255(2): Definition of $\underline{B}^{(3)}$ Field and Spin Connection

In general the free space magnetic flux density and electric field strength are defined by:

$$\underline{B}^a = \underline{\nabla} \times \underline{A}^a - \underline{\omega}^a{}_b \times \underline{A}^b \quad -(1)$$

$$\underline{E}^a = c\left(\underline{\nabla} \times (i\underline{A}^a) - \underline{\omega}^a{}_b \times (i\underline{A}^b)\right) \quad -(2)$$

using the Hodge duality of type (1) of note 255(2).

The original $\underline{B}^{(3)}$ field was defined as:

$$\underline{B}^{(3)*} = -\frac{i\kappa}{A^{(0)}} \underline{A}^{(1)} \times \underline{A}^{(2)} \quad -(3)$$

$$= -\frac{i}{B^{(0)}} \underline{B}^{(1)} \times \underline{B}^{(2)}$$

The definition (3) follows from the frame of reference:

$$\underline{e}^{(1)} \times \underline{e}^{(2)} = i\underline{e}^{(3)*} \quad -(4)$$

in the complex circular basis:

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}}(\underline{i} - i\underline{j})e^{i\phi} \quad -(5)$$

$$\underline{e}^{(2)} = \frac{1}{\sqrt{2}}(\underline{i} + i\underline{j})e^{-i\phi}$$

$$\underline{e}^{(3)} = \underline{k}$$

in rotating and translating space. Here:

$$\phi = \omega t - \kappa Z \quad -(6)$$

where ω is the angular frequency at instant t and κ

is the wavenumber at point Z. Therefore:

$$\underline{e}^{(1)} = \underline{e}^{(2)*} \quad -(7)$$

are complex conjugate plane wave unit vectors. They describe a frame that is rotating and translating. Therefore $\underline{\omega}^{a}{}_{b}$ is the spin connection vector of this frame.

Define:

$$\underline{\omega}^{(3)}{}_{(2)} = \frac{\kappa}{2}\,\underline{e}^{(1)} \quad -(8)$$

$$\underline{\omega}^{(3)}{}_{(1)} = -\frac{\kappa}{2}\,\underline{e}^{(2)} \quad -(9)$$

$$\underline{\omega}^{(3)}{}_{(3)} = -\frac{\kappa}{4}\,\underline{e}^{(3)} \quad -(10)$$

then

$$\boxed{\underline{\omega}^{(3)}{}_{(2)} \times \underline{\omega}^{(3)}{}_{(1)} = i\kappa\,\underline{\omega}^{(3)}{}_{(3)}{}^{*}} \quad -(11)$$

The $\underline{B}^{(3)}$ field is defined from eq. (1) by:

$$\underline{B}^{(3)*} = \underline{\nabla} \times \underline{A}^{(3)*} - i\left(\underline{A}^{(1)} \times \underline{\omega}^{(3)}{}_{(1)} - \underline{A}^{(2)} \times \underline{\omega}^{(3)}{}_{(2)} + \underline{A}^{(3)} \times \underline{\omega}^{(3)}{}_{(3)}\right) \quad -(12)$$

with summation over repeated b indices.

However:

$$\underline{\nabla} \times \underline{A}^{(3)*} = \underline{A}^{(3)} \times \underline{\omega}^{(3)}{}_{(3)} = \underline{0} \quad -(13)$$

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So:

$$\underline{B}^{(3)*} = -i\left(\underline{A}^{(1)} \times \underline{\omega}^{(3)}{}_{(1)} - \underline{A}^{(2)} \times \underline{\omega}^{(3)}{}_{(2)}\right)$$

$$= -i\frac{\kappa}{2}\left(\underline{A}^{(1)} \times \underline{e}^{(2)} - \underline{A}^{(2)} \times \underline{e}^{(1)}\right)$$

$$= -i\kappa\,\underline{A}^{(1)} \times \underline{e}^{(2)} \quad -(14)$$

Finally define: $\underline{A}^{(2)} := A^{(0)}\,\underline{e}^{(2)} \quad -(15)$

to find eq. (3), Q.E.D.

The unit vectors are therefore defined as spin connection vectors:

$$\underline{e}^{(1)} = \frac{2}{\kappa}\,\underline{\omega}^{(3)}{}_{(2)} \quad -(16)$$

$$\underline{e}^{(2)} = -\frac{2}{\kappa}\,\underline{\omega}^{(3)}{}_{(1)} \quad -(17)$$

$$\underline{e}^{(3)} = -\frac{4}{\kappa}\,\underline{\omega}^{(3)}{}_{(3)} \quad -(18)$$

Conversely the spin connection vectors are:

$$\underline{\omega}^{(3)}{}_{(2)} = \frac{\kappa}{2}\,\underline{e}^{(1)} = \frac{\kappa}{2\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{i\phi} \quad -(19)$$

$$\underline{\omega}^{(3)}{}_{(1)} = -\frac{\kappa}{2}\,\underline{e}^{(2)} = -\frac{\kappa}{2\sqrt{2}}\left(\underline{i} + i\underline{j}\right)e^{-i\phi} \quad -(20)$$

$$\underline{\omega}^{(3)}{}_{(3)} = -\frac{\kappa}{4}\,\underline{e}^{(3)} = -\frac{\kappa}{4}\,\underline{k} \quad -(21)$$

Self consistently the spin connections rotate and translate the frame of reference. Therefore this is a theory of general relativity.

Note that:

$$\underline{\nabla} \cdot \underline{B}^{(1)} = \underline{\nabla} \cdot \underline{B}^{(2)} = \underline{\nabla} \cdot \underline{B}^{(3)} = 0, \quad -(22)$$

where the free space magnetic flux densities are spin connection vectors:

$$\underline{B}^{(1)} = B^{(0)} \underline{e}^{(1)} = \frac{2B^{(0)}}{\kappa} \underline{\omega}^{(3)}{}_{(2)} \quad -(23)$$

$$\underline{B}^{(2)} = B^{(0)} \underline{e}^{(2)} = -\frac{2B^{(0)}}{\kappa} \underline{\omega}^{(3)}{}_{(1)} \quad -(24)$$

$$\underline{B}^{(3)} = B^{(0)} \underline{e}^{(3)} = -\frac{4B^{(0)}}{\kappa} \underline{\omega}^{(3)}{}_{(3)} \quad -(25)$$

It follows that:

$$\underline{\nabla} \cdot \underline{\omega}^{(a)}{}_{(b)} \times \underline{A}^{(b)} = 0 \quad -(26)$$

using the ECE hypothesis:

$$\underline{A}^{(b)} = A^{(0)} \underline{q}^{(b)} \quad -(27)$$

Eq. (26) is the Cartan identity in free space of E.D.

Therefore the ECE and $\underline{B}^{(3)}$ theories are rigorously self consistent inter alia

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