248(7): Complex Valued Vector Potential in the Fermion Equation (Chiral Representation of the Dirac Equation)

For a real valued vector potential the fermion equation is:

$$\left((E - e\phi) + c\underline{\sigma}\cdot(\underline{p} - e\underline{A})\right)\phi^L = mc^2\phi^R \quad -(1)$$

$$\left((E - e\phi) - c\underline{\sigma}\cdot(\underline{p} - e\underline{A})\right)\phi^R = mc^2\phi^L \quad -(2)$$

but for a complex valued vector potential:

$$\left((E - e\phi) + c\underline{\sigma}\cdot(\underline{p} - e\underline{A})\right)\phi^L = mc^2\phi^R \quad -(3)$$

$$\left((E - e\phi) - c\underline{\sigma}\cdot(\underline{p} - e\underline{A}^*)\right)\phi^R = mc^2\phi^L \quad -(4)$$

New experimentally measurable effects result from eqns (3) and (4).

The latter can be combined to give:

$$\left((E-e\phi) + c\underline{\sigma}\cdot(\underline{p} - e\underline{A})\right)\left((E-e\phi) - c\underline{\sigma}\cdot(\underline{p} - e\underline{A}^*)\right)\phi^R = m^2c^4\phi^R \quad -(5)$$

Denote:

$$\psi := \phi^R \quad -(6)$$

for ease of notation. Then eq. (5) becomes:

$$\left((E - e\phi)^2 - m^2c^4\right)\psi = c^2\underline{\sigma}\cdot(\underline{p} - e\underline{A})\underline{\sigma}\cdot(\underline{p} - e\underline{A}^*)\psi \quad -(6)$$

$$+\left(c\sigma\cdot(\underline{p} - e\underline{A})(E - e\phi) - (E - e\phi)c\underline{\sigma}\cdot(\underline{p} - e\underline{A}^*)\right)\psi$$

2) The use of a complex $\underline{A}^*$ produces extra terms. If $\underline{A}$ were real valued then eq. (6) would be:

$$\left((E - e\phi)^2 - m^2c^4\right)\phi = c^2\underline{\sigma}\cdot(\underline{p} - e\underline{A})\underline{\sigma}\cdot(\underline{p} - e\underline{A})\phi \quad (7)$$

and eq. (7) was effectively the equation used by Dirac to produce a variety of experimental effects with precision. Therefore eq. (6) will produce hitherto unknown effects.

In order to solve eq. (6) it is assumed in the manner of Dirac that on the left hand side

$$E = i\hbar\frac{\partial}{\partial t} \quad -(8)$$

and on the right hand side:

$$\underline{p} = -i\hbar\underline{\nabla}, \quad E \sim mc^2 \quad -(9)$$

These assumptions are made after linearizing eq. (6) as follows. It is first written as:

$$(E - e\phi - mc^2)(E - e\phi + mc^2)\phi \quad -(10)$$

$$= c^2\underline{\sigma}\cdot(\underline{p} - e\underline{A})\underline{\sigma}\cdot(\underline{p} - e\underline{A}^*)\phi$$

$$+ c(E - e\phi)\left(\underline{\sigma}\cdot(\underline{p} - e\underline{A}) - \underline{\sigma}\cdot(\underline{p} - e\underline{A}^*)\right)\phi$$

$$= c^2\underline{\sigma}\cdot(\underline{p} - e\underline{A})\underline{\sigma}\cdot(\underline{p} - e\underline{A}^*)\phi$$

$$+ ec(E - e\phi)\underline{\sigma}\cdot(\underline{A}^* - \underline{A})\phi$$

Eq. (10) describes the Dirac electron or ECE electron interacting with an electromagnetic field. Dirac considered an electron interacting with a static electric field and a static magnetic field. In ECE physics the $B^{(3)}$ field is produced by the conjugate product $\underline{A} \times \underline{A}^*$. The latter was not considered by Dirac.

