252(7) : Evaluation of Hamiltonians, Part 2.

The main hamiltonian under consideration is:

$$\hat{H}\phi = \frac{e}{4m^2c^2}\left(\underline{r}\cdot\underline{p} + i\underline{\sigma}\cdot\underline{L}\right)\frac{\phi}{r^2}\left(\underline{r}\cdot\underline{p} + i\underline{\sigma}\cdot\underline{L}\right)\phi \quad -(1)$$

consisting of the following four hamiltonians:

$$\hat{H}_1\phi = \frac{e}{4m^2c^2}\,\underline{r}\cdot\underline{p}\left(\frac{\phi}{r^2}\underline{r}\cdot\underline{p}\,\phi\right) \quad -(2)$$

$$\hat{H}_2\phi = \frac{ie}{4m^2c^2}\,\underline{\sigma}\cdot\underline{L}\left(\frac{\phi}{r^2}\underline{r}\cdot\underline{p}\,\phi\right) \quad -(3)$$

$$\hat{H}_3\phi = \frac{ie}{4m^2c^2}\,\underline{r}\cdot\underline{p}\left(\frac{\phi}{r^2}\underline{\sigma}\cdot\underline{L}\,\phi\right) \quad -(4)$$

$$\hat{H}_4\phi = -\frac{e}{4m^2c^2}\,\underline{\sigma}\cdot\underline{L}\left(\frac{\phi}{r^2}\underline{\sigma}\cdot\underline{L}\,\phi\right) \quad -(5)$$

Hamiltonians $\hat{H}_1$ and $\hat{H}_4$ have been worked out in previous notes for UFT 252.

Hamiltonian $\hat{H}_2$

This is:

$$\hat{H}_2\phi = \frac{ie}{4m^2c^2}\frac{\phi}{r^2}\,\underline{r}\cdot\underline{p}\;\underline{\sigma}\cdot\underline{L}\,\phi \quad -(6)$$

where

$$\underline{S} = \frac{1}{2}\hbar\,\underline{\sigma} \quad -(7)$$

so:

2)

$$\hat{H}_2 \psi = \frac{ie}{4m^2c^2} \frac{\phi}{r^2} \underline{r} \cdot \underline{p} \frac{2}{\hbar^2} \underline{S} \cdot \underline{L} \psi \quad - (8)$$

$$= \frac{ie\hbar}{4m^2c^2} \left(j(j+1) - l(l+1) - s(s+1) \right) \frac{\phi}{r^2} \underline{r} \cdot \underline{p} \psi .$$

where

$$\phi = - \frac{e}{4\pi\epsilon_0 r} \quad - (9)$$

and

$$\underline{r} \cdot \underline{p} \psi = - i\hbar r \frac{d\psi}{dr} . \quad - (10)$$

Therefore:

$$\hat{H}_2 = \frac{-e^2\hbar^2}{16\pi\epsilon_0 m^2c^2} \left(j(j+1) - l(l+1) - s(s+1) \right) \frac{1}{r^2} \frac{d\psi}{dr}$$

$$- (11)$$

The energy expectation values are:

$$E = \frac{-e^2\hbar^2}{16\pi\epsilon_0 m^2c^2} \left(j(j+1) - l(l+1) - s(s+1) \right) \int \frac{\psi^*}{r^2} \frac{d\psi}{dr} d\tau$$

$$- (12)$$

These can now be evaluated by computer algebra.