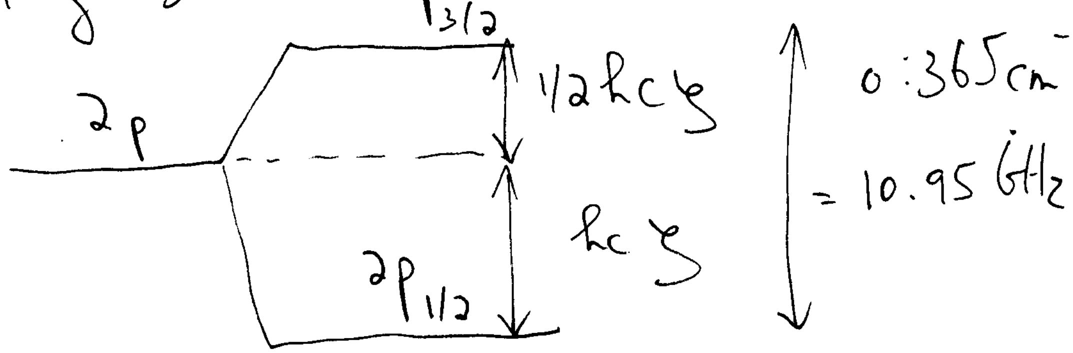
ESO(6): Effect of ESOR on the 2p Doublet of Atomic H

It is well known that the 2p line of atomic H is split by the conventional spin orbit coupling hamiltonian of the form equation to give the energy levels:

$$E_{SO} = \frac{1}{2} hc\,\zeta_{nl} \left(j(j+1) - l(l+1) - s(s+1) \right) \quad (1)$$

where ζ_{nl} is the conventional spin orbit coupling constant. This conventional spin orbit splitting is depicted as in Fig (1):

Fig (1)



The upper level is defined by:

$$j = 3/2,\ l = 1,\ s = 1/2, \quad -(1)$$

$$m_j = -j, \ldots, j = -3/2, 1/2, 3/2$$

and the lower level by

$$j = 1/2,\ l = 1,\ s = -1/2 \quad -(2)$$

$$m_j = -\frac{1}{2}, \frac{1}{2}$$

So the upper level is triply degenerate and the lower level is doubly degenerate. In case (1):

$$j(j+1) - l(l+1) - s(s+1) = 1 \quad -(3)$$

and in case (2):

$$j(j+1) - l(l+1) - s(s+1) = -2 \quad -(4)$$

2) thus giving the results in Fig. (1).

The ESOR Hamiltonian is:

$$\hat{H}\phi = -\frac{e}{2m}\underline{\sigma}\cdot\underline{L}\,\underline{\sigma}\cdot\underline{B}\,\phi \quad -(5)$$

$$= -\frac{e}{\hbar m}\underline{\hat{S}}\cdot\underline{\hat{L}}\,\phi\,\underline{\sigma}\cdot\underline{B}$$

The energy levels from eq. (5) are in consequence:

$$E = -\frac{e\hbar}{2m}\left(j(j+1)-l(l+1)-s(s+1)\right)\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}B_Z \quad -(6)$$

Therefore the ESOR energy levels are:

$$E\left({}^2P_{3/2}\right) = -\frac{e\hbar}{2m}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}B_Z \quad -(7)$$

$$E\left({}^2P_{1/2}\right) = \frac{e\hbar}{m}\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix}B_Z \quad -(8)$$

i.e.

$$E\left({}^2P_{3/2}\right) = -\frac{e\hbar B_Z}{2m},\ \frac{e\hbar B_Z}{2m} \quad -(9)$$

$$E\left({}^2P_{1/2}\right) = \frac{e\hbar B_Z}{m},\ -\frac{e\hbar B_Z}{m} \quad -(10)$$

These additional splittings due to ESOR are sketched in Fig. (2):

3)

