242(9): True Anomaly from Minkowski Force Equation for the General Velocity v.

Consider the fundamental wave equation:

$$\frac{d^2 r}{dt^2} + \Omega^2 r = 0 \quad -(1)$$

The general solution is:

$$r = r_0 e^{i\Omega t} \quad -(2)$$

$$\frac{dr}{dt} = i\Omega r \quad -(3)$$

In general the velocity is:

$$\underline{v} = \frac{dr}{dt} \underline{e}_r + \omega r \underline{e}_\theta \quad -(4)$$

$$\underline{v}^* = \left(\frac{dr}{dt}\right)^* \underline{e}_r + \omega r^* \underline{e}_\theta \quad -(5)$$

So:

$$v^2 = \left(\frac{dr}{dt}\right)\left(\frac{dr}{dt}\right)^* + \omega^2 r r^* \quad -(6)$$

where

$$r r^* = r_0^2 \quad -(7)$$

and

$$\omega^2 = \frac{L_0^2}{m^2 r^4} \quad -(8)$$

So:

$$v^2 = \Omega^2 r^2 + \frac{L_0^2 r_0^2}{m^2 r^4} \quad -(9)$$

2)

Therefore the true anomaly in general is:

$$\theta = \sqrt{2}\,\frac{L_0}{2m}\int \frac{f(r)}{r^2}\,dr \quad -(10)$$

where

$$f(r) = \left(-\int r\,\Omega^2\,dr - x\right)^{-1/2} \quad -(11)$$

and

$$\Omega^2 = -\frac{L_0^2}{m^2 r^4} - \frac{F(r)}{mr} \quad -(12)$$

where

$$F(r) = -\gamma^4\,\frac{mMG}{r^2} - \gamma^2\,\frac{\alpha m M}{r^3}\left(1-\gamma^2\right) \quad -(13)$$

and

$$\gamma^2 = \left(1 - \frac{v^2}{c^2}\right)^{-1} \quad -(14)$$

with

$$v^2 = \Omega^2 r^2 + \frac{L_0^2 r_0^2}{m^2 r^4} \quad -(15)$$

From eqs (12) to (15), Ω^2 can be found in terms of r, m, G and α.

However it is more straightforward to use the Minkowski force equation in the form of eq. (32) of note 238(2):

3)
$$\underline{F} = \left(\gamma^4 m \frac{d^2 r}{dt^2} - \gamma^2 \frac{L_0^2}{mr^3}\right)\underline{e}_r$$
$$+ \left(\frac{\gamma^4}{c^2}\frac{dr}{dt}\frac{d^2 r}{dt^2}\right) m\omega r\, \underline{e}_\theta \quad -(16)$$

in which:
$$\frac{d^2 r}{dt^2} = -\Omega^2 r \quad -(17)$$
$$\frac{dr}{dt} = i\Omega r \quad -(18)$$
$$\omega = \frac{L_0}{mr^2} \quad -(19)$$

Therefore:
$$F = \left(A^2 + B^2\right)^{1/2} \quad -(20)$$

where
$$A^2 = \left(\gamma^4 m\Omega^2 r + \frac{\gamma^2 L_0^2}{mr^3}\right)^2 \quad -(21)$$
$$B^2 = \frac{\gamma^8}{c^4}\Omega^2 r^2 \left(\Omega^2 r\right)^2 m^2 \left(\frac{L_0}{mr}\right)^2 \quad -(22)$$
$$\gamma^2 = \left(1 - \frac{v^2}{c^2}\right)^{-1} \quad -(23)$$
$$v^2 = \Omega^2 r^2 + \frac{L_0^2 r_0^2}{m^2 r^4} \quad -(24)$$

4) From eqs. (13) and (20) to (24), Ω can found in terms of L_0 and r, together with r_0 and m.

Finally θ can be found from eqs. (10) and (11). This is a general scheme that makes no approximation

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