## 247(2): Energy Released in Relativistic Particle Scattering.

It is considered that energy is released in particle scattering in general.

Consider the collision of a particle of mass $m_1$ from an initially stationary particle of mass $m_2$. The equation of conservation of energy is considered to be:

$$\gamma m_1 c^2 + m_2 c^2 = \gamma' m_1 c^2 + \gamma'' m_2 c^2 + E \quad -(1)$$

where $m_1$ and $m_2$ are considered to be the constant rest masses of the particles. Eq. (1) can be written as:

$$\omega + \omega_0 = \omega' + \omega'' + \frac{E}{\hbar} \quad -(2)$$

where

$$\hbar\omega = \gamma m_1 c^2 \quad -(3)$$

$$\hbar\omega_0 = m_2 c^2 \quad -(4)$$

$$\hbar\omega' = \gamma' m_1 c^2 \quad -(5)$$

$$\hbar\omega'' = \gamma'' m_2 c^2 \quad -(6)$$

The equation of conservation of momentum is:

$$\underline{p} = \underline{p}' + \underline{p}'' \quad -(7)$$

where:

2)

$$\underline{p} = \hbar \underline{\kappa} = \gamma m_1 \underline{v} \quad -(8)$$

$$\underline{p}' = \hbar \underline{\kappa}' = \gamma' m_1 \underline{v}' \quad -(9)$$

$$\underline{p}'' = \hbar \underline{\kappa}'' = \gamma'' m_2 \underline{v}'' \quad -(10)$$

As in note 246(6) it follows that:

$$\omega^2 + \omega'^2 - \omega''^2 = 2x_1^2 - x_2^2 + 2(\omega^2 - x_1^2)^{1/2}(\omega'^2 - x_1^2)^{1/2} \cos\theta \quad -(11)$$

where

$$x_1 = m_1 c^2 / \hbar, \quad -(12)$$

$$x_2 = \omega_0 = m_2 c^2 / \hbar. \quad -(13)$$

From eq. (2):

$$\omega'' = \omega - \omega' + \omega_0 - \frac{E}{\hbar} \quad -(14)$$

$$= \omega - \omega' + x_2 - \frac{E}{\hbar}$$

Let

$$x_3 = x_2 - \frac{E}{\hbar} \quad -(15)$$

then

$$\omega''^2 = x_3^2 + 2(\omega - \omega')x_3 + (\omega - \omega')^2 \quad -(16)$$

From eqs. (11) and (16):

$$\omega^2 + \omega'^2 - \left(x_3^2 + 2(\omega - \omega')x_3 + (\omega - \omega')^2\right)$$

$$= 2x_1^2 - x_2^2 + 2(\omega^2 - x_1^2)^{1/2}(\omega'^2 - x_1^2)^{1/2} \cos\theta$$

$$= -x_3^2 - 2(\omega - \omega')x_3 + 2\omega\omega' \quad -(17)$$

So

$$x_3^2 + 2(\omega - \omega')x_3 = 2\omega\omega' + x_2^2 - 2x_1^2 - 2(\omega^2 - x_1^2)^{1/2}(\omega'^2 - x_1^2)^{1/2}\cos\theta \quad -(18)$$

This is a quadratic in $x_3$:

$$x_3^2 + b x_3 + c = 0 \quad -(19)$$

Here:

$$b = 2(\omega - \omega') \quad -(20)$$

$$c = 2(\omega^2 - x_1^2)^{1/2}(\omega'^2 - x_1^2)^{1/2}\cos\theta + 2x_1^2 - x_2^2 + 2\omega\omega' \quad -(21)$$

Therefore:

$$x_3 = \frac{1}{2}\left(-b \pm (b^2 - 4c)^{1/2}\right) \quad -(22)$$

$$= x_2 - E/\hbar$$

Therefore:

$$E = \hbar\left(x_2 - \frac{1}{2}\left(-b \pm (b^2 - 4c)^{1/2}\right)\right) \quad -(23)$$

This is an expression for the energy released

4) In a scattering of mass $m_1$ from an initially stationary $m_2$. Eq. (23) can be written as:

$$\boxed{E = m_2 c^2 \mp \hbar \left( \omega' - \omega \pm \frac{1}{2} \left( b^2 - 4c \right)^{1/2} \right)} \quad -(24)$$

where b and c are defined by eqs. (20) and (21).

## Equal Mass Scattering at Ninety Degrees

In this case eq. (18) reduces to:

$$x_3^2 + 2(\omega - \omega')x_3 + x^2 - 2\omega\omega' = 0 \quad -(25)$$

So:

$$x_3 = \frac{1}{2}\left( 2(\omega' - \omega) \pm \left( 4(\omega - \omega')^2 + 4(2\omega\omega' - x^2) \right)^{1/2} \right) \quad -(26)$$

$$x_3 = \omega' - \omega + \left( \omega^2 + \omega'^2 - x^2 \right)^{1/2} \quad -(27)$$

$$= x - E/\hbar$$

where:

$$x = x_1 = x_2 = mc^2/\hbar \quad -(28)$$

So:

$$\boxed{E = mc^2 + \hbar \left( \omega - \omega' + \left( \omega^2 + \omega'^2 - \omega_0^2 \right)^{1/2} \right)} \quad -(29)$$

where

$$\omega_0 = x = mc^2/\hbar$$