247(3): LENR, Relativistic Collision with Fusion

Consider the fusion equation:

$$\gamma m_1 c^2 + m_2 c^2 = \gamma' m_3 c^2 + E \quad -(1)$$

in which a relativistic particle of mass m_1 collides with a static particle of mass m_2 and fuses with it to form a particle of mass m_3. The energy released during this process is E. The momentum conservation equation is:

$$\underline{p} = \underline{p}' \quad -(2)$$

Here:

$$\hbar\omega = \gamma m_1 c^2 \quad -(3)$$

$$\hbar\omega_0 = m_2 c^2 \quad -(4)$$

$$\hbar\omega' = \gamma' m_3 c^2 \quad -(5)$$

and

$$\underline{p} = \hbar \underline{\kappa} = \gamma m_1 \underline{v} \quad -(6)$$

$$\underline{p}' = \hbar \underline{\kappa}' = \gamma' m_3 \underline{v}' \quad -(7)$$

It follows that:

$$\omega^2 v^2 = \omega'^2 v'^2 \quad -(8)$$

and

$$v^2 = c^2\left(1 - \left(\frac{x_1}{\omega}\right)^2\right), \quad v'^2 = c^2\left(1 - \left(\frac{x_3}{\omega'}\right)^2\right) \quad -(9)$$

where

$$x_1 = m_1 c^2 / \hbar, \quad x_3 = m_3 c^2 / \hbar \quad -(10)$$

From eqs. (8) to (10):

$$\omega^2 - x_1^2 = \omega'^2 - x_3^2 \quad -(11)$$

The energy conservation equation (1) can be written as:

$$\omega + \omega_0 = \omega' + \frac{E}{\hbar} \quad -(12)$$

The frequency ω' can be eliminated between eqs. (11) and (12) to give:

$$\omega^2 - x_1^2 = \left(\omega + \omega_0 - \frac{E}{\hbar}\right)^2 - x_3^2 \quad -(13)$$

So:

$$E = \hbar\left(\omega + \omega_0 - \left(x_3^2 - x_1^2 + \omega^2\right)^{1/2}\right) \quad -(14)$$

where:

$$\omega_0 = \frac{m_2 c^2}{\hbar}, \quad \omega = \frac{\gamma m_1 c^2}{\hbar}, \quad x_1 = \frac{m_1 c^2}{\hbar}, \quad x_3 = \frac{m_3 c^2}{\hbar} \quad -(15)$$

So

$$\boxed{E = \left(\gamma m_1 + m_2 - \left(m_3^2 + (\gamma^2 - 1) m_1^2\right)^{1/2}\right) c^2} \quad -(16)$$

where

$$\gamma^2 = \left(1 - \frac{v^2}{c^2}\right)^{-1} \quad -(17)$$

3)

In order for

$$E > 0 \quad -(18)$$

the mass m_3 must be such that:

$$m_3^2 + (\gamma^2 - 1)m_1^2 < (\gamma m_1 + m_2)^2 \quad -(19)$$

i.e.

$$m_3^2 + (\gamma^2 - 1)m_1^2 < \gamma^2 m_1^2 + 2\gamma m_1 m_2 + m_2^2 \quad -(20)$$

or

$$m_3^2 - m_1^2 - m_2^2 < 2\gamma m_1 m_2 \quad -(21)$$

so

$$\gamma > \frac{m_3^2 - m_1^2 - m_2^2}{2 m_1 m_2} \quad -(22)$$

or

$$\frac{1}{\gamma} = 1 - \frac{v^2}{c^2} < \frac{2 m_1 m_2}{m_3^2 - m_1^2 - m_2^2} \quad -(23)$$

or

$$\frac{v^2}{c^2} > 1 - \frac{2 m_1 m_2}{m_3^2 - m_1^2 - m_2^2} \quad -(24)$$

For

$$v < c \quad -(25)$$

then

$$\boxed{m_3^2 > m_1^2 + m_2^2} \quad -(26)$$

which agrees with the calculation by Dr. Horst Eckardt.

For example, if it is assumed that:

$$m_3 = m_1 + m_2 \quad -(27)$$

4) for the sake of illustration only, then:

$$m_3^2 = m_1^2 + m_2^2 + 2m_1m_2 \qquad (28)$$

which satisfies condition (26).

If:

$$\frac{v^2}{c^2} = 1 - \frac{2m_1m_2}{m_3^2 - m_1^2 - m_2^2} \qquad (29)$$

then

$$E = 0, \qquad (30)$$

therefore in order to get a slightly positive energy E, v^2 must be slightly greater than:

$$v^2 = c^2\left(1 - \frac{2m_1m_2}{m_3^2 - m_1^2 - m_2^2}\right) \qquad (31)$$