247(6): Calculation of the Energy and Momentum of Particles Released in Electron Positron Annihilation

This is considered to be an inelastic collision in which the conservation of energy is:

$$e^- + e^+ = 2\hbar\omega_3 + E \quad -(1)$$

$$= 2\hbar\omega_3 + \hbar\omega_4$$

and the conservation of momentum is:

$$\underline{p_1} + \underline{p_2} = 2\underline{p_3} + \underline{p_4} \quad -(2)$$

Here $\hbar\omega_3$ is the energy of the photon created in the collision and

$$E = \hbar\omega_4 \quad -(3)$$

is the energy of a particle created by the collision. More generally there are many particles created in the collision, but for simplicity only one is considered. The conservation of energy equation gives:

$$\omega_1 + \omega_2 = 2\omega_3 + \omega_4 \quad -(4)$$

where ω_1 is the angular frequency of the electron e^- and ω_4 is the angular frequency of the positron e^+. The initial momentum of the electron is $\underline{p_1}$ and the initial momentum of the positron is $\underline{p_2}$. The momentum of

) the photon created in the collision is $\underline{p_3}$ and the momentum of the particle created by the collision is $\underline{p_4}$.

For ease of notation denote:

$$\omega = \omega_1 + \omega_2 \quad -(5)$$

$$\underline{p} = \underline{p_1} + \underline{p_2}. \quad -(6)$$

Therefore:

$$E = \hbar(\omega - 2\omega_3) = \hbar\omega_4 \quad -(7)$$

and

$$\underline{p_4} = \hbar \underline{k_4} = \underline{p} - 2\underline{p_3} \quad -(8)$$

It follows that:

$$p_4^2 = p^2 + 4p_3^2 - 2pp_3 \cos\theta \quad -(9)$$

and

$$k_4^2 = k^2 + 4k_3^2 - 2kk_3 \cos\theta \quad -(10)$$

Now define:

$$E = \hbar\omega_4 = \gamma_4 m_4 c^2 \quad -(11)$$

$$\underline{p_4} = \hbar \underline{k_4} = \gamma_4 m_4 \underline{v_4} \quad -(12)$$

Here m_4 is the mass of the particle created in the collision, $\underline{p_4}$ its momentum and $\underline{v_4}$ its velocity. The Lorentz factor is:

3)

$$\gamma_4 = \left(1 - \frac{v_4^2}{c^2}\right)^{-1/2} \quad -(13)$$

It follows that:

$$\omega_4^2 v_4^2 = \omega^2 v^2 + 4\omega_3^2 v_3^2 - 2\omega v \omega_3 v_3 \cos\theta \quad -(14)$$

and

$$\omega_4^2 - x_4^2 = \omega^2 - x^2 + 4(\omega_3^2 - x_3^2) - 2(\omega^2 - x^2)^{1/2}(\omega_3^2 - x_3^2)^{1/2}\cos\theta \quad -(15)$$

This equation can be expressed as:

$$E^2 = m_4^2 c^4 + c^2 p_4^2 \quad -(16)$$

where:

$$E = \hbar\omega_4 , \quad x_4 = m_4 c^2/\hbar \quad -(17)$$

and:

$$p_4^2 = \left(\frac{\hbar}{c}\right)^2 \left(\omega^2 - x^2 + 4(\omega_3^2 - x_3^2) - 2(\omega^2 - x^2)^{1/2}(\omega_3^2 - x_3^2)^{1/2}\cos\theta\right) \quad -(18)$$

Here

$$x = mc^2/\hbar , \quad x_3 = m_3 c^2/\hbar \quad -(19)$$

Therefore the momentum $\underline{p_4}$ of the created

4) particle is given by eq. (18).

Eq. (16) is the Einstein energy equation of the particle of mass m_4. Self consistently it is the same equation as:

$$p_4 = \gamma_4 m_4 v_4 \quad -(20)$$

which is eq. (12), QED.

In this notation:

$$\hbar\omega = \hbar(\omega_1 + \omega_2) = \gamma_1 m c^2 + \gamma_2 m c^2 \quad -(21)$$

so

$$\gamma = \gamma_1 + \gamma_2 \quad -(22)$$

and

$$\hbar \underline{\kappa} = \hbar(\underline{\kappa_1} + \underline{\kappa_2}) = \gamma_1 m \underline{v_1} + \gamma_2 m \underline{v_2} = \gamma m \underline{v} \quad -(23)$$

so

$$\underline{v} = \frac{1}{\gamma}\left(\gamma_1 \underline{v_1} + \gamma_2 \underline{v_2}\right) \quad -(24)$$

Here: $\hbar\omega = \gamma m c^2, \; \hbar\kappa = \gamma m v, \; \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(25)$

The energy E_4 and momentum p_4 can be evaluated and graphed by computer.