263(7): Metric for the R Theory and Re-Interpretation of the Conventional Schwarzschild Metric

The Newtonian theory is developed to special relativity with the metric:

$$c^2 d\tau^2 = c^2 dt^2 - v^2 dt^2 \quad -(1)$$

here τ is the proper time, t the observer time, c the speed of light and v is the linear velocity in the observer frame. We have:

$$v^2 dt^2 = dr^2 + r^2 d\theta^2 \quad -(2)$$

in plane polar coordinates, so

$$v^2 = \frac{dr}{dt^2} + r^2 \left(\frac{d\theta}{dt}\right)^2 \quad -(3)$$

$$= \dot{r}^2 + r^2 \dot{\theta}^2$$

Eq. (3) is the well known fundamental result for linear velocity from basic kinematics and the rotation of coordinates of the plane polar system.

From eq. (1):

$$dt = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} d\tau \quad -(4)$$

$$= \gamma . d\tau$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(5)$$

2) is the Lorentz factor of special relativity.

Multiplying eq. (1) by m, the arbitrary mass, gives:

$$mc^2 = mc^2\left(\frac{dt}{d\tau}\right)^2 - mv^2\left(\frac{dt}{d\tau}\right)^2 \quad -(6)$$

$$= \gamma^2 mc^2 - \gamma^2 mv^2$$

so $$m^2c^4 = \gamma^2 m^2 c^4 - \gamma^2 m^2 v^2 c^2 \quad -(7)$$

This is the Einstein energy equation:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(8)$$

where

$$E = \gamma mc^2, \quad -(9)$$

$$p = \gamma mv, \quad -(10)$$

$$E_0 = mc^2 \quad -(11)$$

Eq. (8) is equivalent to:

$$\underline{p} = \gamma m \underline{v}, \quad -(12)$$

the relativistic momentum. The metric (1) therefore generalizes the kinematic theory (3) to the theory of special relativity.

The kinematic equation (3) shows that the Hooke / Newton force law:

$$\underline{F} = -\frac{mMG}{r^2}\,\underline{e}_r \quad -(13)$$

gives the elliptical orbit:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(14)$$

and more generally conical section orbits.

It follows that the metric for R theory is eq. (1) with:

$$v^2 dt^2 = dR^2 + R^2 d\theta^2 \quad -(15)$$

i.e.:

$$\boxed{c^2 d\tau^2 = c^2 dt^2 - dR^2 - R^2 d\theta^2} \quad -(16)$$

Eq. (15) leads to:

$$R = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(17)$$

and

$$\underset{\sim}{F} = -\frac{mMG}{R^2}\,\underline{e}_r \quad -(18)$$

Here:

$$R = r + r_0 \quad -(19)$$

where

$$r_0 = \frac{3MG}{c^2} \quad -(20)$$

The turning point of the Newtonian theory is

$$\frac{d^2r}{dt^2} = 0 \quad -(21)$$

which corresponds to:

$$r = \alpha \quad -(22)$$

where α is the half right latitude.

The turning point of the R theory is:

$$\frac{d^2 R}{dt^2} = 0 \quad -(23)$$

which corresponds to:

$$R = \alpha \quad -(24)$$

i.e.

$$r = \alpha - r_0. \quad -(25)$$

Eq. (25) is also the turning point of the metric:

$$c^2 d\tau^2 = c^2 dt^2 \left(1 - \frac{r_s}{r}\right) - dr^2 \left(1 - \frac{r_s}{r}\right)^{-1} - r^2 d\theta^2 \quad -(26)$$

where

$$r_s = \frac{2MG}{c^2} \quad -(27)$$

Eq. (26) gives eq. (25) as follows. Therefore eq. (26) is merely a complicated way of rewriting eq. (16) to give the same result (25), i.e. to give a precessing ellipse.

By Ockham's Razor, eq. (16) is

s) the preferred theory because it is far simpler than that of the standard model, and is based on a correct geometry, that of the plane polar coordinates, an example of Cartan's geometry.

In ECE theory, eq. (26) has no significance other than the one just described.

