265(2): The Metric of the x Theory

This is the metric of special relativity:

$$c^2 d\tau^2 = (c^2 - v^2) dt^2 \quad -(1)$$

where τ is the proper time of the moving frame and t the time in the fixed or observer frame. It follows that the Lorentz factor is:

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(2)$$

and that:

$$mc^2 = \gamma^2 mc^2 - \gamma^2 mv^2 \quad -(3)$$

Now let:

$$E = \gamma mc^2 \quad -(4)$$

and

$$\underline{p} = \gamma m \underline{v} \quad -(5)$$

It follows from eqns (3) to (5) that:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(6)$$

which is the Einstein energy equation.

The metric (1) is:

$$c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 \quad -(7)$$

so

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2 \left(\frac{d\theta}{dt}\right)^2 \quad -(8)$$

Now use:

$$\frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} \quad -(9)$$

so

$$v^2 = \left(\frac{d\theta}{dt}\right)^2 \left(\left(\frac{dr}{d\theta}\right)^2 + r^2\right) \quad -(10)$$

$$= \omega^2 \left(\left(\frac{dr}{d\theta}\right)^2 + r^2\right)$$

$$= \frac{L^2}{mr^4}\left(\left(\frac{dr}{d\theta}\right)^2 + r^2\right).$$

The Cartan spin connection in this theory is the angular velocity:

$$\omega = \frac{d\theta}{dt} = \frac{L}{mr^2} \quad -(11)$$

where L is the angular momentum, a constant of motion. Note carefully that this is a theory of general relativity because of the presence of a spin connection. The theory reduces to special relativity when:

$$v \longrightarrow \frac{dr}{dt} \quad -(12)$$

i.e. when there is no angular motion.

Therefore the correct way to go from special

3) to general relativity is the obvious way, to replace eq. (12) by eq. (8) in the Minkowski metric.

The function $dr/d\theta$ is derived from an orbit or any angular motion in a plane. The observed planar orbit is the precessing ellipse:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(13)$$

where

$$x = 1 + \frac{3MG}{c^2 \alpha} \quad -(14)$$

This seems to be the case everywhere in the universe.

For the ellipse:

$$\alpha = a(1 - \epsilon^2) \quad -(15)$$

where a is the semi major axis.

For the hyperbola:

$$\alpha = a(\epsilon^2 - 1). \quad -(16)$$

In eq. (15): $\quad 0 < \epsilon < 1. \quad -(17)$

In eq. (16): $\quad \epsilon > 1. \quad -(18)$

The deflection of light due to gravitation is given by:

4)

$$\Delta\theta = 2\int_{R_0}^{\infty} \frac{d\theta}{dr}\,dr - \pi \quad -(19)$$

$$= \frac{2}{\epsilon}$$

where $d\theta/dr$ is calculated from eq. (13).

The gravitational time delay is calculated from

$$\Delta t = \int_{r_1}^{r_2} \frac{dt}{dr}\,dr \quad -(20)$$

where

$$\frac{dt}{dr} = \frac{d\theta}{dr}\frac{dt}{d\theta} = \frac{L}{mr^2}\frac{d\theta}{dr} \quad -(21)$$

where $d\theta/dr$ is again calculated from eq. (13).

The photon velocity at closest approach is calculated from Eq. (8), and the photon mass from:

$$E = \gamma mc^2 = \hbar\omega \quad -(22)$$

The gravitational red shift is given directly by the Lorentz factor:

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(23)$$

where v is given by eq. (8).

5) This is a simple and powerful theory of general relativity.

It would also very interesting to plot various orbits from eqs. (13) to (18) as follows:

1) Orbits as a function of M, both ellipse and hyperbola, for fixed ϵ and α.

2) Orbits as a function of ϵ and α for fixed M.

As M gets very large and α small, the orbits begin to develop a fractal structure.

The philosophical question is why is $x = 3MG/(c^2\alpha)$.