264(4): Estimate of Photon Velocity from x Theory

The orbit of the photon is the hyperbola:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(1)$$

where

$$x = 1 + \frac{r_0}{\alpha} \quad -(2)$$

so

$$\frac{dr}{d\theta} = \frac{x \epsilon r^2}{\alpha} \sin(x\theta). \quad -(3)$$

It follows that the linear velocity of the photon is:

$$v = \frac{dr}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt}, \quad -(4)$$

where

$$\frac{d\theta}{dt} = \omega = \frac{L}{mr^2}. \quad -(5)$$

So

$$v = \frac{x \epsilon L}{m\alpha} \sin(x\theta) \quad -(6)$$

where

$$\cos(x\theta) = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(7)$$

and

$$\sin(x\theta) = \left(1 - \cos^2(x\theta)\right)^{1/2} \quad -(8)$$

so:

2)

$$v = \frac{dr}{dt} = \frac{x \epsilon L}{m \alpha} \left(1 - \frac{1}{\epsilon^2} \left(\frac{\alpha}{r} - 1 \right)^2 \right)^{1/2} \quad -(9)$$

The limits of validity of this formula are defined by:

$$-1 \leq \cos(x\theta) \leq 1 \quad -(10)$$

i.e.

$$-\epsilon \leq \frac{\alpha}{r} - 1 \leq \epsilon \quad -(11)$$

or

$$1 - \epsilon \leq \frac{\alpha}{r} \leq 1 + \epsilon. \quad -(12)$$

which means:

$$r \geq \frac{\alpha}{1+\epsilon} \quad -(13)$$

or

$$r \leq \frac{\alpha}{1-\epsilon}. \quad -(14)$$

The distance of closest approach is defined by:

$$R_0 = \frac{\alpha}{1+\epsilon} \quad -(15)$$

which is compatible with eqn. (13). At closest approach:

$$v = \frac{x \epsilon L}{m \alpha} \left(1 - \frac{1}{\epsilon^2} \left(1 + \epsilon - 1 \right)^2 \right)^{1/2}$$

$$= 0 \quad -(16)$$

3) which means that the linear velocity is zero and the total velocity is:

$$v = \omega R_0 \quad -(17)$$

from

$$\underline{v} = \underline{\omega} \times \underline{r} \quad -(18)$$

which is the angular velocity. In general:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 \quad -(19)$$

Assume that the photon velocity is c at large distances from the sun:

$$v \xrightarrow[r \to \infty]{} c \quad -(20)$$

From eqns. (9) and (20):

$$v \xrightarrow[r \to \infty]{} \frac{x \epsilon L}{m d}\left(1 - \frac{1}{\epsilon^2}\right) \quad -(21)$$

$$= c$$

For light grazing the sun:

$$\epsilon = 235,735.06 \quad -(22)$$

$$d = 1.639992 \times 10^{14}\ m \quad -(23)$$

$$\Delta\theta = 8.4841 \times 10^{-6} \text{ radians} \quad -(24)$$

so to an excellent approximation:

4)
$$\frac{x \epsilon L}{m \alpha} = c \quad -(25)$$

and
$$\frac{L}{m} = \frac{\alpha c}{x \epsilon} \quad -(26)$$

Therefore the linear velocity in the large r limit is:

$$\boxed{v = c \sin(x\theta)} \quad -(27)$$

i.e.
$$\frac{v}{c} = \left(1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2\right)^{1/2} \quad -(28)$$

in the limit:
$$r \geq \frac{\alpha}{1+\epsilon} \quad -(29)$$

Eq. (28) can be used for gravitational time delay. In the infinite r limit:

$$\omega = \frac{L}{mr^2} \longrightarrow 0 \quad -(30)$$

and the velocity is dominated by dr/dt. At closest approach the opposite is true, and

$$v = \omega R_0 = \frac{L}{m R_0} \quad -(31)$$

$$= \frac{\alpha c}{x \epsilon R_0}$$

a result which depends on the radius R_0 of

5) closest approach, the radius of the sun. If we use:

$$R_0 = 6.955 \times 10^8 \, m \quad -(31)$$

and

$$c = 2.998 \times 10^8 \, ms^{-1} \quad -(32)$$

then eq. (31) gives:

$$v = 2.998 \times 10^8 \quad -(33)$$

meaning that the velocity is not slowed very much by the sun.

However, Robitaille's estimate of R_0 could be used and that would determine the photon velocity more accurately. This result makes sense because the orbit is almost a straight line for the sun.