248(4): Particle Collision Theory in the SU(2) Basis

In previous notes it has been shown that a process of the type:

$$\omega + \frac{mc^2}{\hbar} = \omega' + \omega'', \quad -(1)$$

$$\underline{p} = \underline{p}' + \underline{p}'' \quad -(2)$$

can be described by:

$$(E - E' + mc^2)^2 = c^2(\underline{p} - \underline{p}')^2 + m^2c^4 \quad -(3)$$

More generally, any process of the type:

$$E_1 + E = E_2 + E_3 \quad -(5)$$

$$\underline{p}_1 + \underline{p} = \underline{p}_2 + \underline{p}_3 \quad -(6)$$

can be described as:

$$(E_1 - E_2 + E)^2 = c^2(\underline{p}_1 - \underline{p}_2 + \underline{p})^2 + m^2c^4 \quad -(7)$$

i.e.

$$E_3^2 = c^2 p_3^2 + m^2c^4 \quad -(8)$$

Eq. (3) can be written as:

$$(E - E' + mc^2)^2 - m^2c^4 = c^2(\underline{p} - \underline{p}')^2 \quad -(9)$$

$$= (E - E' + mc^2 - mc^2)(E - E' + mc^2 + mc^2)$$

so

$$\boxed{E - E' = \frac{c^2(\underline{p} - \underline{p}')^2}{E - E' + 2mc^2}} \quad -(10)$$

2) A great deal of development of eq. (10) is possible to give a general method of describing processes of type (5) and (6). These may be particle collisions, fusion, atomic and molecular absorption, Rayleigh, Raman and neutron scattering, and so on. Several stages of approximation may be applied to eq. (10).

In the simplest approximation on the classical, relativistic, level:

$$E \sim E' \quad - (11)$$

so:

$$E - E' \sim \frac{1}{2m} \left(\underline{p} - \underline{p}' \right)^2 \quad - (12)$$

For Compton scattering:

$$E = \hbar\omega, \; E' = \hbar\omega', \; \underline{p} = \hbar \underline{\kappa}, \; \underline{p}' = \hbar \underline{\kappa}'$$

$$p = \frac{\hbar\omega}{c}, \; p' = \frac{\hbar\omega'}{c} \quad - (13)$$

so eq. (12) gives:

$$\omega - \omega' \sim \frac{\hbar}{2mc^2} \left(\omega^2 + \omega'^2 - 2\omega\omega' \cos\theta \right) \quad - (14)$$

which is an approximation to the Compton formula:

$$\omega - \omega' = \frac{\hbar}{mc^2} \omega\omega' (1 - \cos\theta) \quad - (15)$$

when

$$\omega^2 + \omega'^2 \sim 2\omega\omega' \quad - (16)$$

3) Eq. (16) follows because

$$\omega^2 + \omega'^2 = (\omega + \omega')^2 - 2\omega\omega' \quad -(17)$$

and if

$$\omega \sim \omega' \quad -(18)$$

then

$$\omega^2 + \omega'^2 \sim 4\omega\omega' - 2\omega\omega' = 2\omega\omega' \quad -(19)$$

QED.

Eq. (10) can always be written using the SU(2) basis as:

$$E - E' = \frac{1}{2m} \underline{\sigma} \cdot (\underline{p} - \underline{p}') \left(1 + \frac{E - E'}{2mc^2}\right)^{-1} \underline{\sigma} \cdot (\underline{p} - \underline{p}') \quad -(20)$$

This equation can be developed in several ways to give new effects which can be tested experimentally. Eq. (20) is similar to the structure of the fermion equation, but for Compton scattering it is written for the photon. By basic geometry, the SU(2) or SU(n) basis can always be used.

