270(1): Transition to Planar Orbit and Derivation of the Experimentally Observed Planetary Precession.

In general the three dimensional planetary orbit is:

$$r = \frac{\alpha}{1+\epsilon\cos\beta} \quad -(1)$$

where

$$\beta = \frac{L}{L_\phi}\,\phi\,\sin\theta. \quad -(2)$$

Here:

$$L^2 = L_\theta^2 + L_\phi^2 \quad -(3)$$

$$L_\theta = mr^2\dot{\theta} \quad -(4)$$

$$L_\phi = mr^2\dot{\phi}\sin\theta \quad -(5)$$

Eq. (1) is the precessing orbit:

$$r = \frac{\alpha}{1+\epsilon\cos(x\phi)} \quad -(6)$$

where

$$x = \frac{L}{L_\phi}\sin\theta. \quad -(7)$$

The transition to a planar orbit occurs as:

$$\frac{d\theta}{dt} \rightarrow 0 \quad -(8)$$

and

$$L_\theta \rightarrow 0 \quad -(9)$$

2) In this case:

$$\beta \rightarrow x\phi \quad -(10)$$

The observed planetary precession in the (r, ϕ) plane is:

$$x = 1 + \frac{3MG}{c^2 \alpha} \quad -(11)$$

so

$$\frac{L}{L_\phi} \sin\theta = 1 + \frac{3MG}{c^2 \alpha} \quad -(12)$$

$$\sim 1$$

This result is satisfied by:

$$\theta \sim \frac{\pi}{2} \quad -(13)$$

and

$$L \sim L_\phi, \quad -(14)$$

and this is self consistent. Rigorously:

$$\sin\theta = \frac{L_\phi}{L}\left(1 + \frac{3MG}{c^2 \alpha}\right) \quad -(15)$$

and

$$\theta = \sin^{-1}\left(\frac{L_\phi}{L}\left(1 + \frac{3MG}{c^2 \alpha}\right)\right) \quad -(16)$$

$$= \text{constant}$$

So

$$\frac{d\theta}{dt} = 0 \quad -(17)$$

3) and:

$$L_\theta = 0, \quad (18)$$

$$L = L_\phi \quad (19)$$

Conclusion

The experimentally observable planetary precession can be explained non-relativistically by fixing the angle θ through eq. (16). Three dimensional orbits produce planetary precession in the (r, ϕ) plane with θ fixed. So planetary precession can be explained with the inverse square law used in three dimensions.

Note

In Eq. (16):

$$L_\phi \sim L \quad (20)$$

to an excellent approximation, so:

$$\boxed{\sin\theta = x} \quad (21)$$
