243(1): Relativistic Correction of the Einstein Theory of Heat Capacity of Solids.

Consider the quantum of energy:

$$E = \hbar \omega \quad -(1)$$

where $\hbar$ is the reduced Planck constant and $\omega$ the angular frequency. The Einstein theory of heat capacity of a crystal lattice assumes that the $3N$ atomic oscillators of a crystal have the same angular frequency $\omega$. The energy of the oscillator is defined by eq. (1). The total vibrational energy of the crystal is $3N\langle E \rangle$, where $\langle E \rangle$ is the mean vibrational energy of an oscillator:

$$\langle E \rangle = E \left( \frac{x}{1-x} \right) \quad -(2)$$

where

$$x = \exp\left( -\frac{\hbar \omega}{kT} \right) \quad -(3)$$

Here $k$ is Boltzmann's constant and $T$ the temperature. The heat capacity is:

$$C_V = 3N \frac{d\langle E \rangle}{dT} \quad -(4)$$

i.e.

$$C_V = \frac{3Nk\left(\hbar\omega/(kT)\right)^2 \exp\left(\hbar\omega/(kT)\right)}{\left(1 - \exp\left(\hbar\omega/(kT)\right)\right)^2} \quad -(5)$$

) This theory was corrected by Debye but is basically a direct expression of eq. (1). The theory does not consider momentum at all.

As in UFT 158 ff. momentum may be included by first assuming the de Broglie / Einstein equations:

$$E = \hbar\omega = \gamma m c^2 \quad -(6)$$

$$\underline{p} = \hbar \underline{\kappa} = \gamma m \underline{v} \quad -(7)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(8)$$

Therefore conservation of energy and momentum may be considered in the context of heat capacity theory, as in UFT 158 ff, studied thousands of times in the past few years. The mean relativistic energy is:

$$\langle E \rangle = \gamma m c^2 \left(\frac{x}{1-x}\right) \quad -(9)$$

where

$$x = \exp\left(-\frac{\gamma m c^2}{kT}\right) \quad -(10)$$

3) The relativistically corrected heat capacity is therefore:

$$C_v = 3N \frac{d\langle E \rangle}{dT} \quad -(11)$$

$$= \frac{3Nk\left(\gamma mc^2/(kT)\right)^2 \exp\left(\gamma mc^2/(kT)\right)}{\left(1 - \exp\left(\frac{\gamma mc^2}{kT}\right)\right)^2} \quad -(12)$$

and this can be plotted by computer algebra. The velocity of the Lorentz factor in eq. (12) is defined by eq. (6):

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} = \frac{\hbar\omega}{mc^2} \quad -(13)$$

so

$$v^2 = c^2\left(1 - \left(\frac{mc^2}{\hbar\omega}\right)^2\right) \quad -(14)$$

Therefore $C_v$ may be calculated as a function of $v$.

Similarly the mean relativistic momentum is

$$\langle \underline{p} \rangle = \underline{p}\left(\frac{x}{1-x}\right) \quad -(15)$$

), where $x$ is given by eq. (10). So:

$$\langle \underline{p} \rangle = \gamma m \underline{v} \left[ \frac{\exp\left(-\frac{\gamma m c^2}{kT}\right)}{1 - \exp\left(-\frac{\gamma m c^2}{kT}\right)} \right] \quad -(16)$$

The mean velocity of the oscillator is:

$$\langle \underline{v} \rangle = \frac{\langle \underline{p} \rangle}{m\gamma} \quad -(17)$$

A Compton effect and photoelectric effect may now be constructed by considering a photon interacting with the oscillator of the solid. After impact of a photon or other particle, the final heat of the solid is changed, so its energy of capacity is changed.

Finally it is reasonable to define a new thermodynamic quantity:

$$C_p = 3N \frac{dp}{dT} \quad -(18)$$

which can be named the "momentum capacity".