280(8): Table of Mean Energies of the Planck Oscillator

The mean energy is:

$$\langle \hbar\omega \rangle = \left(\frac{e^{-x}}{1 - e^{-x}} \right) \hbar\omega \quad -(1)$$

$$= \left(\frac{1}{e^{x} - 1} \right) \hbar\omega$$

where

$$x = \frac{\hbar\omega}{kT} \quad -(2)$$

Here $\hbar$ is the Planck constant, k is the Boltzmann constant and T is the temperature. The angular frequency $\omega = 2\pi f$ where f is the frequency in Hz. We have $1\,cm^{-1} = 30\,GHz$.

ω / rad s^{-1}	f / Hz	$\bar{\nu}$ / cm^{-1}	x	$1/(e^{x}-1)$	Range
10^{10}	1.59×10^{9}	0.053	2.607×10^{-4}	3835.47	microwave
10^{11}	1.59×10^{10}	0.53	0.002607	383.09	microwave
10^{12}	1.59×10^{11}	5.3	0.02607	37.86	far IR
10^{13}	1.59×10^{12}	53	0.2607	3.358	far IR
10^{14}	1.59×10^{13}	530	2.607	0.073	mid IR
10^{15}	1.59×10^{14}	5,300	26.07	4.78×10^{-12}	Infra red
10^{16}	1.59×10^{15}	53,000	260.7	~ 0	ultra violet

The visible range starts with the red at 4×10^{14} Hz and ends with the violet at

2) 7.89×10^{14} Hz. The temperature is $T = 293$ K.

This table shows that the linear approximation used in UFT 279 and notes for UFT 280 is adequate from the microwave to the very far infra red. In the far infra red at 53 cm^{-1} the linear approximation is a rough approximation. At higher frequencies the linear approximation cannot be used. The linear approximation cannot be used in the visible range.

In the microwave range the mean energy $\langle \hbar\omega \rangle$ is much larger than the energy $\hbar\omega$ of one photon in the state $n = 1$. In the far infra-red the mean energy $\langle \hbar\omega \rangle$ is about the same as $\hbar\omega$. From the mid infra-red at 530 cm^{-1} to higher frequencies, the mean energy is much lower than $\hbar\omega$. From the infra red at 5,300 cm^{-1} to higher frequencies the mean energy of the Planck oscillator is effectively zero. This is because the oscillator possesses the energies:

$$E = n\hbar\omega, \quad n = 0, 1, 2, \ldots \quad (3)$$

This is why the Compton theory uses $E = \hbar\omega$, and not $E = 2\langle \hbar\omega \rangle$, because it uses the energy of γ rays.