248(5): Outline of a General Theory of Particle Collision Processes.

The collision of two particles in general is considered to be described by:

$$E + E_1 \rightarrow \text{products} \quad -(1)$$

$$\underline{p} + \underline{p}_1 \rightarrow \text{products} \quad -(2)$$

Equation (1) is the conservation of total energy, and equation (2) is the conservation of total momentum. By cross over symmetry these apply to electron photon and electron positron collisions, and in general to any collision of two particles at any energy. There is also conservation of total angular momentum and conservation of total charge.

In the nineteen twenties Dirac used the minimal prescription to describe the interaction of an electron with the classical electromagnetic field, described by the type of four potential A_μ known to him. The work of AIAS in developing ECE theory shows that this must be generalized to A_μ^a, and the relation between field and potential generalized to:

$$F_{\mu\nu}^a = d \wedge A_\mu^a + \omega \wedge A_\mu^a \quad -(3)$$

So the minimal prescription used by Dirac can be generalized to include the spin connection. This generalized method can be applied to eqns. (1) and (2).

The Dirac equation itself has been corrected and generalized into the fermion equation, which

2) has been derived from the tetrad postulate:

$$D_\mu q^a_{\sim} = 0 \quad -(4)$$

and the ECE wave equation:

$$(\square + R) q^a_\mu = 0 \quad -(5)$$

The ECE wave equation also gives the Schroedinger equation and the Proca equation, and all the wave equations of physics. The fermion equation reduces to the Einstein equation:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(6)$$

in the classical relativistic limit. Eq. (6) is equivalent to the definition of the relativistic linear momentum in special relativity:

$$\underline{p} = \gamma m \underline{v} \quad -(7)$$

where γ is the Lorentz factor:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(8)$$

The ECE duality equations assert that:

$$\underline{p} = \hbar \underline{\kappa} = \gamma m \underline{v} \quad -(9)$$

and

$$E = \hbar \omega = \gamma m c^2 \quad -(10)$$

Eq. (10) reduces for the rest energy to

$$E_0 = \hbar \omega_0 = m c^2 \quad -(11)$$

3) which is the de Broglie equation.

The Planck equation is:

$$E = \hbar \omega \quad -(12)$$

and the de Broglie wave particle dualism is:

$$\underline{p} = \hbar \underline{\kappa} \quad -(13)$$

So ECE theory gives essentially all of physics from Cartan geometry.

Recently in note 247(8) ff. it was shown that a particle collision of the type:

$$E + E_1 = E' + E'' \quad -(14)$$

$$\underline{p} + \underline{p}_1 = \underline{p}' + \underline{p}'' \quad -(15)$$

can be reduced to an Einstein energy equation:

$$E''^2 = c^2 p''^2 + m^2 c^4 \quad -(16)$$

If eq. (16) is applied to an electron then m is the mass of the electron, and in general eq. (16) is:

$$E'' = \gamma'' m c^2 = \hbar \omega'' \quad -(17)$$

and applies to any relativistic particle of mass m.

In eq. (16):

$$E'' = E + E_1 - E' \quad -(18)$$

$$\underline{p}'' = \underline{p} + \underline{p}_1 - \underline{p}' \quad -(19)$$

4) In general:

$$E' = E_1' + E_2' + \dots + E_n' \quad -(20)$$

$$\underline{p}' = \underline{p}_1' + \underline{p}_2' + \dots + \underline{p}_n' \quad -(21)$$

if n particles are created by the collision of two particles such as an electron and positron.

So eq. (16) is:

$$(E + E_1 - E')^2 = c^2(\underline{p} + \underline{p}_1 - \underline{p}')^2 + m^2c^4 \quad -(22)$$

which is very similar to the structure given by the fermion equation, or corrected Dirac equation.

This realization opens up a great number of new possibilities in particle scattering theory, and also in the theory of low energy nuclear reactors.

