280(6): a Photon Theory of Total Internal Reflection.

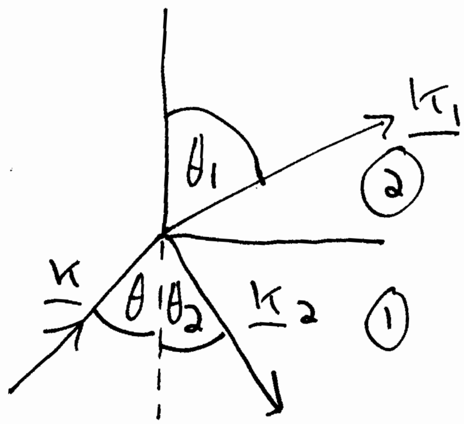
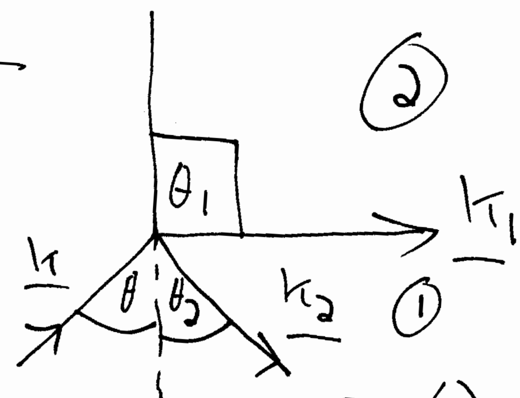


Fig (1)

$\rightarrow$



Total internal reflection is defined in Fig (1), the refracted light travels along the boundary. By conservation of energy and momentum:

$$\langle \omega \rangle = \langle \omega_1 \rangle + \langle \omega_2 \rangle \quad -(1)$$

and

$$\langle \underline{\kappa} \rangle = \langle \underline{\kappa_1} \rangle + \langle \underline{\kappa_2} \rangle \quad -(2)$$

Here:

$$\underline{\kappa} = \kappa \left(\underline{i} \sin\theta + \underline{j} \cos\theta \right) \quad -(3)$$

$$\underline{\kappa_1} = \kappa_1 \left(\underline{i} \sin\theta_1 + \underline{j} \cos\theta_1 \right) \quad -(4)$$

$$\underline{\kappa_2} = \kappa_2 \left(\underline{i} \sin\theta_2 - \underline{j} \cos\theta_2 \right) \quad -(5)$$

It is assumed that medium (1) is glass, in which the phase velocity of light is v, and medium (2) is air, in which the phase velocity is c. So

$$\kappa = \frac{\omega}{v}, \quad \kappa_1 = \frac{\omega_1}{c}, \quad \kappa_2 = \frac{\omega_2}{v} \quad -(6)$$

In the Boltzmann averaging of the Planck theory:

2)
$$\langle \omega \rangle = \left(\frac{x}{1-x}\right)\omega, \quad x = \exp\left(-\frac{\hbar\omega}{kT}\right) \quad -(7)$$
$$\langle \omega_1 \rangle = \left(\frac{x_1}{1-x_1}\right)\omega_1, \quad x_1 = \exp\left(-\frac{\hbar\omega_1}{kT}\right) \quad -(8)$$
$$\langle \omega_2 \rangle = \left(\frac{x_2}{1-x_2}\right)\omega_2, \quad x_2 = \exp\left(-\frac{\hbar\omega_2}{kT}\right) \quad -(9)$$

under the condition:
$$\hbar\omega \ll kT \text{ etc.} \quad -(10)$$

Therefore:
$$\langle k \rangle = \left(\frac{x}{1-x}\right)\frac{\omega}{v} \quad -(11)$$
$$\langle k_1 \rangle = \left(\frac{x_1}{1-x_1}\right)\frac{\omega_1}{c} \quad -(12)$$
$$\langle k_2 \rangle = \left(\frac{x_2}{1-x_2}\right)\frac{\omega_2}{v} \quad -(13)$$

Fm eq. (2):
$$\langle \underline{k_1} \rangle \cdot \langle \underline{k_1} \rangle = \langle \underline{k} \rangle \cdot \langle \underline{k} \rangle + \langle \underline{k_2} \rangle \cdot \langle \underline{k_2} \rangle - 2\langle \underline{k} \rangle \cdot \langle \underline{k_2} \rangle \quad -(14)$$

The refractive index of the glass is:
$$n = \frac{c}{v} \quad -(15)$$

so:
$$\left(\frac{x_1}{1-x_1}\right)^2 \omega_1^2 = n^2\left[\left(\frac{x}{1-x}\right)^2\omega^2 + \left(\frac{x_2}{1-x_2}\right)^2\omega_2^2 - 2\left(\frac{x}{1-x}\right)\left(\frac{x_2}{1-x_2}\right)\omega\omega_2\cos(2\theta)\right] \quad -(16)$$

where we have used

$$\theta = \theta_2 \quad -(17)$$

from Snell's Law. So the angle between $\underline{\kappa}$ and $\underline{\kappa_2}$ is 2θ. By Snell's Law for total internal reflection:

$$\cos(2\theta) = \sin^2\theta - \cos^2\theta$$
$$= 1 - \frac{2}{n^2}. \quad -(18)$$

Finally, from eq. (1):

$$\left(\frac{x}{1-x}\right)\omega = \left(\frac{x_1}{1-x_1}\right)\omega_1 + \left(\frac{x_2}{1-x_2}\right)\omega_2. \quad -(19)$$

Eqns. (16) and (19) may be used to eliminate ω_1 to find ω_2 in terms of ω. From eq. (19):

$$\left(\frac{x_1}{1-x_1}\right)\omega_1 = \left(\frac{x}{1-x}\right)\omega - \left(\frac{x_2}{1-x_2}\right)\omega_2 \quad -(20)$$

So:

$$\left[\left(\frac{x}{1-x}\right)\omega - \left(\frac{x_2}{1-x_2}\right)\omega_2\right]^2$$
$$n^2\left[\left(\frac{x}{1-x}\right)^2\omega^2 + \left(\frac{x_2}{1-x_2}\right)^2\omega_2^2 \quad -(21)\right.$$
$$\left. - 2\left(\frac{x}{1-x}\right)\left(\frac{x_2}{1-x_2}\right)\omega\omega_2\left(1-\frac{2}{n^2}\right)\right]$$