## 2.03(4): Derivation of the Precession Factor

The Thomas precession is summarized as:

$$d\tau^2 = \left(1 - \frac{3v_\theta^2}{c^2}\right) dt^2 \quad -(1)$$

where $v_\theta$ is:

$$v_\theta = \omega r. \quad -(2)$$

From the equivalence principle:

$$v_\theta^2 = \frac{2MG}{r} \quad -(3)$$

so

$$d\tau^2 = \left(1 - \frac{6MG}{c^2 r}\right) dt^2 \quad -(4)$$

i.e.

$$\frac{dt}{d\tau} = \left(1 - \frac{6MG}{c^2 r}\right)^{-1/2} \quad -(5)$$

$$\sim \frac{3MG}{c^2 r} \quad -(6)$$

if

$$\frac{MG}{c^2} \ll r \quad -(7)$$

At turning point:

$$r = \alpha \quad -(8)$$

so

$$\boxed{\frac{dt}{d\tau} = \frac{3MG}{c^2 \alpha}} \quad -(9)$$

QED.