243(6): Some Implications of the Concept of Mean Curvature in Radiation Theory

The mean curvature in radiation theory was defined in the previous note as:

$$\langle R \rangle = \frac{1}{\hbar^2 c^2} \langle E^2 \rangle \quad -(1)$$

where

$$\langle E^2 \rangle = \frac{\hbar^2 \omega^2}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1} \quad -(2)$$

so

$$\langle R \rangle = \frac{\omega^2}{c^2} \left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1 \right)^{-1} \quad -(3)$$

The mean energy in the Planck theory is:

$$\langle E \rangle = \frac{\hbar\omega}{\exp\left(\frac{\hbar\omega}{kT}\right) - 1} \quad -(4)$$

so:

$$\langle E \rangle = \frac{\hbar c^2}{\omega} \langle R \rangle \quad -(5)$$

From eqns. (1) and (5):

$$\langle E^2 \rangle = \hbar\omega \langle E \rangle \quad -(6)$$

From the ECE wave equation:

2)

$$(\Box + R)\, q^{a}_{\mu} = 0 \quad -(7)$$

it may be inferred that:

$$R = \left(\frac{mc}{\hbar}\right)^2 \quad -(8)$$

So

$$\langle R \rangle = \langle m^2 \rangle \frac{c^2}{\hbar^2} \quad -(9)$$

From eqs. (1) and (9):

$$\boxed{\langle m^2 \rangle = \frac{\hbar \omega}{c^4} \langle E \rangle} \quad -(10)$$

In the Planck Einstein theory $\langle m^2 \rangle$ is the mean square photon mass.

For a photon at rest:

$$\omega = \omega_0 \quad -(11)$$

and

$$\langle E \rangle = \hbar \omega_0 \quad -(12)$$

For moving photons:

$$\boxed{\langle m^2 \rangle = \frac{1}{c^4} \left(\frac{\hbar^2 \omega^2}{\exp\left(\frac{\hbar \omega}{kT}\right) - 1} \right)} \quad -(13)$$

This is a new expression for mean square photon mass. In the older de Broglie / Einstein theory:

$$E = \gamma m c^2 = \hbar \omega \quad -(14)$$

$$\underline{p} = \gamma m \underline{v} = \hbar \underline{\kappa} \quad -(15)$$

but as shown in UFT158 ff., eqs. (14) and (15) fail completely in Compton scattering and related effects.

The density of states in the Planck Einstein theory is defined by:

$$\rho(\omega) = \frac{dU}{d\omega} = \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{\left(\exp\left(\frac{\hbar\omega}{kT}\right) - 1\right)} \quad -(16)$$

So mean square photon mass is related to the density of states as follows:

$$\boxed{\omega c \pi^2 \langle m^2 \rangle = \hbar \rho(\omega)} \quad -(17)$$

So the mean square photon mass is quantized in terms of the density of states.

From eqs. (16) and (17):

4)

$$\frac{dU}{d\omega} = \frac{\pi^2 \omega c}{\hbar} \langle m^2 \rangle \quad -(18)$$

$$= \frac{\hbar}{\pi^2 c^3} \frac{\omega^3}{e^{\hbar\omega/kT} - 1} \quad -(19)$$

The total energy density of black body radiation is therefore:

$$U = \frac{\hbar}{\pi^2 c^3} \int_0^\infty \frac{\omega^3}{e^{\hbar\omega/kT} - 1} d\omega$$

$$= \frac{\pi^2 c}{\hbar} \int_0^\infty \omega \langle m^2 \rangle d\omega \quad -(20)$$

$$= \left(\left(\frac{\pi^2}{15} \right) \frac{k^4}{c^3 \hbar^3} \right) T^4$$

which relates the mean square photon mass to the Steffan Boltzmann law as follows:

$$\boxed{\int_0^\infty \omega \langle m^2 \rangle d\omega = \left(\frac{k^4}{15 c^4 \hbar^2} \right) T^4} \quad -(21)$$

This shows that the Steffan Boltzmann law

5) is due to mean square photon mass.

The mean energy of the Planck distribution is

$$\langle E \rangle = \frac{c^4}{\hbar\omega} \langle m^2 \rangle \quad -(22)$$

so the heat capacity is:

$$C_v = 3N \frac{d\langle E \rangle}{dT} = \frac{3Nc^4}{\hbar\omega} \frac{d\langle m^2 \rangle}{dT}$$

$$= 3Nk \left(\frac{\hbar\omega}{kT}\right)^2 \frac{e^{\hbar\omega/kT}}{\left(1 - e^{\hbar\omega/kT}\right)^2} \quad -(23)$$

so

$$\boxed{\frac{d\langle m^2 \rangle}{dT} = \frac{(\hbar\omega)^3}{(kT)^2} \frac{k}{c^4} \frac{e^{\hbar\omega/kT}}{\left(1 - e^{\hbar\omega/kT}\right)^2}}$$

$$-(24)$$

which is the capacity of mean square photon mass.