250(8): Evaluation of the ESOR Hamiltonian under Isotropic Conditions.

The derivation starts with the Hamiltonian from the fermion or chiral Dirac equations:

$$H\phi = -\frac{e}{2m}(\underline{p}\cdot\underline{A}+\underline{A}\cdot\underline{p})\phi \quad -(1)$$

Now use the Pauli algebra:

$$\underline{p}\cdot\underline{A} = \frac{1}{r^2}\,\underline{\sigma}\cdot\underline{L}\;\underline{\sigma}\cdot\underline{A}\times\underline{r} \quad -(2)$$

$$\underline{A}\cdot\underline{p} = \frac{1}{r^2}\,\underline{\sigma}\cdot\underline{A}\times\underline{r}\;\underline{\sigma}\cdot\underline{L} \quad -(3)$$

to find that:

$$H\phi = -\frac{e}{mr^2}\,\underline{\sigma}\cdot\underline{A}\times\underline{r}\;\underline{\sigma}\cdot\underline{L}\,\phi \quad -(4)$$

Introduce:

$$\underline{S} = \frac{1}{2}\hbar\,\underline{\sigma} \quad -(5)$$

so that:

$$H\phi = -\frac{2e}{\hbar mr^2}\,\underline{\sigma}\cdot\underline{A}\times\underline{r}\;\underline{S}\cdot\underline{L}\,\phi \quad -(6)$$

where

$$\underline{S}\cdot\underline{L}\,\phi = \frac{\hbar^2}{2}\left(j(j+1)-l(l+1)-s(s+1)\right)\phi \quad -(7)$$

The energy eigenvalues are therefore:

$$E = -\frac{e\hbar}{m}\left(j(j+1)-l(l+1)-s(s+1)\right)\frac{\underline{\sigma}\cdot\underline{A}\times\underline{r}}{r^2} \quad -(8)$$

2) where:

$$j = l + s, \ldots, |l - s| \quad -(9)$$

In deriving eqs. (2) and (3) use has been made of:

$$\underline{A} = \frac{1}{2} \underline{B} \times \underline{r} \quad -(10)$$

So

$$\underline{A} \times \underline{r} = (\underline{B} \times \underline{r}) \times \underline{r}$$

$$= \underline{r}(\underline{r} \cdot \underline{B}) - \underline{B} r^2 \quad -(11)$$

So:

$$\boxed{\begin{aligned} H\phi &= \frac{e}{2\pounds m} \underline{\sigma} \cdot \left(\underline{B} - \frac{\underline{r}}{r^2} (\underline{r} \cdot \underline{B}) \right) \underline{\hat{S}} \cdot \underline{\hat{L}} \phi \\ &= -\frac{e}{2m} (\underline{p} \cdot \underline{A} + \underline{A} \cdot \underline{p}) \phi \end{aligned}} \quad -(12)$$

If the magnetic field is aligned in Z then:

$$\underline{\sigma} \cdot \frac{\underline{r}}{r^2} (\underline{r} \cdot \underline{B}) = \frac{\sigma_Z Z^2 B_Z}{x^2 + y^2 + Z^2} \quad -(13)$$

Finally assume that the sample is isotropic so on average:

$$\frac{Z^2}{x^2 + y^2 + Z^2} = \frac{1}{3} \quad -(14)$$

4) So

$$H\phi = \frac{1}{3}\frac{e}{\hbar m}\sigma_Z B_Z \underline{S}\cdot\underline{L}\,\phi \qquad -(15)$$

$$= \frac{1}{6}\frac{e\hbar}{m}\sigma_Z B_Z\left(j(j+1) - l(l+1) - s(s+1)\right)\phi$$

Resonance occurs at $\qquad -(16)$

$$\boxed{\omega = \frac{1}{3}\frac{eB_Z}{m}\left(j(j+1) - l(l+1) - s(s+1)\right)}$$

where

$$j = l+s, \ldots, |l-s| \qquad -(17)$$

This is electron spin orbit resonance (ESOR) in an isotropic sample.

The electric dipole selection rules are the same as for the conventional description of the Zeeman effect:

$$\Delta l = \pm 1, \; \Delta m_l = 0, \pm 1 \qquad -(18)$$

The conventional method of developing eq. (1) is:

$$H\phi = -\frac{e}{2m}\underline{L}\cdot\underline{B}\,\phi \qquad -(19)$$

$$= -\frac{e\hbar}{2m} m_L B_Z\,\phi$$

$$m_L = -L, \ldots, L$$