280(5) : Final Version of Note 280(4)

Correcting equation (30) of note 280(4) gives:

$$\cos 2\theta = \cos^2\theta - \sin^2\theta \quad -(1)$$

$$= 1 - \frac{1}{2n^2}$$

Therefore eq. (33) becomes:

$$(\omega - \omega_2)^2 = n^2(\omega - \omega_2)^2 + \omega\omega_2 \quad -(2)$$

i.e.

$$(\omega - \omega_2)^2(1 - n^2) = \omega\omega_2 \quad -(3)$$

This equation can be written as:

$$\omega^2 - (2 + A)\omega\omega_2 + \omega_2^2 = 0 \quad -(4)$$

where

$$A := \frac{1}{1 - n^2} \quad -(5)$$

so

$$\omega_2 = \frac{\omega}{2}\left[A + 2 \pm \left((A+2)^2 - 4\right)^{1/2}\right] \quad -(6)$$

and

$$\omega = \frac{\omega_2}{2}\left[A + 2 \pm \left((A+2)^2 - 4\right)^{1/2}\right] \quad -(7)$$

From eq. (3):

$$\omega \neq \omega_2 \quad -(8)$$

unless

$$\omega = \omega_2 = 0. \quad -(9)$$

Also A is negative valued because:

$$n > 1. \quad -(10)$$

Therefore there are two possible solutions:

2)

$$\omega_2 = \frac{\omega}{2}\left[A + 2 \pm \left((A+2)^2 - 4\right)^{1/2}\right]$$

$$= \frac{\omega}{2}\left[A + 2 \pm \left(A^2 + 2A\right)^{1/2}\right]$$

$$= \frac{\omega}{2}\left[2 + \frac{1}{1-n^2} \pm \left(\left(\frac{1}{1-n^2}\right)^2 + 2\left(\frac{1}{1-n^2}\right)\right)^{1/2}\right] \quad -(11)$$

If $$n = 1.5 \quad -(12)$$

then $$\frac{1}{1-n^2} = -0.8 \quad -(13)$$

so $A^2 + 2A$ is negative valued, and:

$$\omega_2 = \omega' \pm i\omega'' \quad -(14)$$

where $$\omega' = \frac{1}{2}(A+2) \quad -(15)$$

and $$\omega'' = \frac{1}{2}\left(A^2 + 2A\right)^{1/2} \quad -(16)$$

If it is assumed that:

$$\omega_2 = \text{Real } \omega_2 \quad -(17)$$

is the physical result then:

$$\omega_2 = \frac{\omega}{2}\left(2 + \frac{1}{1-n^2}\right) \quad -(18)$$

If it is assumed that the real energy is:

$$E_2 = \hbar(\omega\omega^*)^{1/2} \quad -(19)$$

in analogy with the classical energy density:

$$E_n = \epsilon_0 EE^* + \frac{1}{\mu_0} BB^* \quad -(20)$$

then

$$\omega_2 = (\omega'^2 + \omega''^2)^{1/2} \quad -(21)$$

$$= \frac{1}{2}\left[(A+2)^2 + A^2 + 2A\right]\omega$$

$$= \frac{1}{2}\left[2A^2 + 4A + 4\right]\omega$$

$$= (A^2 + 2A + 2)\omega$$

If

$$A = \frac{1}{1-n^2} = -0.8 \quad -(22)$$

then:

$$\omega_2 = (2.64 - 1.6)\omega \quad -(23)$$

$$= 1.04\omega$$

Eq. (18) gives:

$$\omega_2 = 0.6\omega \quad -(24)$$

The choice between eqs (22) and (23) must be determined experimentally.