270(8): Corrections to UFT 238

The Lagrangian is:

$$\mathcal{L} = T - V = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2 + r^2 \sin^2\theta \, \dot{\phi}^2 \right) + \frac{k}{r} \quad -(1)$$

There are three Euler Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{r}} \right) \quad -(2)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\theta}} \right) \quad -(3)$$

$$\frac{\partial \mathcal{L}}{\partial \phi} = \frac{d}{dt} \left(\frac{\partial \mathcal{L}}{\partial \dot{\phi}} \right) \quad -(4)$$

Eq. (2) gives:

$$F = m\ddot{r} - \frac{L^2}{mr^3} = -\frac{k}{r^2}. \quad -(5)$$

Eq. (4) gives:

$$\frac{d}{dt} \left(mr^2 \dot{\phi} \sin^2\theta \right) = 0. \quad -(6)$$

Eq. (3) gives:

$$\frac{d}{dt} \left(mr^2 \dot{\theta} \right) = mr^2 \sin\theta \cos\theta \, \dot{\phi}^2 \quad -(7)$$

So the conserved angular momenta are:

2)
$$L_Z = L_\phi = m r^2 \dot{\phi} \sin^2\theta \quad -(8)$$

and
$$L^2 = m^2 r^4 \left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta \cos^2\theta + \dot{\phi}^2 \sin^4\theta \right)$$
$$= m^2 r^4 \left(\dot{\theta}^2 + \dot{\phi}^2 \sin^2\theta \right) \quad -(9)$$

In UFT 238 the term on the right hand side of eqn (7) was missing. Therefore eq. (46) of UFT 238 is

$$\frac{d\beta}{dt} = \frac{L}{mr^2}, \quad \frac{d\phi}{dt} = \frac{L_\phi}{mr^2 \sin^2\theta} \quad -(10)$$

and
$$\frac{d\beta}{d\phi} = \frac{L}{L_\phi} \sin^2\theta \quad -(11)$$

Therefore
$$\int d\beta = \frac{L}{L_\phi} \int \sin^2\theta \, d\phi \quad -(12)$$

In eq (12), θ and ϕ are independent lagrangian variables, so:
$$\int \sin^2\theta \, d\phi = \sin^2\theta \int d\phi \quad -(13)$$

It follows that:
$$\boxed{\beta = \frac{L}{L_\phi} \sin^2\theta \, \phi} \quad -(14)$$

3) So the β ellipse is:

$$r = \frac{\alpha}{1 + \epsilon \cos \beta} \quad - (15)$$

i.e.

$$\boxed{r = \frac{\alpha}{1 + \epsilon \cos\left(\frac{L}{L_\phi} \sin^2\theta \, \phi\right)}} \quad - (16)$$

This is a precessing ellipse.

Eq. (16) should be graphed to give the correct graphics. The precession constant is now:

$$x = \frac{L}{L_\phi} \sin^2\theta \quad - (17)$$

$$= \frac{L}{L_Z} \sin^2\theta$$

All the rest of UFT 238 is correct, including the graphics. The problem with collapse to a 2-D orbit is removed.