + \frac{2}{3\omega} d\omega \quad -(9)$$

So

$$dN = \left(1 + \frac{2}{3\omega} d\omega\right) dN_1 \quad -(10)$$

$$= dN_1 + \left(\frac{2}{3\omega} d\omega\right) dN_1$$

In the usual theory:

$$dN = dN_1 \quad -(11)$$

and

$$N = N_1 \quad -(12)$$

This is the number of photons in a volume V of radiation.

However, from eq. (10):

$$\int dN = \int dN_1 + \int \left(\frac{2}{3\omega} d\omega\right) dN_1 \quad -(13)$$

i.e.

$$\boxed{N = N_1 + \int \left(\frac{2}{3\omega} d\omega\right) dN_1} \quad -(14)$$

3) The correction to the Rayleigh Jeans density of states is:

$$N = N_1 + N_c \quad -(15)$$

where

$$N_c = \int \left(\frac{2}{3\omega} d\omega \right) dN_1 \quad -(16)$$

Assume that:

$$N_c = \int \left(\frac{2}{3\omega} d\omega \right) \int dN_1 \quad -(17)$$

$$= \frac{2N_1}{3} \log_e \omega$$

The number of photons is increased by:

$$\boxed{N_c = \frac{2}{3} N_1 \log_e \omega} \quad -(18)$$

and

$$\frac{dN}{V} = \frac{dN_1}{V} \left(1 + \frac{2}{3} \log_e \omega \right) \quad -(19)$$