265(1): Derivation of the Gravitational Red Shift.

Consider the metric of special relativity:

$$c^2 d\tau^2 = c^2 dt^2 - v^2 dt^2 \quad -(1)$$

where

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2, \quad -(2)$$

so

$$\left(\frac{d\tau}{dt}\right)^2 = 1 - \frac{v^2}{c^2} \quad -(3)$$

and

$$\frac{dt}{d\tau} = \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(4)$$

Eq. (4) defines the Lorentz factor in terms of velocity defined in eq. (2).

The gravitational red shift can be defined by eq. (4) in terms of this velocity v. Note carefully that there is no need of the failed Einstein equation or Schwarzschild metric. It is a common misconception that the gravitational red shift is due to Einstein. In fact it can be derived straightforwardly from Newtonian dynamics as follows.

Consider the equivalence principle in the conventional form:

2)

$$\underset{\sim}{F} = m \frac{d\underline{v}}{dt} = m\underline{g} = -\frac{mMG}{r^2} \quad -(5)$$

The work done on a particle by a force $\underset{\sim}{F}$ in transforming the particle from condition 1 to condition 2 is:

$$W_{12} = \int_1^2 \underset{\sim}{F} \cdot d\underset{\sim}{r} \quad -(6)$$

in which:

$$\underset{\sim}{F} \cdot d\underset{\sim}{r} = m \frac{d\underline{v}}{dt} \cdot \frac{d\underline{r}}{dt} dt = m \frac{d\underline{v}}{dt} \cdot \underline{v}\, dt$$

$$= \frac{m}{2} \frac{d}{dt} \left(\underset{\sim}{v} \cdot \underset{\sim}{v} \right) dt = \frac{m}{2} \frac{d}{dt} \left(v^2 \right) dt$$

$$= d\left(\frac{1}{2} m v^2 \right) \quad -(7)$$

So

$$\int \underset{\sim}{F} \cdot dr = \frac{1}{2} m v^2 \quad -(8)$$

(Marion and Thornton, 3rd edition, page 71).

From eqs. (5) and (8):

$$\int \underset{\sim}{F} \cdot d\underset{\sim}{r} = \frac{1}{2} m v^2 = -\int \frac{mMG}{r^2} dr$$

$$= \frac{mMG}{r} \quad -(9)$$

So:

$$\boxed{v^2 = \frac{2MG}{r}} \quad -(10)$$

3)

The velocity defined in Eq. (10) is

$$v^2 = \left(\frac{dr}{dt}\right)^2 \quad -(11)$$

The gravitational red shift can therefore be defined as:

$$\frac{dt}{d\tau} = \gamma = \left(1 - \frac{2mG}{c^2 r}\right)^{-1/2} \quad -(12)$$

In eq. (12), the old Schwarzschild radius is defined as:

$$r_0 = \frac{2mG}{c^2} \quad -(13)$$

and is a consequence of Newtonian dynamics, not of Einsteinian dynamics.

If

$$r_0 \ll r \quad -(14)$$

then

$$\boxed{\frac{dt}{d\tau} \sim \frac{mG}{c^2 r}} \quad -(15)$$

which is the gravitational red shift, Q.E.D.

Note carefully that eq. (15) is an approximation, and is valid only if the velocity is defined as radial, eq. (11). This is the case when an object falls directly to the

4) earth's surface. This was tested by the Harvard tower experiment, but it is a test of Newtonian dynamics incorporated into the Minkowski metric. It is not a test of the Einstein theory at all.

More generally v is defined by eq. (2). For example if:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(16)$$

then:

$$v^2 = MG\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(17)$$

where a is the semi major axis of the ellipse:

$$a = \frac{\alpha}{1-\epsilon^2}. \quad -(18)$$

The gravitational red shift is then:

$$\frac{dt}{d\tau} = \gamma = \left(1 - \frac{MG}{c^2}\left(\frac{2}{r} - \frac{1}{a}\right)\right)^{-1/2} \quad -(19)$$

The precessing ellipse observed in planetary orbits is:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(20)$$

where

$$x = 1 + \frac{3MG}{c^2\alpha}. \quad -(21)$$

From eqs. (2) and (20):

$$v^2 = x^2 MG\left(\frac{2}{r} - \frac{1}{a}\right) + \frac{L^2}{m^2 r^2}\left(1 - x^2\right) \quad -(22)$$

where

$$L^2 = \alpha m^2 MG \quad -(23)$$

so

$$v^2 = x^2 MG\left(\frac{2}{r} - \frac{1}{a}\right) + \alpha\frac{MG}{r^2}\left(1 - x^2\right)$$

$$= MG\left[x^2\left(\frac{2}{r} - \frac{1}{a}\right) + \frac{\alpha}{r^2}\left(1 - x^2\right)\right] \quad -(24)$$

The expected gravitational red shift in the experimentally observed precessing orbit is:

$$\frac{t}{\tau} = \gamma = \left(1 - \frac{MG}{c^2}\left[x^2\left(\frac{2}{r} - \frac{1}{a}\right) + \frac{\alpha}{r^2}\left(1 - x^2\right)\right]\right)^{-1/2} \quad -(25)$$

where

$$x = 1 + \frac{3MG}{c^2\alpha} \quad -(26)$$

This result can be tested experimentally.