258(2): Conditions for a Beltrami Potential.

The Beltrami potential is defined as:

$$\underline{\nabla} \times \underline{A}^a = \kappa \underline{A}^a \quad -(1)$$

Therefore:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a) = \kappa \underline{\nabla} \times \underline{A}^a = \kappa^2 \underline{A}^a. \quad -(2)$$

From vector analysis:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a) = \underline{\nabla} (\underline{\nabla} \cdot \underline{A}^a) - \nabla^2 \underline{A}^a. \quad -(3)$$

If

$$\underline{\nabla} \cdot \underline{A}^a = 0 \quad -(4)$$

then:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a) = -\nabla^2 \underline{A}^a \quad -(4).$$

From eqns. (2) and (4):

$$(\nabla^2 + \kappa^2) \underline{A}^a = \underline{0} \quad -(5)$$

Eq. (5) is the Helmholtz wave equation. It is part of the Proca equation:

$$(\square + \kappa^2) A^a_\mu = 0 \quad -(6)$$

where the d'Alembertian operator is:

$$\square = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 \quad -(7)$$

Eq. (6) is derived in ECE theory from the

2) tetrad postulate:

$$D_\mu q^a_\nu = 0 \quad -(8)$$

The ECE hypothesis is:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(9)$$

So

$$D_\mu q^a_\nu = \partial_\mu q^a_\nu + \omega^a_{\mu\nu} - \Gamma^a_{\mu\nu} = 0 \quad -(10)$$

where

$$\omega^a_{\mu\nu} = \omega^a_{\mu b} q^b_\nu \quad -(11)$$

$$\Gamma^a_{\mu\nu} = \Gamma^\lambda_{\mu\nu} q^a_\lambda \quad -(12)$$

It follows that:

$$\partial_\mu A^a_\nu = A^{(0)} \left(\Gamma^a_{\mu\nu} - \omega^a_{\mu\nu} \right) \quad -(13)$$

Therefore:

$$\Box A^a_\nu = \partial^\mu \partial_\mu A^a_\nu = A^{(0)} \partial^\mu \left(\Gamma^a_{\mu\nu} - \omega^a_{\mu\nu} \right) \quad -(14)$$

$$:= -\kappa^2 A^a_\nu$$

So

$$\boxed{\kappa^2 = q^\nu_a \partial^\mu \left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu} \right)} \quad -(15)$$

In the Proca equation:

$$\kappa^2 = \left(\frac{m_0 c}{\hbar} \right)^2 \quad -(16)$$

where m_0 is the photon mass. So the photon

) mass is defined by geometry:

$$m_0^2 = \left(\frac{\hbar}{c}\right)^2 \tilde{q}^{\nu}{}_{a}\, \partial^{\mu}\left(\omega^{a}_{\mu\nu} - \Gamma^{a}_{\mu\nu}\right) \quad -(17)$$

The photon mass is also defined by the Einstein energy equation in the limit of special relativity:

$$E^2 = c^2 p^2 + m_0^2 c^4 \quad -(18)$$

where

$$E = i\frac{\hbar}{c}\frac{\partial}{\partial t}, \quad \underline{p} = -i\hbar\underline{\nabla} \quad -(19)$$

from which it follows that:

$$\left(\Box + \left(\frac{m_0 c}{\hbar}\right)^2\right) A^{a}_{\mu} = 0 \quad -(20)$$

Eq. (20) is also obtained from eq. (14) if:

$$\kappa_0 = \frac{m_0 c}{\hbar} \quad -(21)$$

Using:

$$E = \hbar\omega, \quad \underline{p} = \hbar\underline{\kappa} \quad -(22)$$

eq. (18) becomes:

$$\boxed{\frac{\omega^2}{c^2} - \kappa^2 = \kappa_0^2} \quad -(23)$$

In this notation:

$$A^{a}_{\mu} = \left(A^{a}_{0}, -\underline{A}^{a}\right) \quad -(24)$$

4) Now consider a potential of the type:

$$A^a_\mu = A^a_\mu(0) \exp(i(\omega t - \kappa Z)) \quad -(25)$$

For the Proca equation the relation between ω and κ in eq. (25) is given by eq. (23). In general it is given by:

$$\boxed{\frac{\omega^2}{c^2} - \kappa^2 = q \tilde{V}_a \partial^\mu (\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu})} \quad -(26)$$

so the phase can always be written as in eq. (25). The d'Alembertian can always be written as:

$$\square = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2 \quad -(27)$$

$$= \frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \underline{\nabla} \times (\nabla \times \ )$$

