240(3): Calculation of Precession for The True Precessing Ellipse.

The true precessing ellipse is:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(1)$$

and its force law is:

$$F = -\frac{L_0^2(1-x^2)}{mr^3} - \frac{L_0^2 x^2}{mr^2\alpha} \quad -(2)$$

For:

$$\epsilon \rightarrow 0 \quad -(3)$$

then

$$r \rightarrow \alpha \quad -(4)$$

and in this approximation the apsidal angle is:

$$\phi = \pi\left(3 + \frac{r}{F}\frac{\partial F}{\partial r}\right)^{-1/2} \quad -(5)$$

Here:

$$\frac{\partial F}{\partial r} = \frac{3L_0^2(1-x^2)}{mr^4} + \frac{2L_0^2 x^2}{mr^3 x} \quad -(6)$$

$$= \frac{L_0^2}{mr^3\alpha}\left(\frac{3}{r}(1-x^2)\alpha + 2x^2\right)$$

so:

$$\phi = \pi\left(3 - \frac{\left(\frac{3\alpha}{r}(1-x^2) + 2x^2\right)}{\frac{\alpha}{r}(1-x^2) + x^2}\right)^{-1/2}$$

$$= \pi \left(\frac{x^2}{\frac{\alpha}{r}(1-x^2) + x^2} \right)^{-1/2}$$

$$\boxed{\phi = \pi \left(1 + \frac{\alpha}{x^2 r}(1-x^2) \right)^{-1/2}} \quad -(7)$$

In the solar system:

$$x^2 \rightarrow 1 \quad -(8)$$

so

$$\phi \sim \pi \left(1 + \frac{(x^2-1)\alpha}{2x^2 r} \right) \quad -(9)$$

The perihelion advance is:

$$\Delta\theta = 2\Delta\phi = \frac{2\pi(x^2-1)\alpha}{2x^2 r} \quad -(10)$$

In previous work it was shown that:

$$\Delta\theta = 2\pi(x-1) \quad -(11)$$

so:

$$x - 1 = \frac{(x^2-1)}{2x^2}\left(\frac{\alpha}{r}\right) \quad -(12)$$

$$= \frac{(x-1)(x+1)}{2x^2}\left(\frac{\alpha}{r}\right)$$

and:

3)

$$\frac{\alpha}{r} = \frac{2x^2}{x+1} \quad -(13)$$

and
$$2x^2 = \frac{\alpha}{r}(x+1) \quad -(14)$$

The solution of eqn. (14) is:

$$\boxed{x = \frac{1}{4}\frac{\alpha}{r}\left(1 \pm \left(1 + 8\frac{r}{\alpha}\right)^{1/2}\right)} \quad -(15)$$

and as:
$$\frac{\alpha}{r} \to 1, \quad x \to 1 \quad -(16)$$

QED. The equivalent result from the Einstein theory is:

$$2\pi(x-1) = \frac{6\pi GM}{ac^2} \quad -(17)$$

as shown in note 240(2), i.e.

$$x(\text{Einstein}) = 1 + \frac{3GM}{ac^2} \quad -(18)$$

If the positive root is taken in eq. (15), then using:

$$\frac{r}{\alpha} \to 1 \quad -(19)$$

4) $$\boxed{x\,(\text{true processing ellipse}) \doteq \frac{\alpha}{r}} \quad -(20)$$

Finally use:

$$r \sim a, \quad \alpha = \frac{L_0^2}{m^2 M G} \quad -(21)$$

to find

$$x \sim \frac{L_0^2}{m^2 M G a} \quad -(22)$$

which is completely different from the Einstein result:

$$x\,(\text{Einstein}) = 1 + \frac{3L_0^2}{\alpha m^2 a c^2} \quad -(23)$$

Therefore EGR never gives the true precessing ellipse.
