1) 255(3): Proof that Zero Torsion means Zero Curvature

Consider the vector format of the Cartan identity derived in UFT 254:

$$\underline{\nabla} \cdot \underset{\sim}{T}^a + \underline{\omega}^a{}_b \cdot \underset{\sim}{T}^b := \underline{q}^b \cdot \underline{R}^a{}_b \quad - (1)$$

If torsion is neglected:

$$\underset{\sim}{T}^a =? \underline{0} \quad - (2)$$

then

$$\underline{q}^b \cdot \underline{R}^a{}_b = 0 \quad - (3)$$

A possible solution of eq. (3) is:

$$\underline{R}^a{}_b = \underline{0} \quad - (4)$$

So if torsion is zero, curvature is zero, QED.

This conclusion is alone enough to refute the entire era of Einsteinian general relativity.

If it is argued that

$$\underline{q}^b \cdot \underline{R}^a{}_b = 0 \quad - (5)$$

then in tensor notation this is equivalent to the obsolete first Bianchi identity:

$$R^a{}_{\mu\nu\rho} + R^a{}_{\rho\mu\nu} + R^a{}_{\nu\rho\mu} = 0 \quad - (6)$$

Eq. (6) means that in the obsolete Einsteinian era:

$$\underline{q}^b \cdot \underline{R}^a{}_b = 0, \quad - (7)$$

$$\underline{R}^a{}_b \neq \underline{0} \quad - (8)$$

$$\underset{\sim}{T}^a = \underline{0} \quad - (9)$$

By definition eq. (7) means that:

2)

$$q^b \cdot \left(\underline{\nabla} \times \underline{\omega}^a{}_b - \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \right) = 0. \quad -(10)$$

The first Bianchi identity is therefore:

$$\underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b = \underline{q}^b \cdot \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \quad -(11)$$

In tensor notation eq. (10) is:

$$R^a{}_{\mu\nu\rho} + R^a{}_{\rho\mu\nu} + R^a{}_{\nu\rho\mu} \quad -(12)$$
$$= q^b{}_\mu \left(R^a{}_{b\nu\rho} + R^a{}_{b\mu\nu} + R^a{}_{b\rho\mu} \right)$$

which is the antisymmetrized tensor product equivalent to

$$R^a{}_b \wedge q^b = q^b \wedge R^a{}_b = 0 \quad -(13)$$

In short hand eq. (13) in differential form notation is

$$R \wedge q = q \wedge R = 0. \quad -(14)$$

Note carefully that the correct Cartan identity is:

$$D \wedge T := q \wedge R \quad -(15)$$

In vector notation eq. (15) is:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{q}^b = \underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{q}^b - \underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b \quad -(16)$$

The Bianchi identity (14) assume zero torsion:

$$\underset{\sim}{T}^a = \underline{\nabla} \times \underline{q}^a - \underline{\omega}^a{}_b \times \underline{q}^b = \underline{0} \quad -(17)$$

i.e.

$$\underline{\nabla} \times \underline{q}^b = \underline{\omega}^b{}_c \times \underline{q}^c \quad -(18)$$

From eq. (18) in eq. (16):

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{q}^b = \underline{\omega}^a{}_b \cdot \underline{\omega}^b{}_c \times \underline{q}^c - \underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b \quad -(19)$$

in which:

$$\underline{\omega}^a{}_b \cdot \underline{\omega}^b{}_c \times \underline{q}^c = \underline{q}^b \cdot \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b . \quad -(20)$$

From eqs. (19) and (20):

$$\underline{q}^b \cdot \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b = \underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{q}^b + \underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b \quad -(21)$$

but from eq. (11):

$$\underline{q}^b \cdot \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b = \underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b \quad -(22) .$$

Comparing eqs. (21) and (22) gives:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{q}^b = 0 \quad -(23)$$

Eq. (23) means that the first Bianchi identity implies:

$$\underline{\nabla} \cdot \underline{B}^a = 0 \quad -(24)$$

which is the result also obtained by:

$$\underline{\omega}^a{}_b \cdot \underset{\sim}{T}^b = \underline{q}^b \cdot \underline{R}^a{}_b \quad -(25)$$

in previous notes.

The first Bianchi identity implies that

$${}^a{}_b \cdot \left(\underline{\nabla} \times \underline{q}^b - \underline{\omega}^b{}_c \times \underline{q}^c \right) = 0 \quad -(26)$$

4) So:

$$\underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{q}^b = \underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b \quad -(27)$$

which is eq. (23), QED.

The analysis is therefore rigorously self consistent.

