248(2): Inelastic Compton Scattering with Finite Photon Mass.

Following upon note 248(1) the equations of conservation of energy and momentum are:

$$\omega + x_2 = \omega' + \omega'' + E_1/\hbar \quad -(1)$$

and

$$\underline{p} = \underline{p}' + \underline{p}'' + \underline{p_1} \quad -(2)$$

As in previous notes these result in the equation:

$$(E - E' - E_1 + mc^2)^2 = c^2(\underline{p} - \underline{p}' - \underline{p_1})^2 + m^2c^4 \quad -(3)$$

which again has a Dirac like structure.

It is assumed that:

$$E_1^2 = c^2 p_1^2 + m_2^2 c^4 \quad -(4)$$

so that a new particle is created of mass m_2.

Now write:

$$E_2 = E - E' \quad -(5)$$

$$\underline{p_2} = \underline{p} - \underline{p}' \quad -(6)$$

with:

$$\underline{p} = \hbar \underline{\kappa}, \quad \underline{p}' = \hbar \underline{\kappa}' \quad -(7)$$

As in previous notes:

$$\kappa^2 = \frac{1}{c^2}(\omega^2 - \omega_0^2); \quad \kappa'^2 = \frac{1}{c^2}(\omega'^2 - \omega_0^2) \quad -(8)$$

with

$$E_1 = \hbar\omega_1, \quad \kappa_1^2 = \frac{1}{c^2}(\omega_1^2 - \omega_{10}^2) \quad -(9)$$

where ω_{10} is the rest frequency of the created particle. Therefore:

$$E_2 = \hbar(\omega - \omega'), \quad \underline{p}_2 = \hbar(\underline{k} - \underline{k}') \quad -(10)$$

We now derive an expression for the rest frequency of the photon, ω_0.

From eq. (3):

$$\left(\omega - \omega' - \omega_1 + \frac{mc^2}{\hbar}\right)^2 = \frac{c^2}{\hbar^2}\left(\underline{p}_2 - \underline{p}_1\right)^2 + \left(\frac{mc^2}{\hbar}\right)^2 \quad -(11)$$

i.e.

$$(\omega - \omega' - \omega_1)^2 + 2\left(\frac{mc^2}{\hbar}\right)(\omega - \omega' - \omega_1) = \frac{c^2}{\hbar^2}\left(\underline{p}_2 - \underline{p}_1\right)^2. \quad -(12)$$

$$= \frac{c^2}{\hbar^2}\left(\underline{p} - \underline{p}' - \underline{p}_1\right)^2$$

$$= \frac{c^2}{\hbar^2}\left((\underline{p} - \underline{p}')^2 - 2\underline{p}_1 \cdot (\underline{p} - \underline{p}') + p_1^2\right)$$

$$= c^2\left((\underline{k} - \underline{k}')^2 - 2\underline{k}_1 \cdot (\underline{k} - \underline{k}') + k_1^2\right)$$

$$= c^2\big(k^2 + k'^2 - 2kk'\cos\theta + k_1^2$$
$$- 2\underline{k}_1 \cdot (\underline{k} - \underline{k}')\big)$$

$$= c^2\big(k^2 + k'^2 - 2kk'\cos\theta + k_1^2$$
$$- 2|\underline{k}_1||\underline{k} - \underline{k}'|\cos\theta_1\big)$$

3) where:

$$|\underline{\kappa} - \underline{\kappa}'| = \left(\kappa^2 + \kappa'^2 - 2\kappa\kappa'\cos\theta\right)^{1/2} \quad -(13)$$

Here:

$$\kappa^2 = \frac{1}{c^2}\left(\omega^2 - \omega_0^2\right) \quad -(14)$$

$$\kappa'^2 = \frac{1}{c^2}\left(\omega'^2 - \omega_0^2\right) \quad -(15)$$

$$\kappa_1^2 = \frac{1}{c^2}\left(\omega_1^2 - \omega_{10}^2\right) \quad -(16)$$

It follows that:

$$\omega_0^2 = \omega\omega' + \omega_1(\omega - \omega') - \left(\frac{mc^2}{\hbar}\right)(\omega - \omega' - \omega_1)$$
$$- 2(\omega^2 - \omega_0^2)^{1/2}(\omega'^2 - \omega_0^2)^{1/2}\cos\theta - \frac{\omega_{10}^2}{2}$$
$$-(\omega_1^2 - \omega_{10}^2)^{1/2}\left(\omega^2 + \omega'^2 - 2\omega_0^2 - 2(\omega^2 - \omega_0^2)^{1/2}(\omega'^2 - \omega_0^2)^{1/2}\cos\theta\right)^{1/2}\cos\theta_1 \quad -(17)$$

For zero photon mass and with particle created then:

$$\omega_0 = 0, \quad E_1 = 0, \quad \underline{p}_1 = 0 \quad -(18)$$

and

$$\omega_{10} = 0, \quad \omega_1 = \omega_{10} = 0 \quad -(19)$$

In this case eq. (17) reduces to the Compton formula:

$$\omega - \omega' = \frac{\hbar}{mc^2}\,\omega\omega'(1-\cos\theta) \quad -(20)$$

QED. Eq. (17) simplifies considerably for:

$$\cos\theta = \cos\theta_1 = 0 \quad -(21)$$

to give:

$$\omega_0^2 = \omega\omega' + \omega_1(\omega-\omega') - \frac{\omega_{10}^2}{2} - \left(\frac{mc^2}{\hbar}\right)(\omega-\omega'-\omega_1) \quad -(22)$$

It is seen that the photon mass depends on the properties of the created particle, these properties are represented by the energy of the created particle:

$$E_1 = \hbar\omega_1 \quad -(23)$$

and its momentum:

$$\underline{p_1} = \hbar\,\underline{\kappa_1} \quad -(24)$$

with

$$E_1^2 = c^2 p_1^2 + m_2^2 c^4 \quad -(25)$$

where m_2 is the mass of the created particle.