In the standard model, eq. (26) is described as the Schwarzschild metric and a solution of the Einstein field equation. However the latter is incorrect geometrically and Schwarzschild did not derive eq. (26).

Defining:

$$E = mc^2 \left(1 - \frac{r_s}{r} \right) \frac{dt}{d\tau}, \quad L = mr^2 \frac{d\theta}{d\tau} \quad -(28)$$

Eq. (26) gives:

$$m \left(\frac{dr}{d\tau} \right)^2 = \frac{E^2}{mc^2} - \left(1 - \frac{r_s}{r} \right) \left(mc^2 + \frac{L^2}{mr^2} \right) \quad -(29)$$

and

$$\left(\frac{dr}{d\theta} \right)^2 = r^4 \left(\frac{1}{b^2} - \left(1 - \frac{r_s}{r} \right) \left(\frac{1}{a^2} + \frac{1}{r^2} \right) \right) \quad -(30)$$

where

$$a = \frac{L}{mc}, \quad b = \frac{cL}{E} \quad -(31)$$

Eq. (26) also gives:

$$\Delta\theta = \int_{R_0}^{\infty} \frac{1}{r^2}\left(\frac{1}{b^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{a^2} + \frac{1}{r^2}\right)\right)^{-1/2} dr$$

$$- \pi \qquad -(32)$$

for the deflection of the orbit of a parabola near an attracting mass M.

Now denote:

$$f = \frac{dr}{d\tau} \qquad -(33)$$

then

$$\frac{df}{d\tau} = \frac{df}{dr}\frac{dr}{d\tau} = \frac{d^2 r}{d\tau^2} \qquad -(34)$$

and

$$\frac{d}{d\tau}\left(\frac{dr}{d\tau}\right)^2 = 2\,\frac{d^2 r}{d\tau^2} \qquad -(35)$$

The turning point for eq. (29) occurs at:

$$\frac{d^2 r}{d\tau^2} = 0 \qquad -(36)$$

i.e. $$\frac{d}{d\tau}\left(\frac{dr}{d\tau}\right)^2 = 0 \qquad -(37)$$

So:

$$\frac{d}{d\tau}\left(\frac{E^2}{mc^2} - \left(1 - \frac{r_0}{r}\right)\left(mc^2 + \frac{L^2}{mr^2}\right)\right) = 0 \qquad -(38)$$

Using:

7)

$$\frac{d}{d\tau}\left(\frac{E^2}{mc^2} - mc^2\right) = 0 \quad -(39)$$

we have:

$$\frac{d}{d\tau}\left(\frac{r_s mc^2}{r} - \frac{L^2}{mr^2} + \frac{r_s L^2}{mr^3}\right) = 0 \quad -(40)$$

Divide by mc^2 and use:

$$\frac{df}{d\tau} = \frac{df}{dr}\frac{dr}{d\tau} \quad -(41)$$

and

$$\frac{dr}{d\tau} \neq 0 \quad -(42)$$

implies that

$$\frac{d}{dr}\left(\frac{r_s}{r} - \frac{L^2}{m^2c^2r^2} + \frac{r_s L^2}{m^2c^2r^3}\right) = 0 \quad -(43)$$

i.e.

$$-\frac{MG}{r^2} + \frac{L^2}{m^2r^3} - \frac{3MGL^2}{m^2c^2r^4} = 0 \quad -(44)$$

This is eq. (16) of note 262(6). It is:

$$r^2 - \alpha r + r_0\alpha = 0 \quad -(45)$$

with roots:

$$r = \alpha - r_0 \quad -(46)$$

and

$$r = r_0 \quad -(47)$$

Eq. (46) is eq. (25) QED.

8)

Conclusion

The theory of orbital precession and light deflection due to gravitation is due to the metric:

$$c^2 d\tau^2 = c^2 dt^2 - v^2 dt^2 \quad -(48)$$

of special relativity, with:

$$v^2 dt^2 = dR^2 + R^2 d\theta^2 \quad -(49)$$

and

$$R = r + \frac{3MG}{c^2} \quad -(50)$$

Everything attributed to the Einstein theory is due to the R theory of Cartan geometry.