248(10): Translation of Particle Collision Theory into the Dirac and Schroedinger Equations

As in note 248(5) consider the general particle collision process:

$$E + E_1 = E' + E'' \quad -(1)$$

and

$$\underline{p} + \underline{p_1} = \underline{p}' + \underline{p}'' \quad -(2)$$

in which

$$E''^2 = c^2 p''^2 + m^2 c^4 \quad -(3)$$

i.e.

$$E'' = \gamma'' m c^2 = \hbar \omega'' . \quad -(4)$$

If many particles are produced by the collision then:

$$E' = E_1' + E_2' + \dots + E_n' \quad -(5)$$

$$\underline{p}' = \underline{p_1}' + \underline{p_2}' + \dots + \underline{p_n}' \quad -(6)$$

In Compton scattering:

$$E_1 = mc^2 \quad -(7)$$

$$p_1 = \underline{0} \quad -(8)$$

for an initially stationary electron.

Schroedinger Approximation

Eq. (3) is written as:

$$E''^2 - m^2 c^4 = c^2 p''^2 \quad -(9)$$

i.e.

$$(E'' - mc^2)(E'' + mc^2) = c^2 p''^2 \quad -(10)$$

So

$$E_2 = E'' - mc^2 = \frac{c^2 p''^2}{E'' + mc^2} \quad -(11)$$

The non-relativistic kinetic energy is obtained in the low energy approximation:

$$E'' \sim mc^2 \quad -(12)$$

so

$$E_2 = \frac{1}{2m} p''^2 \quad -(13)$$

Finally use:

$$p'' = -i\hbar \underline{\nabla} \quad -(14)$$

to obtain <u>the</u> <u>Schroedinger equation</u>:

$$-\frac{\hbar^2 \nabla^2}{2m} \phi = E_2 \phi \quad -(15)$$

Here:

$$E_2 = E'' - mc^2$$
$$= E + E_1 - E' - mc^2 \quad -(16)$$

For Compton scattering:

$$E_1 = mc^2 \quad -(17)$$

So

$$E_2 = E - E' \quad -(18)$$

and

$$\boxed{-\frac{\hbar^2}{2m} \nabla^2 \phi = (E - E')\phi} \quad -(19)$$

Eqn (19) describes the process:

3)

$$E + mc^2 = E' + E'' \quad -(20)$$

and

$$\underline{p} = \underline{p}' + \underline{p}'' \quad -(21)$$

so the translation rule is: —(22)

$$\boxed{\begin{aligned} E + mc^2 &= E' + E'' \\ \underline{p} &= \underline{p}' + \underline{p}'' \end{aligned}} \longrightarrow \boxed{-\frac{\hbar^2}{2m}\nabla^2\phi = (E - E')\phi}$$

in the non relativistic quantum approximation.

It is well known that eq. (22) leads to quantum tunnelling. So the translation (22) can be used for low energy nuclear reactions described as particle collision processes.

Fermion Equation

In this case eq. (10) is written in the SU(2) basis:

$$(E'' - mc^2)(E'' + mc^2) = c^2 \underline{\sigma}\cdot\underline{p}''\,\underline{\sigma}\cdot\underline{p}'' \quad -(23)$$

so:

$$E_2 = E'' - mc^2 = \frac{c^2\,\underline{\sigma}\cdot\underline{p}''\,\underline{\sigma}\cdot\underline{p}''}{E'' + mc^2} \quad -(24)$$

In the low energy approximation (12) eq. (24) is:

4)

$$E_2 = \frac{1}{2m} \underline{\sigma} \cdot \underline{p}'' \underline{\sigma} \cdot \underline{p}'' \quad -(25)$$

If $\underline{p}''$ is real valued:

$$\underline{\sigma} \cdot \underline{p}'' \underline{\sigma} \cdot \underline{p}'' = p''^2 + i \underline{\sigma} \cdot \underline{p}'' \times \underline{p}''$$

$$= p''^2 \quad -(26)$$

So eq. (22) is obtained again in the low energy approximation.

However, if p'' is complex valued, then:

$$E_2 = \frac{1}{2m} \underline{\sigma} \cdot \underline{p}'' \underline{\sigma} \cdot p''^* + \frac{i\underline{\sigma}}{2m} \cdot \underline{p}'' \times \underline{p}''^* \quad -(27)$$

which quantizes to:

$$\boxed{-\frac{\hbar^2}{2m} \nabla^2 \psi + \frac{i\underline{\sigma}}{2m} \cdot \underline{p}'' \times \underline{p}''^* \psi = (E - E')\psi \quad -(28)}$$

In principle, a new observable effect occurs, via the hamiltonian:

$$H_2 = \frac{i\underline{\sigma}}{2m} \cdot \underline{p}'' \times \underline{p}''^* \quad -(29)$$

related to the RFR hamiltonian:

$$H_{RFR} = \frac{ie^2\underline{\sigma}}{2m} \cdot \underline{A} \times \underline{A}^* \quad -(30)$$

5) In eq. (28), $\underline{p}''$ is the momentum of the scattered electron, and if the electron is scattered in a circularly polarized trajectory:

$$\underline{p}'' = \frac{p^{(0)}}{\sqrt{2}}(\underline{i} - i\underline{j}) \quad -(31)$$

$$\underline{p}''^{*} = \frac{p^{(0)}}{\sqrt{2}}(\underline{i} + i\underline{j}) \quad -(32)$$

and

$$H_2 = \frac{p^{(0)2}}{2m}\,\underline{\sigma}\cdot\underline{k} = \frac{p^{(0)2}}{2m}\,\sigma_Z. \quad -(33)$$

Resonance can be observed between the two states of σ_Z:

$$\sigma_Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad -(34)$$

Finally, using:

$$p''^2 = p^2 + p'^2 - 2pp'\cos\theta \quad -(35)$$

the Schrodinger equation (22) reduces to the Compton formula:

$$\frac{1}{2m}\left(p^2 + p'^2 - 2pp'\cos\theta\right) = E - E' \quad -(36)$$

in the non-relativistic approximation (12). For the "massless" photon:

$$E = \hbar\omega, \quad \underline{p} = \hbar\underline{\kappa}, \quad p = \frac{\hbar\omega}{c} \quad -(37)$$

6) so eq. (36) becomes:

$$\frac{\hbar}{mc^2}\left(\frac{1}{2}\left(\omega^2+\omega'^2\right)-\omega\omega'\cos\theta\right) = \omega-\omega' \quad -(38)$$

and if $\omega \sim \omega'$ – (39)

$$\frac{1}{2}\left(\omega^2+\omega'^2\right) \sim 2\omega\omega' \quad -(40)$$

so eq. (38) is the Compton formula:

$$\frac{\hbar}{mc^2}\,\omega\omega'(1-\cos\theta) = \omega-\omega' \quad -(41)$$

in the approximation: $\omega \sim \omega'$ – (42)

equivalent to the approximation (12), QED.

In the next series of notes and papers, many new phenomena like this will be derived.