If:

$$E - E' << 2mc^2 \quad -(21)$$

then:

$$E - E' \sim \frac{1}{2m} \underline{\sigma} \cdot (\underline{p} - \underline{p}') \left(1 - \frac{E - E'}{2mc^2}\right) \underline{\sigma} \cdot (\underline{p} - \underline{p}') \quad -(22)$$

4) In the approximation:

$$E \sim E' \quad -(23)$$

then:

$$E - E' \sim \frac{1}{2m} \underline{\sigma} \cdot (\underline{p} - \underline{p}') \underline{\sigma} \cdot (\underline{p} - \underline{p}') \quad -(24)$$

$$= \frac{1}{2m} (\underline{p} - \underline{p}') \cdot (\underline{p} - \underline{p}') + i \underline{\sigma} \cdot (\underline{p} - \underline{p}') \times (\underline{p} - \underline{p}')$$

If $\underline{p}$ and $\underline{p}'$ are real valued ~~then~~ the use of the SU(2) basis has no effect on the approximate Compton formula (14). However if p' and $\underline{p}'$ are complex valued then:

$$E - E' = \frac{1}{2m} (\underline{p} - \underline{p}') \cdot (\underline{p} - \underline{p}')^* \quad -(25)$$

$$+ i \underline{\sigma} \cdot (\underline{p} - \underline{p}') \times (\underline{p} - \underline{p}')^*$$

and there are physical effects due to the conjugate product. If the incoming photon and scattered photon are circularly polarized then:

$$\underline{p} = \frac{1}{\sqrt{2}} p_0 (\underline{i} - i\underline{j}) e^{i(\omega t - kZ)} \quad -(26)$$

$$\underline{p}^* = \frac{1}{\sqrt{2}} p_0 (\underline{i} + i\underline{j}) e^{-i(\omega t - kZ)} \quad -(27)$$

5) and

$$\underline{p}' = p_0 (\underline{i} - i\underline{j}) e^{i(\omega' t - \kappa' Z)} \quad -(28)$$

$$\underline{p}^{*\prime} = p_0 (\underline{i} + i\underline{j}) e^{-i(\omega' t - \kappa' Z)} \quad -(29)$$

Then for example:

$$i\underline{\sigma} \cdot \underline{p} \times \underline{p}^* = -p_0 \, \underline{k} \cdot \underline{\sigma} \quad (30)$$

$$i\underline{\sigma} \cdot \underline{p}' \times \underline{p}'^* = -p_0 \, \underline{k} \cdot \underline{\sigma} \quad (31)$$

and new physical effects appear in Compton scattering by changing the basis from O(3) to SU(2).

These can be worked out more accurately from eq. (3), which becomes:

$$(E - E' + mc^2)^2 = c^2 \, \underline{\sigma} \cdot (\underline{p} - \underline{p}') \, \underline{\sigma} \cdot (\underline{p} - \underline{p}')^* + m^2 c^4$$

$$-(32)$$

This will be the subject of the next note.

Finally in this note, eq. (20) is developed by quantization to give:

$$\left(E - E'\right)\psi = \frac{1}{2m}\left(\underline{\sigma}\cdot\left(\underline{p}-\underline{p}'\right)\left(1+\frac{E-E'}{2mc^2}\right)^{-1}\underline{\sigma}\cdot\left(\underline{p}-\underline{p}'\right)\right)\psi \quad -(33)$$

giving a variety of new effects well known from the fermion equation analysis of previous UFT papers. One method of quantizing this equation is to use on the left hand side:

$$E = i\hbar\frac{\partial}{\partial t} \quad -(34)$$

and on the right hand side:

$$\underline{p} = -i\hbar\underline{\nabla} \quad -(35)$$

This gives:

$$\left(i\hbar\frac{\partial}{\partial t} - E'\right)\psi = -\frac{\hbar^2}{2m}\left(\underline{\sigma}\cdot\left(\underline{\nabla}-\underline{p}'\right)\left(1+\frac{E-E'}{2mc^2}\right)^{-1}\underline{\sigma}\cdot\left(\underline{\nabla}-\underline{p}'\right)\right)\psi \quad -(36)$$

and various new effects emerge related to the effects from the fermion equation. Note carefully however that eq. (36) is for the photon.