264(5): The Relation of Orbital Precession to a Combination of the Gravitational Red Shift and the de Sitter or Geodetic Precession.

The experimentally observed precessing planar orbit is:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(1)$$

where

$$x = \frac{3MG}{c^2 \alpha} \quad -(2)$$

The force law for the orbit (1) is:

$$F = -\frac{L^2}{mr^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right) \quad -(3)$$

$$= m\left(\ddot{r} - r\dot{\theta}^2\right),$$

$$F = (x^2 - 1)\frac{L^2}{mr^3} - \frac{x^2 L^2}{\alpha m r^2}. \quad -(3a)$$

Here

$$m r \dot{\theta}^2 = \frac{L^2}{mr^3}. \quad -(4)$$

Therefore:

$$F(r) + m r \dot{\theta}^2 = \frac{x^2 L^2}{mr^2}\left(\frac{1}{r} - \frac{1}{\alpha}\right)$$

$$= m\ddot{r} \quad -(5)$$

In the near Newtonian limit:

$$\alpha = \frac{L^2}{m^2 M G} \quad -(6)$$

So:

$$\frac{1}{x^2}\frac{d^2 r}{dt^2} = \frac{L^2}{mr^3} - \frac{mMG}{r^2} \quad -(7)$$

Eq. (7) is:

$$\frac{d^2 r}{dt_1^2} = \frac{L^2}{mr^3} - \frac{mMG}{r^2} \quad -(8)$$

where

$$t_1 = \left(1 + \frac{3MG}{c^2\alpha}\right)t \quad -(9)$$

for a clockwise precession and

$$t_1 = \left(1 - \frac{3MG}{c^2\alpha}\right)t \quad -(10)$$

for an anticlockwise precession.

At the turning point:

$$\frac{d^2 r}{dt_1^2} = 0 \quad -(11)$$

and

$$\alpha = r \quad -(12)$$

So the turning point is a special case of:

3)

$$t_1 = \left(1 - \frac{3mG}{c^2 r}\right) t \quad -(13)$$

for an anticlockwise precession, and

$$t_1 = \left(1 + \frac{3mG}{c^2 r}\right) t \quad -(14)$$

for a clockwise precession.

Eq. (13) is the combination of the gravitational red shift and the de Sitter preccession:

$$d\tau = \left(1 - \frac{3mG}{c^2 r}\right) dt \quad -(15)$$

$$= \left(1 - \frac{2mG}{c^2 r} - \frac{mG}{c^2 r}\right)$$

Q.E.D.

At the turning point $\alpha = r$ defines other turning points. For the ellipse it defines the perihelion:

$$\alpha = r_{min}(1 + \epsilon) \quad -(16)$$

and the aphelion:

$$\alpha = r_{max}(1 - \epsilon) \quad -(17)$$

In the hyperbola the perihelion is the distance of closest approach:

$$\alpha = R_0(1 + \epsilon) \quad -(18).$$

4) Note that $3r_0/\alpha$ is a constant of motion:

$$\frac{3r_0}{\alpha} = 3\left(\frac{mMG}{c^2L}\right)^2 \quad -(19)$$

At the various turning points of an orbit the combined gravitational red shift and de Sitter precession are constants of motion:

$$\frac{3r_0}{\alpha} = \left(\frac{2MG}{c^2r} + \frac{MG}{c^2r}\right)_{r=\alpha} \quad -(20)$$

Discussion

The gravitational red shift and de Sitter precession are known experimentally with high accuracy. The orbital precession is also known with high accuracy and is the same thing as the sum of the gravitational red shift and the de Sitter precession at the turning point $r = \alpha$. Light deflection by gravitation is also known with accuracy and is also described by eq. (1) of x theory.