Eq. (10) can be written as:

$$(E - e\phi - mc^2)\phi = \left(\frac{c^2 \underline{\sigma} \cdot (\underline{p} - e\underline{A})\, \underline{\sigma} \cdot (\underline{p} - e\underline{A}^*)}{E - e\phi + mc^2} \right) \phi$$
$$+ \left(\frac{ec(E - e\phi)}{E - e\phi + mc^2} \right) \underline{\sigma} \cdot (\underline{A}^* - \underline{A})\, \phi \quad -(11)$$

Rearranging eq. (11) gives:

$$E\phi = (e\phi + mc^2)\phi$$
$$+ \frac{1}{2m} \left(\underline{\sigma} \cdot (\underline{p} - e\underline{A}) \left(1 - \frac{e\phi}{2mc^2} \right)^{-1} \underline{\sigma} \cdot (\underline{p} - e\underline{A}^*) \right) \phi$$
$$+ \frac{e}{2mc} (mc^2 - e\phi) \left(1 - \frac{e\phi}{2mc^2} \right)^{-1} \underline{\sigma} \cdot (\underline{A}^* - \underline{A})\, \phi \quad -(12)$$

In the Dirac type approximation:

4)

$$e\phi << mc^2 \quad -- (13)$$

so eq. (12) becomes:

$$\hat{E}\psi = i\hbar \frac{d\psi}{dt} = (H_1 + H_2 + H_3)\psi. \quad -(14)$$

In the Schroedinger type representation:

$$(\hat{H}_1 + \hat{H}_2 + \hat{H}_3)\psi = E\psi \quad -(15)$$

where:

$$\hat{H}_1 = e\phi + mc^2 \quad -(16)$$

$$\hat{H}_2 = \frac{1}{2m} \underline{\sigma} \cdot (\underline{p} - e\underline{A})\left(1 + \frac{e\phi}{2mc^2}\right) \underline{\sigma} \cdot (\underline{p} - e\underline{A}^*) \quad -(17)$$

$$\hat{H}_3 = \frac{1}{2} ec \left(1 + \frac{e\phi}{2mc^2}\right) \underline{\sigma} \cdot (\underline{A}^* - \underline{A}) \quad -(18)$$

with:

$$\underline{p} = -i\hbar \underline{\nabla} \quad -(19)$$

In general, $\underline{\nabla}$ can operate on both ϕ and $\underline{A}$, so new observable effects are produced only writing eq. (1) as it is.

Note carefully that eq. (18) leads to a new type of Zeeman, ESR or NMR effect of a fermion in an electromagnetic field.

5) This term will be developed later in this note.

Firstly however develop eq. (17) as:

$$\hat{H}_{21}\phi = \frac{1}{2m}\left(\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla} - e\underline{A}\right)\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla} - e\underline{A}^*\right)\right)\phi$$

$$= \frac{1}{2m}\Big[\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla}\right)\underline{\sigma}\cdot\left(-e\underline{A}^*\right)$$
$$+ \underline{\sigma}\cdot\left(-i\hbar\underline{\nabla}\right)\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla}\right)$$
$$+ \underline{\sigma}\cdot\left(-e\underline{A}\right)\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla}\right)$$
$$+ \underline{\sigma}\cdot\left(-e\underline{A}\right)\underline{\sigma}\cdot\left(-e\underline{A}^*\right)\Big]\phi \quad -(20)$$

$$= \frac{1}{2m}\Big[ie\hbar\left(\underline{\nabla}\cdot\underline{A}^* + i\underline{\sigma}\cdot\left(\underline{\nabla}\times\underline{A}^*\right)\right)$$
$$-\hbar^2\left(\nabla^2 + i\underline{\sigma}\cdot\underline{\nabla}\times\underline{\nabla}\right)$$
$$+e^2\left(\underline{A}\cdot\underline{A}^* + i\underline{\sigma}\cdot\underline{A}\times\underline{A}^*\right)$$
$$+ie\hbar\left(\underline{A}\cdot\underline{\nabla} + i\underline{\sigma}\cdot\left(\underline{A}\times\underline{\nabla}\right)\right)\Big]\phi \quad -(21)$$

There is also:

$$\hat{H}_{22}\phi = \frac{e}{4m^2c^2}\left(\underline{\sigma}\cdot\left(\underline{p} - e\underline{A}\right)\phi\,\underline{\sigma}\cdot\left(\underline{p} - e\underline{A}^*\right)\right)\phi$$

to be developed in other notes. When $\underline{A}$ is real $\hat{H}_{22}$ gives the Thomas factor, spin orbit coupling and the Darwin term.

