255(7): Extended Engineering Model, Part 2, The Inhomogeneous Equations

The analysis starts with the Evans Identity:

$$D_\mu \tilde{T}^a_{\nu\rho} + D_\rho \tilde{T}^a_{\mu\nu} + D_\nu \tilde{T}^a_{\rho\mu} := \tilde{R}^a_{\mu\nu\rho} + \tilde{R}^a_{\rho\mu\nu} + \tilde{R}^a_{\nu\rho\mu} \quad -(1)$$

which is:

$$D_\mu T^{a\mu\nu} := R^{a\ \mu\nu}_{\ \mu} \quad -(2)$$

In the ECE engineering model the inhomogeneous field equation is:

$$\boxed{\partial_\mu T^{a\mu\nu} = j^{a\nu}} \quad -(3)$$

where

$$j^{a\nu} = R^{a\ \mu\nu}_{\ \mu} - \omega^a_{\ \mu b} T^{b\mu\nu} \quad -(4)$$

In general:

$$j^{a\nu} \neq 0 \quad -(5)$$

in electromagnetism and as observed experimentally in gravitation.

Note carefully that the experimental constraint of the homogeneous field equation:

$$\omega^a_{\ \mu b} \tilde{T}^{b\mu} = q^b_{\ \mu} \tilde{R}^{a\ \ \mu\nu}_{\ b} \quad -(6)$$

leads to the Hodge dual:

$$\omega^a_{\ \mu b} T^b_{\ \nu\rho} + \omega^a_{\ \rho b} T^b_{\ \mu\nu} + \omega^a_{\ \nu b} T^b_{\ \rho\mu} = R^a_{\ \mu\nu\rho} + R^a_{\ \rho\mu\nu} + R^a_{\ \nu\rho\mu} \quad -(7)$$

2) and does not imply that $j^{a\nu}$ is zero. However in the free field:

$$\left(\omega^a{}_{\mu b} T^{b\mu\nu} = q^b{}_\mu R^a{}_b{}^{\mu\nu}\right)_{\text{free field}} \quad -(8)$$

and:

$$\left(\partial_\mu T^{a\mu\nu}\right)_{\text{free field}} = 0 \quad -(9)$$

The electromagnetic field tensor is for each a:

$$T^{\mu\nu} = \begin{bmatrix} 0 & -T^1(\text{orb}) & -T^2(\text{orb}) & -T^3(\text{orb}) \\ T^1(\text{orb}) & 0 & -T^3(\text{spin}) & T^2(\text{spin}) \\ T^2(\text{orb}) & T^3(\text{spin}) & 0 & -T^1(\text{spin}) \\ T^3(\text{orb}) & -T^2(\text{spin}) & T^1(\text{spin}) & 0 \end{bmatrix} \quad -(10)$$

Therefore in vector notation eq. (3) is:

$$\boxed{\begin{aligned} \underline{\nabla} \cdot \underset{\sim}{T}^a(\text{orb}) &= j^{a0} \quad -(11) \\ \underline{\nabla} \times \underline{T}^a(\text{spin}) - \frac{1}{c}\frac{\partial \underline{T}^a}{\partial t}(\text{orb}) &= \underline{j}^a \quad -(12) \end{aligned}}$$

In electromagnetism they become the Coulomb law:

$$\underline{\nabla} \cdot \underline{E}^a = \rho^a / \epsilon_0 \quad -(13)$$

and the Ampère Maxwell law:

$$\underline{\nabla} \times \underline{B}^a - \frac{1}{c^2}\frac{\partial \underline{E}^a}{\partial t} = \mu_0 \underline{J}^a \quad -(14)$$

) in S.I. units.

The ECE theory generalizes the original laws to any spacetime from Minkowski spacetime.

