241(2): Calculation of Precessions for Various Force Laws

In this note it is shown that precession of the perihelion can be caused by force laws of the type:

$$F(r) = -\frac{k}{r^2} - \frac{A_n}{r^{2+n}} \quad -(1)$$

where

$$n = 1, 2, 3, \ldots \quad -(2)$$

This means that the force law of the Einsteinian general relativity (EGR) is not unique.

1) Inverse Cubed

$$F(r) = -\frac{k}{r^2} - \frac{A_1}{r^3} \quad -(3)$$

The apsidal angle for an approximately circular orbit is:

$$\phi = \pi \left(3 + \frac{r}{F}\frac{dF}{dr} \right)^{-1/2} \quad -(4)$$

where

$$\frac{dF(r)}{dr} = \frac{2k}{r^3} + \frac{3A_1}{r^4} \quad -(5)$$

so

$$\phi = \pi \left(3 - \frac{\left(\frac{2k}{r^2} + \frac{3A_1}{r^3} \right)}{\left(\frac{k}{r^2} + \frac{A_1}{r^3} \right)} \right)^{-1/2} \quad -(6)$$

$$= \pi \left(\frac{\frac{k}{r^2}}{\frac{k}{r^2} + \frac{A_1}{r^3}} \right)^{-1/2}$$

2)
$$= \pi\left(1+\frac{A_1}{rk}\right)^{1/2}$$

If $A_1 << rk$ — (7)

then:
$$\psi \sim \pi\left(1+\frac{A_1}{2rk}\right) \quad -(8)$$

so
$$\boxed{\Delta\theta_1 = \frac{\pi A_1}{rk}} \quad -(9)$$

2) Inverse Fourth
$$F(r) = -\frac{k}{r^2} - \frac{A_2}{r^4} \quad -(10)$$

so
$$\boxed{\Delta\theta_2 = \frac{2\pi A_2}{kr^2}} \quad -(11)$$

3) Inverse Fifth
$$F(r) = \frac{-k}{r^2} - \frac{A_3}{r^5} \quad -(12)$$

so
$$\frac{dF}{dr} = \frac{2k}{r^3} + \frac{5A_3}{r^6} \quad -(13)$$

so
$$\psi = \pi\left(3 - \frac{\left(\frac{2k}{r^2}+\frac{5A_3}{r^5}\right)}{\left(\frac{k}{r^2}+\frac{A_3}{r^5}\right)}\right)^{-1/2}$$

$$= \pi \left(\frac{\frac{k}{r^2} - \frac{5A_3}{r^5}}{\frac{k}{r^2} + \frac{A_3}{r^5}} \right)^{-1/2}$$

$$= \pi \left(1 - \frac{5A_3}{kr^3}\right)^{-1/2} \left(1 + \frac{A_3}{kr^3}\right)^{1/2} \quad -(14)$$

If

$$5A_3 \ll kr^3 \quad -(15)$$

$$\psi \sim \pi \left(1 + \frac{5A_3}{2kr^3}\right)\left(1 + \frac{A_3}{2kr^3}\right)$$

$$\sim \pi \left(1 + \frac{3A_3}{kr^3}\right) \quad -(16)$$

So

$$\boxed{\Delta\theta_3 = \frac{6\pi A_3}{kr^3}} \quad -(17)$$

Therefore:

$$\Delta\theta_1 = \frac{\pi A_1}{mMGr}, \quad \Delta\theta_2 = \frac{2\pi A_2}{mMGr^2}, \quad \Delta\theta_3 = \frac{6\pi A_3}{mMGr^3}$$

$$-(18)$$

In EGR:

$$A_2 = \frac{3MGL^2}{mc^2} \quad -(19)$$

where:

$$L_0^2 = \alpha m^2 \underline{M} G \quad -(20)$$

so:

$$\Delta\theta_2 = \frac{2\pi}{r^2}\left(\frac{3\underline{M}G}{mc^2}\right)\left(\frac{\alpha m^2 \underline{M} G}{m\underline{M}G}\right)$$

$$= \frac{6\pi\alpha \underline{M} G}{c^2 r^2}$$

$$= \frac{3\pi \alpha r_0}{r^2} \quad -(21)$$

where

$$r_0 = \frac{2\underline{M}G}{c^2} \quad -(22)$$

For a nearly circular orbit:

$$\alpha \sim r \quad -(23)$$

so

$$\boxed{\Delta\theta_2 = 3\pi \frac{r_0}{r}} \quad -(24)$$

Consider:

$$\Delta\theta_1 = \frac{\pi A_1}{m\underline{M}Gr} \quad -(25)$$

then:

$$\Delta\theta_1 = \Delta\theta_2 \quad -(26)$$

if:

$$\frac{A_1}{m\underline{M}G} = 3r_0 = \frac{6\underline{M}G}{c^2} \quad -(27)$$

i.e. if

$$\boxed{A_1 = \frac{6m\underline{M}^2G^2}{c^2}} \quad -(28)$$

5) This means that the results of EGR are obtained by a force law:

$$F(r) = -\frac{mMG}{r^2} - \frac{3r_0 mMG}{r^3} \quad -(29)$$

$$\boxed{F(r) = -\frac{mMG}{r^2}\left(1 + 3\frac{r_0}{r}\right)} \quad -(30)$$

It is known that this force law produces the true precessing ellipse:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(31)$$

Eq. (31) produces the force law:

$$F(r) = -\frac{L_0^2 x^2}{mr^2\alpha} - \frac{L_0^2(1-x^2)}{mr^3} \quad -(32)$$

$$= -\frac{mMGx^2}{r^2} - \alpha(1-x^2)\frac{mMG}{r^3}$$

Eqs. (30) and (32) are the same if:

$$x \to 1 \quad -(33)$$

and

$$\alpha(1-x^2) = 3r_0 = \frac{6MG}{c^2} \quad -(34)$$

6) i.e if:

$$x^2 \sim 1 - \frac{6MG}{ac^2} \quad -(35)$$

because for a nearly circular orbit:

$$d \sim a, \quad -(36)$$

where d is the half right magnitude and where a is the semi major axis.

From eq. (36):

$$x \sim 1 - \frac{3MG}{ac^2} \quad -(37)$$

and this is the same result as note 24(4), eq. (11), for EGR.

Conclusion

For nearly circular orbits the true force law of a precessing ellipse is eq. (30) to an excellent approximation. This is another refutation of EGR, which uses the incorrect force law (10).