6) Eq. (20) reduces to:

$$\hat{H}_{21}\phi = \frac{1}{2m}\Big[-\hbar^2\nabla^2\phi + e^2(\underline{A}\cdot\underline{A}^* + i\underline{\sigma}\cdot\underline{A}\times\underline{A}^*)\phi$$

$$+ ie\hbar\,\underline{\nabla}\cdot(\underline{A}^*\phi) + ie\hbar\,\underline{A}\cdot\underline{\nabla}\phi$$

$$- e\hbar\,\underline{\sigma}\cdot\underline{A}\times\underline{\nabla}\phi - e\hbar\,\underline{\sigma}\cdot\underline{\nabla}\times(\underline{A}^*\phi)\Big] \quad -(22)$$

This can be written as:

$$\hat{A}_{21}\phi = \frac{1}{2m}\Big[ie^2\underline{\sigma}\cdot\underline{A}\times\underline{A}^*\phi - e\hbar\,\underline{\sigma}\cdot\underline{A}\times\underline{\nabla}\phi$$

$$- e\hbar\,\underline{\sigma}\cdot\underline{\nabla}\phi\times\underline{A}^* - e\hbar\,\underline{\sigma}\cdot(\underline{\nabla}\times\underline{A}^*)\phi$$

$$+ \dots\Big] \quad -(23)$$

There are four terms from eq. (23) that can give novel Zeeman type effects. In addition there is a fifth term from eq. (18) as discussed already.

If the wavefunction has no gradient:

$$\underline{\nabla}\phi = \underline{0} \quad -(24)$$

then eq. (23) simplifies. In the standard physics:

$$\underline{B}^* = \underline{\nabla}\times\underline{A}^*. \quad -(25)$$

7) From eqs. (24) and (25), eq. (23) reduces to:

$$\hat{H}_{21}\phi = \frac{1}{2m}\left[i e^2 \underline{\sigma} \cdot \underline{A} \times \underline{A}^* \phi - e\hbar \underline{\sigma} \cdot \underline{B}^* \phi \right] + \ldots \quad -(26)$$

There is a term due to the magnetic component of the electromagnetic field:

$$\hat{H}_{211} = -\frac{e\hbar}{2m} \underline{\sigma} \cdot \underline{B}^*, \quad -(27)$$

and a term due to the conjugate product of the electromagnetic field:

$$\hat{H}_{212} = \frac{ie^2}{2m} \underline{\sigma} \cdot \underline{A} \times \underline{A}^* \quad -(28)$$

In ECE physics the $\underline{B}^{(3)}$ field is defined by:

$$\underline{B}^{(3)*} = -ig\, \underline{A} \times \underline{A}^* \quad -(29)$$
$$= -ig\, \underline{A}^{(1)} \times \underline{A}^{(2)}$$

So:

$$\underline{A} \times \underline{A}^* = \frac{i}{g} \underline{B}^{(3)*} = \frac{i}{g} \underline{B}^{(3)} \quad -(30)$$

Eq. (28) is therefore the hamiltonian that

8) defines radiatively induced fermion resonance (RFR)

$$\boxed{\hat{H}_{RFR} = -\frac{e^2}{2mg}\,\underline{\sigma}\cdot\underline{B}^{(3)}} \quad -(31)$$