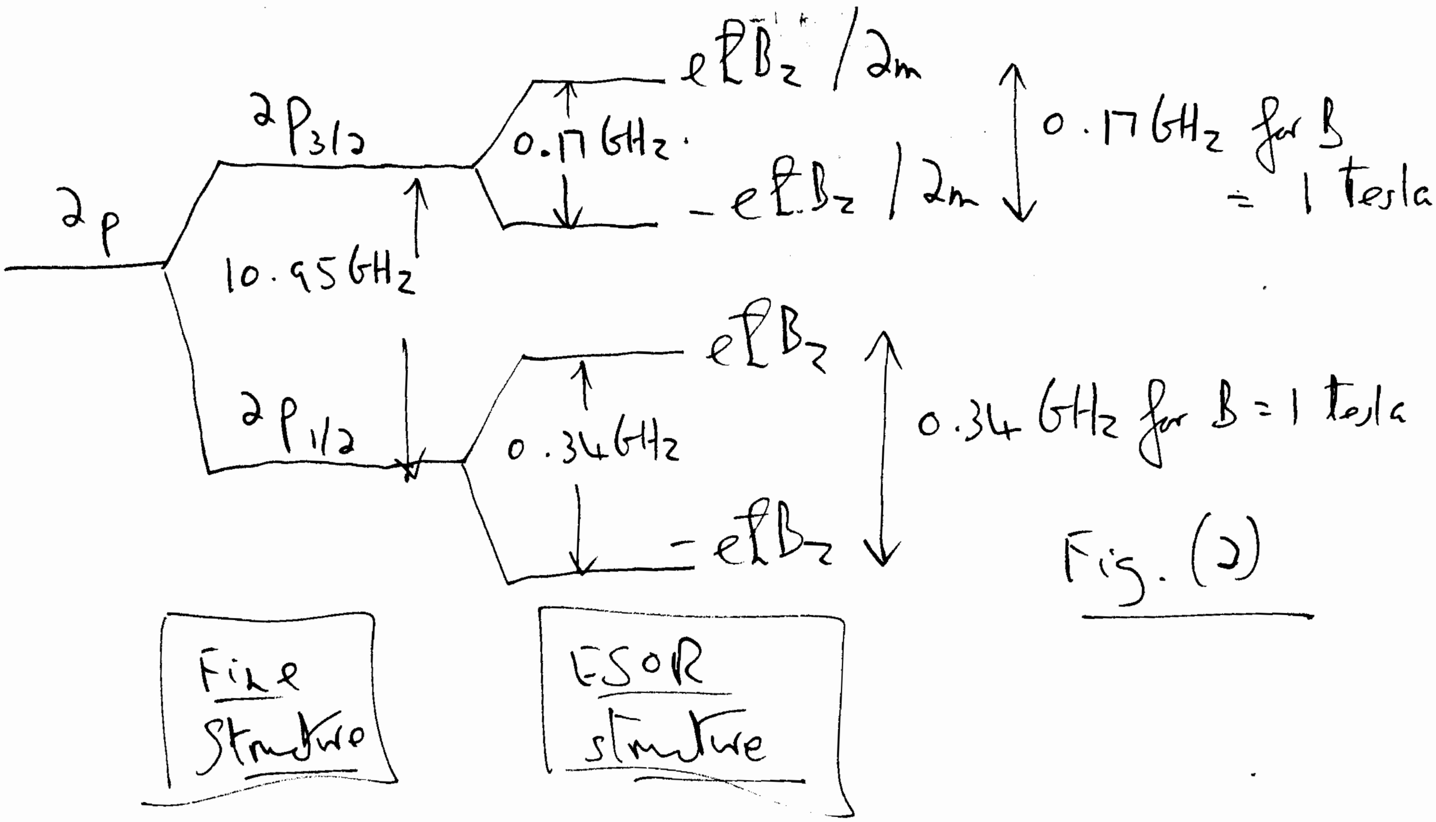


Fig. (2)

It is seen that the ESOR resonance frequency for $^2P_{3/2}$ is the same as the ESR frequency of an electron in a static magnetic field:

$$^2P_{3/2}: \quad \omega_{ESOR} = \frac{eB}{m} \quad -(11)$$

but for $^2P_{1/2}$ the ESOR frequency is twice the ESR frequency

$$^2P_{1/2}: \quad \omega_{ESOR} = \frac{2eB}{m} \quad -(12)$$

These results are capable of great development