247(8): General Theory of Particle Scattering and Annihilation and Comparison with Dirac Formalism

As in note 247(7), in particle physics if:

$$A + B \rightarrow C + D \quad -(1)$$

then

$$A + \bar{C} \rightarrow \bar{B} + D \quad -(2)$$

by cross over symmetry. So the same general theory should describe:

$$\gamma + e^- \rightarrow \gamma + e^- \quad -(3)$$

and

$$e^+ + e^- \rightarrow \gamma + \gamma \quad -(4)$$

by the cross over symmetry:

$$\gamma \rightarrow e^+ \quad -(5)$$

$$e^- \rightarrow \gamma \quad -(6)$$

assuming that the photon is its own anti particle. Eq. (3) is Compton scattering and eq. (4) is positron electron annihilation.

As argued in previous notes a more complete and correct theory requires:

$$\gamma + e^- \rightarrow \gamma + e^- + E \quad -(7)$$

$$e^+ + e^- \rightarrow \gamma + \gamma + E \quad -(8)$$

however, for the purposes of the development in this note we consider eqs. (3) and (4).

The equations of conservation of energy and momentum describing the processes (1) and (2) are:

$$\omega + x_2 = \omega' + \omega'' \quad -(9)$$

$$\underline{p} = \underline{p}' + \underline{p}'' \quad -(10)$$

where
$$x_2 = mc^2/\hbar \quad -(11)$$

and m is the mass of the initially static particle. Therefore:

$$\omega'' = x_2 + \omega - \omega' \quad -(12)$$

and
$$\underline{p}'' = \underline{p} - \underline{p}' \quad -(13)$$

Defining:
$$E'' = \hbar\omega'' \quad -(14)$$

then:
$$E''^2 = c^2 p''^2 + m^2c^4 \quad -(15)$$

and
$$E'' = E - E' + mc^2 \quad -(16)$$

where
$$E' = \hbar\omega', \quad E = \hbar\omega. \quad -(17)$$

From these equations:

$$(E - E' + mc^2)^2 = c^2(\underline{p} - \underline{p}')^2 + m^2c^4 \quad -(18)$$

It follows that:

$$E - E' + mc^2 = \frac{c^2(\underline{p} - \underline{p}')^2}{E - E' + mc^2} + \frac{m^2c^4}{E - E' + mc^2} \quad -(19)$$

This is similar to the formalism obtained from the Dirac equation developed into the fermion equation.

The momenta are defined by:

$$\underline{p} = \hbar \underline{\kappa}, \quad \underline{p}' = \hbar \underline{\kappa}' \quad -(20)$$

For the "massless photon"

$$p = \hbar \kappa = \hbar \frac{\omega}{c}, \quad p' = \hbar \frac{\omega'}{c} \quad -(21)$$

It follows that:

$$\left(\omega - \omega' + \frac{mc^2}{\hbar}\right)^2 = \frac{c^2}{\hbar^2}\left(p^2 + p'^2 - 2pp'\cos\theta\right) + \left(\frac{mc^2}{\hbar}\right)^2 \quad -(22)$$

$$= \omega^2 + \omega'^2 - 2\omega\omega'\cos\theta + \left(\frac{mc^2}{\hbar}\right)^2$$

$$= \omega^2 + \omega'^2 - 2\omega\omega' + 2\left(\frac{mc^2}{\hbar}\right)\left(\omega - \omega'\right) \quad -(23)$$

i.e.

$$\boxed{\omega - \omega' = \left(\frac{\hbar}{mc^2}\right)\omega\omega'\left(1 - \cos\theta\right)} \quad -(24)$$

which is the Compton formula.

The Compton formula follows from the relativistic equation (15) for the scattered electron. This is:

$$\boxed{E'' = \gamma'' mc^2 = \hbar\omega''} \quad -(25)$$

Eq. (19) may be written as:

$$-E' + mc^2 = \frac{1}{m}\left(\underline{p} - \underline{p}'\right)^2\left(1 + \frac{E - E'}{mc^2}\right)^{-1} + mc^2\left(1 + \frac{E - E'}{mc^2}\right)^{-1} \quad -(26)$$

4)

If
$$E - E' \ll mc^2 \quad - (27)$$

then:

$$E - E' + mc^2 = \frac{1}{m}\left(\underline{p} - \underline{p}'\right)^2\left(1 - \frac{E-E'}{mc^2}\right) \quad - (28)$$
$$+ mc^2\left(1 - \frac{E-E'}{mc^2}\right)$$

i.e.

$$E - E' = \frac{1}{2m}\left(\underline{p} - \underline{p}'\right)^2\left(1 - \frac{E-E'}{mc^2}\right)$$

$$\boxed{E - E' \sim \frac{1}{2m}\left(\underline{p} - \underline{p}'\right)^2} \quad - (29)$$

In this limit it is clear that E and E' are kinetic energies. They are defined by:

$$E = \hbar\omega = \gamma m_1 c^2 \quad - (30)$$
$$E' = \hbar\omega' = \gamma' m_1 c^2 \quad - (31)$$

where m_1 is the photon mass. It has been assumed for the sake of argument that:

$$m_1 = 0, \quad \gamma = \gamma' \rightarrow \infty \quad - (32)$$

because

$$v \rightarrow c; \quad v' \rightarrow c \quad - (33)$$

So eq. (29) describes the transfer of kinetic energy from the electron to the photon in the non-relativistic

2) Eq. (29) may be written as:

$$\hbar(\omega-\omega') = \frac{1}{2m}\left(\frac{\hbar}{c}\right)^2\left(\omega^2+\omega'^2-2\omega\omega'\cos\theta\right) \quad -(34)$$

i.e.

$$\omega-\omega' = \frac{\hbar}{mc^2}\left(\frac{1}{2}\left(\omega^2+\omega'^2\right)-\omega\omega'\cos\theta\right) \quad -(35)$$

From eqs. (24) and (35), it is seen that the Compton formula has been approximated with:

$$\omega^2+\omega'^2 = 2\omega\omega' \quad -(36)$$

Note that eq. (36) is equivalent to:

$$(\omega+\omega')^2 = \omega^2+\omega'^2+2\omega\omega' \sim 4\omega\omega' \quad -(37)$$

i.e.

$$\omega \sim \omega' \quad -(38)$$

From eq. (61) of UFT 172 the fermion eqn. produces:

$$(E-e\phi)^2-m^2c^4 = c^2\underline{\sigma}\cdot(\underline{p}-e\underline{A})\,\underline{\sigma}\cdot(\underline{p}-e\underline{A}) \quad -(39)$$

Eqs. (18) and (39) are the same if:

$$e\phi = E' - mc^2 \quad -(40)$$

and

$$\underline{p}' = e\underline{A} \quad -(41)$$

Eqs. (18) and (39) are the quantized version of eq. (15) with:

$$E'' \rightarrow E - E' + mc^2 = E - e\phi \quad - (42)$$

and

$$\underline{p}'' \rightarrow \underline{p} - \underline{p}' = \underline{p} - e\underline{A} \quad - (43)$$

and the interaction of the scattered electron with the scattered photon is treated with a minimal prescription. This process describes the time reversal of eq. (10), i.e.

$$\underline{p}' + \underline{p}'' \longrightarrow \underline{p} \quad - (44)$$

The time reversal of eq. (9) is:

$$E'' + E' = E + mc^2 \quad - (45)$$

Eqs. (44) and (45) describe a moving electron interacting with a moving photon to produce a static electron and moving photon.

This theory develops particle scattering theory into a relativistic quantum theory