Eq. (27) defines the optical Zeeman effect to first order in the magnetic flux density of the electromagnetic field. It also defines first order optical ESR, optical NMR and optical MRI. In free space the electromagnetic potentials may be defined as:

$$\underline{A} = \underline{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}}(\underline{i} - i\underline{j})\,e^{i\Phi}, \quad -(32)$$

$$\underline{A}^{*} = \underline{A}^{(2)} = \frac{A^{(0)}}{\sqrt{2}}(\underline{i} + i\underline{j})\,e^{-i\Phi}, \quad -(33)$$

$$\Phi = \omega t - \kappa Z \quad -(34)$$

where ω is the angular frequency of the field at instant t and $\underline{\kappa}$ is its wave vector at point Z. Therefore:

$$\underline{A}\times\underline{A}^{*} = \frac{A^{(0)2}}{2}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -i & 0 \\ 1 & i & 0 \end{vmatrix} \quad -(34)$$

i.e. $\underline{A} \times \underline{A}^* = -iA^{(0)2}\,\underline{k}$ — (35)

From eqs. (28) and (35) the RFR Hamiltonian is:

$$\boxed{\hat{H}_{212} = \frac{e^2 A^{(0)2}}{2m}\,\underline{\sigma} \cdot \underline{k}} \quad -(36)$$

This leads to radiatively induced Zeeman effects at second order, radiatively induced ESR, NMR and MRI. It is the result of the Dirac or fermion equation, and was first observed by van der Ziel et al. in 1964 from the Bloembergen group at Harvard using an induction method in doped paramagnetic solids and liquids.

Its resonant equivalent, RFR, exists from the Dirac equation at all electromagnetic frequencies.

The great advantage of eq. (36) becomes apparent when the power density I of an electromagnetic beam is introduced, in watts per square metre:

$$I = cU = \frac{c}{\mu_0} B^{(0)2} \quad -(37)$$

where $\bar{U}$ is the magnetic part of the electromagnetic energy density in joules per cubic metre. Now use:

$$B^{(0)} = \kappa A^{(0)} = \frac{\omega}{c} A^{(0)} \quad -(38)$$

so

$$\boxed{\underline{I} = \frac{\omega^2}{\mu_0 c} A^{(0)2}} \quad -(39)$$

From eqs. (36) and (39) it follows that the RFR Hamiltonian is:

$$\hat{H}_{212} = \hat{H}_{RFR} = \left(\frac{\mu_0 c e^2}{2m}\right)\left(\frac{I}{\omega^2}\right)\sigma_Z \quad -(40)$$

where

$$\sigma_Z = \underline{\sigma} \cdot \underline{k} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Eq. (40) was confirmed by Warren et al.:
W. S. Warren et al. Science, $\underline{255}$, 1983 (1992) and Mol. Phys. $\underline{93}$, 371 (1998). As described by M. W. Evans and S. Crowell, pp. 29 ff (2001), electron resonance from eq. (40) occurs at:

$$\omega_{res} = 1.007 \times 10^{28} \frac{I}{\omega^2} \quad -(41)$$

and proton resonance at:

$$\omega_{res} = 1.532 \times 10^{25} \frac{I}{\omega^2} \quad -(42)$$

Warner et al. used a circularly polarized argon laser with a FTNMR device at 528.7 nm, 488 nm and 476.5 nm, with power density of 10 W cm^{-2}. Under these conditions the resonance from eq. (42) occurs at 0.12, 0.10 and 0.098 Hz. Tiny shifts in this range were observed by Warner et al. It must be realized that eq. (39) gives the free space relation between I and $A^{(0)2}$. Inside material matter this result may be modified by the well known internal field effect of dielectrics.

In developing eq. (18), it may be possible to set up an experiment in which the root mean square (average) of $\underline{A}^* - \underline{A}$ is used:

$$\left\langle \left((\underline{A}^* - \underline{A}) \cdot (\underline{A} - \underline{A}^*) \right)^{1/2} \right\rangle \quad -(43)$$

$$= \left\langle (\underline{A}^* \cdot \underline{A})^{1/2} + (\underline{A} \cdot \underline{A}^*)^{1/2} \right\rangle$$

$$+ \text{ phase dependent terms}$$

$$= \left\langle \left(A^{(0)2} \right)^{1/2} \right\rangle + \dots$$

and this would lead to new fermion resonance effects