241(4): Definition of the Factor x from the Minkowski Equation

In the limit of a nearly circular orbit the force of the Minkowski equation is:

$$\underline{F} = \left(-\gamma^4 \frac{mMG}{r^2} - \gamma^2 \frac{\alpha mMG(1-\gamma^2)}{r^3} \right) \underline{e}_r \quad -(1)$$

For the true precessing ellipse:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(2)$$

$$\underline{F} = \left(-\frac{mMGx^2}{r^2} - \frac{\alpha(1-x^2)mMG}{r^3} \right) \underline{e}_r \quad -(3)$$

Comparing eqs. (1) and (3):

$$x^2 = \frac{r\gamma^4 + \alpha\gamma^2(1-\gamma^2) - \alpha}{r-\alpha} \quad -(4)$$

where

$$\gamma^2 = \left(1 - \frac{v^2}{c^2}\right)^{-1} \quad -(5)$$

and

$$v^2 = MG\left(\frac{2}{r} - \frac{1}{a}\right), \quad -(6)$$

$$a = \frac{\alpha}{\epsilon^2 - 1}. \quad -(7)$$

Now calculate t as a function of r as follows.

First, note that:

2)
$$L_0 = mr^2 \frac{d\theta}{dt} = mr^2 \frac{d\theta}{dr}\frac{dr}{dt}. \quad -(8)$$

From eq. (2): $$\frac{dr}{d\theta} = \frac{\epsilon x r^2}{\alpha}\sin(x\theta) \quad -(9)$$

and $$\cos(x\theta) = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(10)$$

with $$\cos^2(x\theta) + \sin^2(x\theta) = 1 \quad -(11)$$

So $$\sin(x\theta) = \left(1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2\right)^{1/2} \quad -(12)$$

and $$\frac{dr}{d\theta} = \frac{\epsilon x r^2}{\alpha}\left(1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2\right)^{1/2} \quad -(13)$$

and $$\frac{d\theta}{dr} = \frac{\alpha}{\epsilon x r^2}\left(1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2\right)^{-1/2} \quad -(14)$$

From eqs. (8) and (14):
$$L_0 = \frac{m\alpha}{\epsilon x}\left(1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2\right)^{-1/2}\frac{dr}{dt}$$
$$-(15)$$

So:

$$dt = \frac{m\alpha}{\epsilon x L_0}\left(1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2\right)^{-1/2} dr \quad - (16)$$

and

$$\boxed{t = \frac{m\alpha}{\epsilon L_0}\int \frac{1}{x}\left(1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2\right)^{-1/2} dr} \quad - (17)$$

in which x is given by eqs. (4) and (5).

In the limit of a nearly circular orbit:

$$a \sim r \quad - (18)$$

and

$$v^2 \sim \frac{MG}{r} \quad - (19)$$

so

$$\gamma^2 = \left(1 - \frac{MG}{c^2 r}\right)^{-1} \quad - (20)$$

Computer algebra is needed to integrate eq. (7) and find r in terms of t.