In eqs (11) and (12):

$$\underline{T}^a(\text{spin}) = \underline{\nabla} \times \underline{q}^a - \underline{\omega}^a{}_b \times \underline{q}^b \quad -(15)$$

$$\underline{T}^a(\text{orb}) = -\underline{\nabla} q^a_0 - \frac{1}{c}\frac{\partial \underline{q}^a}{\partial t} - \omega^a{}_{0b}\,\underline{q}^b + q^b_0\,\underline{\omega}^a{}_b \quad -(16)$$

Eqs. (11) and (12) must always be solved simultaneously with the homogeneous field equations:

$$\underline{\nabla} \cdot \underset{\sim}{T}^a(\text{spin}) = 0 \quad -(17)$$

$$\frac{1}{c}\frac{\partial \underset{\sim}{T}^a(\text{spin})}{\partial t} + \underline{\nabla} \times \underset{\sim}{T}^a(\text{orb}) = \underline{0} \quad -(18)$$

The Cartan identity provides the following constraints:

$$\underline{\nabla} \cdot \underline{\omega}^b{}_c \times \underline{q}^c = 0 \quad -(19)$$

$$\underline{\omega}^a{}_b \cdot \underset{\sim}{T}^b(\text{spin}) = \underline{q}^b \cdot \underline{R}^a{}_b(\text{spin}) \quad -(20)$$

$$\omega^a{}_{0b}\,\underline{T}^b(\text{spin}) + \underline{\omega}^a{}_b \times \underset{\sim}{T}^b(\text{orbital}) \quad -(21)$$

$$= q^b_0\,\underline{R}^a{}_b(\text{spin}) + \underline{q}^b \times \underline{R}^a{}_b(\text{orbital})$$

where:

$$\underline{R}^{a}{}_{b}(\text{spin}) = \underline{\nabla} \times \underline{\omega}^{a}{}_{b} - \underline{\omega}^{a}{}_{c} \times \underline{\omega}^{c}{}_{b} \quad -(22)$$

and:

$$\underline{E}^{a}{}_{b}(\text{orb}) = -\underline{\nabla}\,\omega^{a}{}_{0b} - \frac{1}{c}\frac{\partial \underline{\omega}^{a}{}_{b}}{\partial t} - \omega^{a}{}_{0c}\,\underline{\omega}^{c}{}_{b} + \omega^{c}{}_{0b}\,\underline{\omega}^{a}{}_{c} \quad -(23)$$

There is also a set of antisymmetry constraints.

Example

From eqs. (20), (15) and (22):

$$\underline{\omega}^{a}{}_{b} \cdot \left(\underline{\nabla} \times \underline{q}^{b} - \underline{\omega}^{a}{}_{c} \times \underline{q}^{c} \right) \quad -(24)$$

$$= \underline{q}^{b} \cdot \left(\underline{\nabla} \times \underline{\omega}^{a}{}_{b} - \underline{\omega}^{a}{}_{c} \times \underline{\omega}^{c}{}_{b} \right).$$

Using:

$$\underline{\omega}^{a}{}_{b} \cdot \underline{\omega}^{a}{}_{c} \times \underline{q}^{c} = \underline{q}^{b} \cdot \underline{\omega}^{a}{}_{c} \times \underline{\omega}^{c}{}_{b} \quad -(25)$$

it is found that:

$$\underline{\omega}^{a}{}_{b} \cdot \underline{\nabla} \times \underline{q}^{b} = \underline{q}^{b} \cdot \underline{\nabla} \times \underline{\omega}^{a}{}_{b} \quad -(26)$$

The Cartan Identity is:

$$\underline{\nabla} \cdot \underline{\omega}^{b}{}_{c} \times \underline{q}^{c} = \underline{\omega}^{a}{}_{b} \cdot \underline{\nabla} \times \underline{q}^{b} - \underline{q}^{b} \cdot \underline{\nabla} \times \underline{\omega}^{a}{}_{b} \quad -(27)$$

So eq. (26) implies:

$$\underline{\nabla} \cdot \underline{\omega}^{b}{}_{c} \times \underline{q}^{c} = 0 \quad -(28)$$

self consistently, QED