Therefore:

$$\square \underline{A}^a = -\kappa_1^2 \underline{A}^a \quad -(28)$$

or

$$(\square + \kappa_1^2)\underline{A}^a = \underline{0} \quad -(29)$$

where

$$\kappa_1^2 = q \tilde{V}_a \partial^\mu (\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}) \quad -(30)$$

Eq. (29) is:

$$\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \kappa_1^2\right)\underline{A}^a + \underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a) = \underline{0} \quad -(31)$$

Therefore:

5)

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a) = \kappa^2 \underline{A}^a$$
$$= -\left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} + \kappa_1^2\right)\underline{A}^a \quad -(32)$$

If:

$$\underline{A}^a = \underline{A}^a(0) \exp\left(i(\omega t - \kappa Z)\right) \quad -(33)$$

then:

$$\boxed{\begin{aligned}\underline{\nabla} \times (\underline{\nabla} \times \underline{A}^a) &= \left(\frac{\omega^2}{c^2} - \kappa_1^2\right)\underline{A}^a \\ &= \kappa^2 \underline{A}^a\end{aligned}} \quad -(34)$$

Eq. (34) is the result of the tetrad postulate and the ECE hypothesis. So the potential can always be defined as a Beltrami equation (QED). Known analytical solutions emerge when κ^2 is a constant.

The conditions under which:

$$\underline{\nabla} \cdot \underline{A}^a = 0 \quad -(35)$$

must now be found.

Consider the tetrad postulate in the form:

$$D_\mu q^{a\nu} = 0 \quad -(36)$$

$$= \partial_\mu q^{a\nu} + \omega^a_{\mu b} q^{b\nu} - \Gamma^\nu_{\mu\lambda} q^{a\lambda}$$

$$= \partial_\mu q^{a\nu} + \omega^{a\nu}_\mu - \Gamma^{a\nu}_\mu$$

6) Therefore:

$$\partial_\mu A^{a\mu} = A^{(0)} \left(\Gamma^{a\mu}_{\ \mu} - \omega^{a\mu}_{\ \mu} \right) \quad -(37)$$

This means:

$$\frac{1}{c} \frac{\partial}{\partial t} A^{a0} + \underline{\nabla} \cdot \underline{A}^a = A^{(0)} \left(\Gamma^{a\mu}_{\ \mu} - \omega^{a\mu}_{\ \mu} \right) \quad -(38)$$

i.e.

$$\underline{\nabla} \cdot \underline{A}^a = A^{(0)} \left(\Gamma^{a\mu}_{\ \mu} - \omega^{a\mu}_{\ \mu} \right) - \frac{1}{c} \frac{\partial A^{a0}}{\partial t} \quad -(39)$$

Therefore if:

$$\boxed{\frac{1}{c} \frac{\partial A^{a0}}{\partial t} = A^{(0)} \left(\Gamma^{a\mu}_{\ \mu} - \omega^{a\mu}_{\ \mu} \right)} \quad -(40)$$

then

$$\underline{\nabla} \cdot \underline{A}^a = 0. \quad -(41)$$

Under condition (41):

$$\underline{\nabla} \times \left(\underline{\nabla} \times \underline{A}^a \right) = -\nabla^2 \underline{A}^a \quad -(42)$$

In the standard model, the Lorenz condition is

$$\partial_\mu A^\mu = 0 \quad -(43)$$

but in ECE theory this is true only if:

$$\Gamma^{a\mu}_{\ \mu} = \omega^{a\mu}_{\ \mu} \quad -(43)$$

However, if eq. (43) is true, then:

$$\kappa_1^2 = q \tilde{V}_a \partial^\mu \left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu} \right) \quad - (44)$$
$$= 0$$

and from eq. (34):

$$\frac{\omega^2}{c^2} = \kappa^2 \quad - (45)$$

which is true only for the massless photon.

So the Lorenz gauge is incompatible with photon mass. The Proca equation is not gauge invariant.

Note that the assumption of the Beltrami condition:

$$\underline{\nabla} \times \underline{A}^a = \kappa \underline{A}^a \quad - (46)$$

where κ is a constant, means that:

$$\underline{A}^a = \frac{1}{\kappa} \underline{\nabla} \times \underline{A}^a \quad - (47)$$

so:

$$\underline{\nabla} \cdot \underline{A}^a = \frac{1}{\kappa} \underline{\nabla} \cdot \underline{\nabla} \times \underline{A}^a$$
$$= 0 \quad - (48)$$

Therefore condition (48) is always compatible with:

$$\underline{\nabla} \times \left(\underline{\nabla} \times \underline{A}^a \right) = \kappa^2 \underline{A}^a$$
$$= \left(\frac{\omega^2}{\kappa^2} - \kappa_1^2 \right) \underline{A}^a \quad - (49)$$