1) 269(6) : Time Dependence of Angle in an Elliptical Orbit.

To begin with consider the simple ellipse:

$$r = \frac{\alpha}{1+\epsilon\cos\phi}, \quad -(1)$$

then

$$\frac{d\phi}{dt} = \frac{L}{mr^2}$$

$$= \frac{L}{m\alpha^2}\left(1+\epsilon\cos\phi\right)^2 \quad -(2)$$

so

$$dt = \left(\frac{m\alpha^2}{L}\right)\frac{d\phi}{(1+\epsilon\cos\phi)^2} \quad -(3)$$

Therefore:

$$t = \left(\frac{m\alpha^2}{L}\right)\int\frac{d\phi}{(1+\epsilon\cos\phi)^2} \quad -(4)$$

$$= \left(\frac{m\alpha^2}{L}\right)\left[\frac{\epsilon\sin\phi}{(\epsilon^2-1)(1+\epsilon\cos\phi)} - \frac{1}{(\epsilon^2-1)}\int\frac{d\phi}{1+\epsilon\cos\phi}\right]$$

where

$$\int\frac{d\phi}{1+\epsilon\cos\phi} = \frac{2}{(1-\epsilon^2)^{1/2}}\tan^{-1}\left[\frac{(1-\epsilon)\tan(\phi/2)}{(1-\epsilon^2)^{1/2}}\right]$$

$$-(5)$$

2)

for $\epsilon^2 < 1$.

Therefore at a given ϕ, the time t can be found and the orbit can be animated. For a given ϕ, r can be found from Eq. (1), so an animation can be made of r as a function of t and ϕ as a function of t.

<u>Three Dimensional Animation</u>

For a given ϕ, r and t, the half right latitude is given by:

$$\alpha_1 = \frac{L_1^2}{mk} = \frac{L_2^2}{mk \sin^4\theta} \quad -(6)$$

and

$$\epsilon_1^2 = 1 + \frac{2EL_2^2}{mk^2 \sin^4\theta} \quad -(7)$$

Therefore t can be plotted as a function of θ and ϕ from eqs. (4) to (7), and r can also be plotted as a function of θ and ϕ.
