38(3): Some Consequences of Assuming the Beltrami Equation in the Standard Model.

The Beltrami equation is:

$$\underline{\nabla} \times \underline{B} = \kappa \underline{B} \quad -(1)$$

where κ is a constant. In the standard model:

$$\underline{B} = \underline{\nabla} \times \underline{A} \quad -(2)$$

so

$$\frac{1}{\kappa} \underline{\nabla} \times \underline{B} = \underline{\nabla} \times \underline{A} \quad -(3)$$

and

$$\boxed{\underline{B} = \kappa \underline{A}} \quad -(4)$$

It is immediately obvious that the Beltrami equation contradicts U(1) gauge invariance. This is because $\underline{B}$ is observable, and if:

$$\underline{A} \rightarrow \underline{A} + \underline{\nabla}\chi \quad -(5)$$

then $\underline{B}$ changes. The only possible solution is:

$$\underline{\nabla}\phi = \underline{0} \quad -(6)$$

and $\underline{A}$ is a physical quantity. The U(1) gauge invariance of the standard model collapses completely and with it the Higgs boson theory. The standard explanation of the Aharonov Bohm effect also collapses because solutions of eq. (1) are observable experimentally.

It follows from eqs. (1) to (4) that:

$$\underline{\nabla} \times \underline{A} = \kappa \underline{A} \quad -(7)$$

and

$$\underline{\nabla} \times \underline{B} = \kappa^2 \underline{A} \quad -(8)$$

In the Ampère Maxwell law:

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(9)$$

it follows that:

$$\kappa^2 \underline{A} = \underline{J} + \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} \quad -(10)$$

where

$$\underline{E} = -\underline{\nabla} \phi - \frac{\partial \underline{A}}{\partial t} \quad -(11)$$

Therefore:

$$\kappa^2 \underline{A} = \mu_0 \underline{J} + \frac{1}{c^2} \frac{\partial}{\partial t} \left(-\underline{\nabla} \phi - \frac{\partial \underline{A}}{\partial t} \right) \quad -(12)$$

The standard model assumes the Lorenz condition:

$$\underline{\nabla} \cdot \underline{A} + \frac{1}{c^2} \frac{\partial \phi}{\partial t} = 0 \quad -(13)$$

It follows from eq. (7) that:

$$\underline{\nabla} \cdot \underline{A} = \frac{1}{\kappa} \underline{\nabla} \cdot \underline{\nabla} \times \underline{A} = 0 \quad -(14)$$

so

$$\frac{\partial \phi}{\partial t} = 0 \quad -(15)$$

Therefore eq. (12) becomes:

3)

$$\frac{1}{c^2}\frac{\partial^2 \underline{A}}{\partial t^2} + \kappa^2 \underline{A} = \mu_0 \underline{J} \quad -(16)$$

Using:

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}) = \underline{\nabla}(\underline{\nabla} \cdot \underline{A}) - \nabla^2 \underline{A} = \kappa^2 \underline{A} \quad -(17)$$

eq. (16) becomes the d'Alembert equation:

$$\Box \underline{A} = \left(\frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2\right)\underline{A} = \mu_0 \underline{J} \quad -(18)$$

The solutions of eq. (18) are consistent with the Lorenz condition those of eq. (7) if and only if the Lorenz condition is not is assumed. However, neither is eq. (18). This is U(1) gauge invariant, neither is eq. (18) data because solutions the result of experimental data because solutions of eq. (1) are observed experimentally.

The standard model equations are:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(19)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(20)$$

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(21)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(22)$$

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} \quad -(23)$$

$$\underline{B} = \underline{\nabla} \times \underline{A} \quad -(24)$$

4) From eqs. (21) and (23):

$$\nabla^2 \phi + \frac{\partial}{\partial t} \underline{\nabla} \cdot \underline{A} = -\rho/\epsilon_0 \quad -(25)$$

However, the Beltrami condition implies

$$\underline{\nabla} \cdot \underline{A} = 0 \quad -(26)$$

so it implies the Poisson equation:

$$\nabla^2 \phi = -\rho/\epsilon_0 \quad -(27)$$

From eqs. (22) to (24):

$$\underline{\nabla} \times (\underline{\nabla} \times \underline{A}) - \frac{1}{c^2} \frac{\partial}{\partial t} \left(-\underline{\nabla} \phi - \frac{\partial \underline{A}}{\partial t} \right) = \mu_0 \underline{J}. \quad -(28)$$

i.e.

$$\Box \underline{A} + \underline{\nabla} \left(\underline{\nabla} \cdot \underline{A} + \frac{1}{c^2} \frac{\partial \phi}{\partial t} \right) = \mu_0 \underline{J} \quad -(29)$$

The Beltrami condition implies the Lorenz condition

$$\underline{\nabla} \cdot \underline{A} + \frac{1}{c^2} \frac{\partial \phi}{\partial t} = 0 \quad -(30)$$

if

$$\frac{\partial \phi}{\partial t} = 0 \quad -(31)$$

In general, the Lorenz condition in eq. (25) produces:

$$\Box \phi = \rho / \epsilon_0 \quad -(32)$$

5) and the Lorenz condition in eq. (29) produces:

$$\Box \underline{A} = \mu_0 \underline{J} \quad -(33)$$

With the definitions:

$$A^\mu = (c\phi, \underline{A}) \quad -(34)$$

$$J^\mu = \left(\frac{\rho}{c}, \underline{J}\right) \quad -(35)$$

$$\epsilon_0 \mu_0 = \frac{1}{c^2} \quad -(36)$$

the standard model produces:

$$\Box A^\mu = \mu_0 J^\mu \quad -(37)$$

if and only if:

$$\partial_\mu A^\mu = 0 \quad -(38)$$

which is the covariant form of the Lorenz condition.

However, the Beltrami condition means that the only possible solution of eq. (38) is:

$$\boxed{\underline{\nabla} \cdot \underline{A} = 0, \quad \frac{\partial \phi}{\partial t} = 0} \quad -(39)$$

In the Ampère Maxwell law (22):

$$\underline{\nabla} \times \underline{B} = \kappa^2 \underline{A} \quad -(40)$$

and

$$\frac{\partial \underline{E}}{\partial t} = -\frac{\partial}{\partial t}\left(\underline{\nabla}\phi + \frac{\partial \underline{A}}{\partial t} \right) \quad -(41)$$

leading to

$$\Box \underline{A} = \mu_0 \underline{J} \quad -(42)$$

However:

$$\Box \phi = -\nabla^2 \phi = \rho / \epsilon_0 \quad -(43)$$

because:

$$\frac{\partial \phi}{\partial t} = 0 \quad -(44)$$

The Beltrami condition means that the scalar potential is time independent and that the vector potential is no longer U(1) gauge invariant. The next note will discuss what happens to these equations in ECE theory.
