242(7): Baseline Check by Computer

From note 242(6) the true anomaly is:

$$\theta = \sqrt{2}\,\frac{A}{T}\int \frac{f(r)}{r^2}\,dr \quad -(1)$$

where

$$f(r) = \left(-\int r\,\Omega^2\,dr - x\right)^{-1/2} \quad -(2)$$

and

$$\Omega^2 = -\frac{L_0^2}{m^2 r^4} - \frac{F(r)}{mr}. \quad -(3)$$

The basic wave equation is:

$$\frac{d^2 r}{dt^2} + \Omega^2 r = 0 \quad -(4)$$

The Newtonian force is:

$$F(r) = -\frac{L_0^2}{\alpha m r^2} \quad -(5)$$

in which case:

$$f(r) = \left(\frac{2L_0^2 r - L_0^2\alpha}{2\alpha m^2 r^2} - A\right)^{-1/2} \quad -(6)$$

For the square root to be real:

$$L_0^2 > 2\alpha^2 x m^2 \quad -(7)$$

The Newtonian result from eq. (1) is therefore:

2)

$$\boxed{\theta = \gamma - \frac{2mA}{L_0 T} \sin^{-1}\left(\frac{\frac{\alpha}{r} - 1}{(1 - 2\alpha^2 x m^2)^{1/2}}\right) \quad -(8)}$$

If:

$$x = -\frac{L_0^2}{2\alpha^2 m^2}(\epsilon - 1)(\epsilon + 1), \quad -(9)$$

$$\gamma = \pi \quad -(10)$$

then:

$$\theta = \frac{mA}{L_0 T}\left(\pi - 2\sin^{-1}\left(\frac{\frac{\alpha}{r} - 1}{((\epsilon-1)(\epsilon+1)+1)^{1/2}}\right)\right) \quad -(11)$$

i.e.:

$$r = \frac{\alpha}{1 + \epsilon \cos\left(\frac{L_0 T}{2mA}\theta\right)} \quad -(12)$$

For any curve in Lagrangian dynamics:

$$dt = \frac{2m}{L_0} dA \quad -(13)$$

3) (J. B. Marion and S. T. Thornton, "Classical Dynamics" (Harcourt, 1988, 3rd ed.))

The area of the ellipse is swept out in time T, so

$$\int_0^T dt = \frac{2m}{L_0}\int_0^A dA \quad -(14)$$

so

$$T = \frac{2m}{L_0}A \quad -(15)$$

From eqs. (12) and (15):

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(16)$$

QED. In Newtonian dynamics, the orbit is an ellipse with area:

$$A = \pi ab \quad -(17)$$

where

$$a = \frac{\alpha}{1-\epsilon^2} = \frac{mMG}{2|E|} \quad -(18)$$

$$b = \frac{\alpha}{(1-\epsilon^2)^{1/2}} = \frac{L_0}{(2m|E|)^{1/2}} \quad -(19)$$

So:

$$T = \frac{2m}{L_0}\cdot\pi ab = \pi mMG\left(\frac{m}{2}\right)^{1/2}E^{-3/2} \quad -(20)$$

4) The semiminor axis can be written as:

$$b = (\alpha a)^{1/2} \quad -(21)$$

where

$$\alpha = \frac{L_0^2}{m^2 \underline{M} G} \quad -(22)$$

so:

$$T^2 = \left(\frac{4\pi^2}{\underline{M} G}\right) a^3 \quad -(23)$$

which is Kepler's third law of planetary motion.

Eq (9) can be simplified by noting that:

$$a = \frac{\alpha}{1-\epsilon^2} = \frac{\alpha}{(1-\epsilon)(1+\epsilon)} \quad -(24)$$

so

$$x = \frac{L_0^2}{2a\alpha m^2} \quad -(25)$$

where:

$$L_0^2 = m^2 \underline{M} G \alpha \quad -(26)$$

so

$$\boxed{x = \frac{\underline{M} G}{2a}} \quad -(27)$$

which has the correct units of the square of

5) velocity.

The true anomaly may now be calculated for any force law and any planar orbit. For example if:

$$F(r) = -\frac{mMGx_1^2}{r^2} - \alpha(1-x_1^2)\frac{mMG}{r^3} \quad -(28)$$

the orbit from Lagrangian dynamics is:

$$r = \frac{\alpha}{1+\epsilon\cos(x_1\theta)} \quad -(29)$$

where x_1 is the precession factor. By using eq. (28) in eq. (1), eq. (29) should result, and x_1 can be calculated exactly.

Einsteinian general relativity claims that:

$$F(r) = -\frac{mMG}{r^2} - \frac{3L_0^2 MG}{m^2c^2r^4} \quad -(30)$$

The force law eq. (30) can now be used in eq. (1) to find the orbit, r as a function of θ.

6) The claim of EGR is that eq. (30) produces:

$$\theta = 2\pi(1 + x_1) \quad -(31)$$

per revolution of 2π, where:

$$x_1 = \frac{3GM}{ac^2(1-\epsilon^2)} \quad -(32)$$

From eqs. (15) and (11) the Newtonian result is:

$$\theta = \frac{1}{2}\left(\pi - 2\sin^{-1}\left(\frac{\frac{\alpha}{r} - 1}{((\epsilon-1)(\epsilon+1)+1)^{1/2}}\right)\right) \quad -(33)$$

If

$$\theta = 2\pi \quad -(34)$$

the Newtonian result from eq. (16) is:

$$r = \frac{\alpha}{1+\epsilon} \quad -(35)$$

According to EGR, the force law (30) must produce:

$$\theta \rightarrow 2\pi\left(1 + \frac{3GM}{ac^2(1-\epsilon^2)}\right) \quad -(36)$$

So using the force law (30) in eq. (1) must change eq. (35) to:

7)

$$r = \frac{\alpha}{1 + \epsilon \cos(x_1 \theta)} \quad (37)$$

If we consider the general equation (1), a change is forced from eq. (5) to eq. (30) should produce the result (37) if EGR is true. This is therefore a test of EGR.