263(6): Equivalent Theories of Orbital Precession

1) R Theory

The orbit is described by:

$$R = \frac{\alpha}{1+\epsilon\cos\theta} \quad - (1)$$

where

$$R = r + r_0 \quad - (2)$$

Here α is the half right latitude, ϵ is the eccentricity, (r, θ) is the plane polar coordinate system and

$$r_0 = \frac{3MG}{c^2} \quad - (3)$$

where M is the mass at the centre of the solar system for example, the focus of the ellipse. G is Newton's constant and c the speed of light. This choice of r_0 gives the precession angle

$$\theta = \frac{r_0}{\alpha} \quad - (4)$$

per radian. For one complete orbit of 2π radians

$$\theta = 2\pi\frac{r_0}{\alpha} = \frac{6\pi MG}{\alpha c^2} \quad - (5)$$

It is claimed that this is always observed precisely for all precessions. If this claim is accepted, then eq. (1) is a precise equation of all planar orbital precessions in the universe.

The force law for eq. (1) is:

2)

$$F(R) = -\frac{L^2}{mR^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{R}\right) + \frac{1}{R}\right) = -\frac{mMG}{R^2} \quad -(6)$$

and

$$m\ddot{R} = -\frac{mMG}{R^2} + \frac{L^2}{mR^3} \quad -(7)$$

The force law (6) can be expanded as:

$$F = -\frac{mMG}{(r+r_0)^2} = -\frac{mMG}{r^2}\left(1+\frac{r_0}{r}\right)^{-2}$$

$$\sim -\frac{mMG}{r^2}\left(1-2\frac{r_0}{r}\right) \quad -(8)$$

$$= -\frac{mMG}{r^2} + \frac{2mMGr_0}{r^3}$$

for:

$$r_0 << r. \quad -(9)$$

2) x Theory

The orbit is described by:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(10)$$

with force law:

$$F(r) = -\frac{L^2}{mr^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right)$$

$$= (x^2 - 1)\frac{L^2}{mr^3} - \frac{x^2 L^2}{\alpha m r^2} \quad -(11)$$

where

$$\alpha = \frac{L^2}{m^2 M G} \quad -(12)$$

So

$$F = (x^2 - 1)\frac{L^2}{mr^3} - x^2\frac{mMG}{r^2} \quad -(13)$$

If

$$x \sim 1 \quad -(14)$$

as in the solar system, then comparing eqns (8) and (13) gives:

$$x^2 - 1 = 2\frac{r_0}{\alpha} \quad -(15)$$

So

$$x^2 = 1 + 2\frac{r_0}{\alpha} \quad -(16)$$

If

$$r_0 \ll \alpha \quad -(17)$$

as in the solar system then:

$$x = \left(1 + 2\frac{r_0}{\alpha}\right)^{1/2} \quad -(18)$$

$$\sim 1 + \frac{r_0}{\alpha}$$

So

$$\boxed{x = 1 + \frac{r_0}{\alpha}} \quad -(19)$$

4)

The two theories are the same if:

$$x = 1 + \frac{3MG}{c^2 d} \quad -(20)$$

per radian of orbital rotation. For an angle θ of orbital rotation:

$$x\theta = \left(1 + \frac{3MG}{c^2 d}\right)\theta \quad -(21)$$

For one complete orbit:

$$\theta = 2\pi = 360^\circ \quad -(22)$$

Discussion

The R and x theories are preferred to the Einstein ~~theory~~ because the R and x theories are much simpler and are mathematically correct. The Einstein theory is hugely over complicated, neglects torsion, and is incorrect. The way in which Einstein worked out the orbital precession in 1915 was rejected immediately by K. Schwarzschild in Dec. 1915. A. A. Vankov has recently discovered several more errors in the 1915 Einstein paper.