242(10): General Expression for Velocity

From the wave equation:

$$\frac{d^2 r}{dt^2} + \Omega^2 r = 0 \quad -(1)$$

it is found that:

$$\frac{dr}{dt} = -\int \Omega^2 r \, dt + A \quad -(2)$$

The velocity is:

$$\underline{v} = \frac{dr}{dt} \underline{e}_r + \frac{L_0}{mr} \underline{e}_\theta \quad -(3)$$

so

$$v^2 = \left(\frac{dr}{dt}\right)^2 + \left(\frac{L_0}{mr}\right)^2 \quad -(4)$$

The Lorentz factor is:

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(5)$$

From eq. (1):

$$t = \frac{1}{\sqrt{2}} \int \left(-\int r\Omega^2 dr - x\right)^{-1/2} dr + y \quad -(6)$$

so:

$$\frac{dt}{dr} = \frac{1}{\sqrt{2}} \left(-\int r\Omega^2 dr - x\right)^{-1/2} \quad -(7)$$

and

$$\frac{dr}{dt} = \sqrt{2} \left(-\int r\Omega^2 dr - x\right)^{1/2} \quad -(8)$$

2) Therefore:

$$\Omega^2 = -\frac{L_0^2}{m^2 r^4} - \frac{F(r)}{mr} \quad -(9)$$

$$\underline{F} = \left(\gamma^4 m \frac{d^2 r}{dt^2} - \frac{\gamma^2 L_0^2}{m r^3}\right) \underline{e}_r \quad -(10)$$
$$+ \left(\frac{\gamma^4}{c^2} \frac{dr}{dt} \frac{d^2 r}{dt^2}\right) \frac{L_0}{mr} \underline{e}_\theta$$

$$\gamma^2 = \left(1 - \frac{v^2}{c^2}\right)^{-1} \quad -(11)$$

$$v^2 = \left(\frac{dr}{dt}\right)^2 + \left(\frac{L_0}{mr}\right)^2 \quad -(12)$$

Eqs. (8) to (12) express Ω^2 in terms of dr/dt, $\frac{d^2 r}{dt^2}$, r and constants. This set must be solved for Ω.

From eq. (8), the true anomaly is:

$$\theta = \sqrt{2}\,\frac{L_0}{2m} \int \frac{f(r)}{r^2}\, dr \quad -(13)$$

$$f(r) = \left(-\int r\Omega^2\, dr - x\right)^{-1/2} \quad -(14)$$
$$= \sqrt{2}\, \frac{dt}{dr}$$

3) so $\theta = \frac{L_0}{m}\int \frac{dt}{r^2}$ — (15)

i.e $\frac{d\theta}{dt} = \frac{L_0}{mr^2}$ — (16)

Q.E.D.

The calculations are self consistent but complicated.