From eqs. (11) and (27):

$$\underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b = \underline{q}^b \cdot \underline{\omega}^a{}_c \times \underline{\omega}^c{}_b \quad -(28)$$
$$= \underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{q}^b$$

Therefore the first Bianchi identity can be written as:

$$\underline{q}^b \cdot \underline{R}^a{}_b = 2\,\underline{\omega}^a{}_b \cdot \underline{\nabla} \times \underline{q}^b$$
$$= 2\,\underline{q}^b \cdot \underline{\nabla} \times \underline{\omega}^a{}_b = 0 \quad -(29)$$

If there is no gravitomagnetic field then

$$\underline{\nabla} \times \underline{q}^b = \underline{\omega}^b{}_c \times \underline{q}^c \quad -(30)$$

and

$$\underline{q}^b \cdot \underline{R}^a{}_b = 0. \quad -(31)$$

The commutator method can be used to prove

) Flat:

$$\underline{R}^a{}_b = \underline{0} \quad -(32)$$

if

$$\underset{\sim}{T}^a = \underline{0} \quad -(33)$$

as in note 254(4) and UFT139. This leads to the result (4), which is consistent with:

$$\underline{v}^b \cdot \underline{R}^a{}_b = 0 \quad -(34)$$

The first Bianchi identity may also be written as:

$$\underline{\nabla} \cdot \underline{\omega}^a{}_b \times \underline{v}^b = 0 \quad -(35)$$

which implies the absence of the gravitational equivalent of the magnetic monopole.

Eq. (1) is a very clear demonstration that if the torsion is zero the curvature is zero, thus confirming that the commutator method is correct. The assumption:

$$\underset{\sim}{T}^a = \underline{0} \quad -(36)$$

implies eq. (35), which also implies:

$$\underline{\omega}^a{}_b \cdot \underset{\sim}{T}^b = \underline{v}^b \cdot \underline{R}^a{}_b \quad -(37)$$
$$= \underline{0}$$

self consistently.

The first Bianchi identity was devised in an era when torsion was not known and [illegible] be written as:

$$R^a{}_{\mu\nu\rho} = R^a{}_{\rho\mu\nu} = R^a{}_{\nu\rho\mu} = 0 \quad -(38\ \cancel{23})$$

The correct identities must always be the Cartan identity:

$$D_\mu T^a{}_{\nu\rho} + D_\rho T^a{}_{\mu\nu} + D_\nu T^a{}_{\rho\mu} := R^a{}_{\mu\nu\rho} + R^a{}_{\rho\mu\nu} + R^a{}_{\nu\rho\mu} \quad -(39\ \cancel{24})$$

and the Evans identity:

$$D_\mu \tilde{T}^a{}_{\nu\rho} + D_\rho \tilde{T}^a{}_{\mu\nu} + D_\nu \tilde{T}^a{}_{\rho\mu} := \tilde{R}^a{}_{\mu\nu\rho} + \tilde{R}^a{}_{\rho\mu\nu} + \tilde{R}^a{}_{\nu\rho\mu} \quad -(40\ \cancel{25})$$

These have been proven in all detail in the UFT papers.

The vector notation for eqs. (39 $\cancel{24}$) and (40 $\cancel{25}$) is:

$$\underline{\nabla} \cdot \underline{T}^a + \underline{\omega}^a{}_b \cdot \underline{T}^b := \underline{q}^b \cdot \underline{R}^a{}_b \quad -(41\ \cancel{26})$$

and

$$\underline{\nabla} \cdot \underline{\tilde{T}}^a + \underline{\omega}^a{}_b \cdot \underline{\tilde{T}}^b := \underline{q}^b \cdot \underline{\tilde{R}}^a{}_b \quad -(42\ \cancel{27})$$

for spacelike indices:

$$\mu, \nu, \rho = i, j, k = 1, 2, 3 \quad -(43\ \cancel{28})$$

i.e.

$$D_1 T^a{}_{23} + D_2 T^a{}_{31} + D_3 T^a{}_{12} := R^a{}_{123} + R^a{}_{231} + R^a{}_{312} \quad -(44\ \cancel{29})$$

5) and:

$$D_1 \tilde{T}^a_{\;23} + D_2 \tilde{T}^a_{\;31} + D_3 \tilde{T}^a_{\;12} := \tilde{R}^a_{\;123} + \tilde{R}^a_{\;231} + \tilde{R}^a_{\;312} \qquad -\left(\overset{45}{\cancel{30}}\right)$$

When the time like or 0 index of eqs. $(\overset{39}{\cancel{34}})$ and $(\overset{40}{\cancel{25}})$ is considered, the cyclic sum $(\overset{44}{\cancel{29}})$ becomes:

$$D_0 T^a_{\;23} + D_2 T^a_{\;30} + D_3 T^a_{\;02} := R^a_{\;023} + R^a_{\;230} + R^a_{\;302} \qquad -\left(\overset{46}{\cancel{31}}\right)$$

$$D_0 T^a_{\;31} + D_1 T^a_{\;03} + D_3 T^a_{\;10} := R^a_{\;031} + R^a_{\;103} + R^a_{\;310}$$

$$D_0 T^a_{\;12} + D_1 T^a_{\;20} + D_2 T^a_{\;01} := R^a_{\;012} + R^a_{\;120} + R^a_{\;201}$$

and its Hodge dual equivalent.

For eqs. $(\overset{46}{\cancel{34}})$ it is again true that if torsion is zero, curvature is zero. This will be discussed in the next note.