Eq. (22) can be written as:

$$(E + E_1 - E')^2 - m^2c^4 = c^2(\underline{p} + \underline{p}_1 - \underline{p}')^2$$

$$= (E + E_1 - E' + mc^2)(E + E_1 - E' - mc^2) \quad -(23)$$

so eq. (23) can be linearized as follows:

$$E + E_1 - E' + mc^2 = \frac{c^2(\underline{p} + \underline{p}_1 - \underline{p}')^2}{E + E_1 - E' - mc^2} \quad -(24)$$

5) and can be prepared for semi classical quantization in the SU(2) basis. It is well known that Dirac used this method to give the g factor of the electron, the Landé factor and the correct description of spin-orbit coupling. He considered the interaction of an electron with a classical electromagnetic field. The energy E and momentum $\underline{p}$ of the electron were quantized:

$$E = i\hbar \frac{\partial}{\partial t}, \quad \underline{p} = -i\hbar \underline{\nabla} \quad -(25)$$

and he used the minimal prescription in the usual format given by the standard physics:

$$p^\mu \rightarrow p^\mu - eA^\mu \quad -(26)$$

i.e.

$$E \rightarrow E - e\phi \quad -(27)$$

$$\underline{p} \rightarrow \underline{p} - e\underline{A} \quad -(28)$$

The Einstein equation gives:

$$(E - e\phi)^2 = c^2(\underline{p} - e\underline{A})^2 + m^2c^4 \quad -(29)$$

so

$$(E - e\phi - mc^2)(E - e\phi + mc^2) = c^2(\underline{p} - e\underline{A})^2 \quad -(30)$$

This can be linearized as:

$$E - e\phi - mc^2 = \frac{c^2(\underline{p} - e\underline{A})^2}{E - e\phi + mc^2} \quad -(31)$$

Eqs. (24) and (31) are the same if:

$$e\phi = E' - E_1 \quad -(32)$$

$$e\underline{A} = \underline{p}' - \underline{p}_1 \quad -(33)$$

Eqs. (32) and (33) are written for the sake of showing that the structure of particle collision ~~theory~~ can be reduced to the structure of the semi-classical Dirac type theory. More generally, particle collisions and interactions are of many different types, the Dirac theory was applied to the interaction of an electron with a classical electromagnetic field in the minimal prescription (26) of the now obsolete standard physics.

Eq. (31) (or eq. (24)) is quantized as follows. First write:

$$E = mc^2 + e\phi + \frac{c^2(\underline{p} - e\underline{A})^2}{E - e\phi + mc^2} \quad -(34)$$

which can be expressed as:

7)
$$\hat{E} = mc^2 + e\phi + \frac{c^2(\hat{\underline{p}} - e\underline{A})^2}{E - e\phi + mc^2} \quad -(35)$$

where the operators $\hat{E}$ and $\hat{p}$ are defined by eq. (25). Eq. (35) is now written in the ( ) basis as follows:

$$\hat{E} = mc^2 + e\phi + c^2(\hat{p} - e\underline{A})(E - e\phi + mc^2)^{-1}(\hat{p} - e\underline{A}) \quad -(36)$$

At this point it is assumed that the particle of mass m, the electron in Dirac theory, is slow moving, so:

$$E = \gamma mc^2 \sim mc^2 \quad -(37)$$

Eq. (36) becomes:

$$\hat{E} = mc^2 + e\phi + c^2(\hat{\underline{p}} - e\underline{A})(2mc^2 - e\phi)^{-1}(\hat{\underline{p}} - e\underline{A})$$

$$= mc^2 + e\phi + \frac{1}{2m}(\hat{\underline{p}} - e\underline{A})\left(1 - \frac{e\phi}{2mc^2}\right)^{-1}(\hat{\underline{p}} - e\underline{A}) \quad -(38)$$

It is now assumed that:

$$e\phi << 2mc^2 \quad -(39)$$

So:

$$\hat{E} = mc^2 + e\phi + \frac{1}{2m}(\underline{\hat{p}} - e\underline{A})\left(1 + \frac{e\phi}{2mc^2}\right)(\underline{\hat{p}} - e\underline{A}) \quad -(40)$$

$$= mc^2 + e\phi + \frac{1}{2m}(\underline{\hat{p}} - e\underline{A})(\underline{\hat{p}} - e\underline{A}) \quad -(41)$$

$$+ \frac{e}{4m^2c^2}(\underline{\hat{p}} - e\underline{A})\,\phi\,(\underline{\hat{p}} - e\underline{A})$$

The SU(2) basis is now introduced and the operators act on a wave function ψ. In the Dirac theory this is the wave function of an electron, but in general particle collision theory it can be any wave function. So eq. (41) becomes:

$$\hat{E}\psi = (mc^2 + e\phi)\psi + \frac{1}{2m}\left(\underline{\sigma}\cdot(\underline{\hat{p}} - e\underline{A})\,\underline{\sigma}\cdot(\underline{\hat{p}} - e\underline{A})\right)\psi \quad -(42)$$

$$+ \frac{e}{4m^2c^2}\left(\underline{\sigma}\cdot(\underline{\hat{p}} - e\underline{A})\,\phi\,\underline{\sigma}\cdot(\underline{\hat{p}} - e\underline{A})\right)\psi$$

Note that the wave function ψ is acted upon by the complete brackets on the RHS of eq. (43). This produces the g factor of the electron, the

9) Landé factor, ESR, NMR, MRI, Spin orbit coupling in atoms and molecules, the Thomas factor and the Darwin term.

In ECE theory there are many more effects because A_μ is replaced by A^a_μ, and in particle collision theory there are very many new effects. The minimal prescription can be extended to gravitation, weak and strong nuclear fields, and all permutations and combinations of such fields.

Using eq. (25) in eq. (42):

$$
\begin{aligned}
i\hbar \frac{\partial \phi}{\partial t} &= (mc^2 + e\phi)\phi \\
&+ \frac{1}{2m}\left(\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla} - e\underline{A}\right)\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla} - e\underline{A}\right)\right)\phi \\
&+ \frac{e}{4m^2c^2}\left(\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla} - e\underline{A}\right)\phi\,\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla} - e\underline{A}\right)\right)\phi
\end{aligned}
\quad -(43)
$$

The overall structure of this equation is:

$$(\hat{H}_1 + \hat{H}_2 + \hat{H}_3)\phi = E\phi \quad -(44)$$

where the hamiltonian operators are defined as follows:

$$\hat{H}_1 = mc^2 + e\phi \quad -(45)$$

10)

$$\hat{H}_2 = \frac{1}{2m}\left(\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla} - e\underline{A}\right)\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla} - e\underline{A}\right)\right) \quad -(46)$$

and

$$\hat{H}_3 = \frac{e}{4m^2c^2}\left(\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla} - e\underline{A}\right)\phi\,\underline{\sigma}\cdot\left(-i\hbar\underline{\nabla} - e\underline{A}\right)\right) \quad -(47)$$

The $\hat{H}_2$ hamiltonian gives the g factor of the electron at the Dirac level, the Landé factor, ESR, NMR and MRI. The $\hat{H}_3$ hamiltonian gives the Thomas factor, the structure of spin-orbit effects in atomic and molecular theory and the Darwin term.

In order to evaluate the hamiltonians, the following SU(2) algebra is used:

$$\underline{\sigma}\cdot\underline{\Pi}\,\underline{\sigma}\cdot\underline{\Pi} = \Pi^2 + i\underline{\sigma}\cdot\underline{\Pi}\times\underline{\Pi} \quad -(48)$$

All of this well known theory can be greatly developed now within the context of ECE theory. The most well known result of $\hat{H}_2$ is probably the interaction of an electron with a magnetic field, giving the Zeeman effect and Landé factor, ESR, NMR and MRI. This will be developed in the next note.