240(8): Precession due to Nutation of the Earth.

This is an example of a standard calculation in astronomy. We will to examine the effect on this calculation of EGR. The standard potential energy is eq. (8.74) of R. Fitzpatrick, "Introduction to Celestial Mechanics" (C.U.P. 2012 and online):

$$\phi = -\frac{mMG}{r} + \frac{MG(I_{\parallel} - I_{\perp})}{2r^3}\left(\frac{3}{2}\sin^2\theta - 1\right) \quad -(1)$$

where m is the mass of the earth, M is the mass of the sun, G the Newton constant, r the mean distance between Earth and sun, and:

$$\theta = 23.44^\circ \quad -(2)$$

The moments of inertia $I_{\parallel}$ and $I_{\perp}$ are:

$$I_{\parallel} = 8.034 \times 10^{37}\ \text{kg m}^2 \quad -(3)$$

$$I_{\perp} = 8.008 \times 10^{37}\ \text{kg m}^2 \quad -(4)$$

because the earth is a symmetric top (K. Lambeck, "The Earth's Variable Rotation" (C.U.P. 1980)).

The force from eq. (1) is:

$$F = -\frac{\partial \phi}{\partial r} = -\frac{k}{r^2} - \frac{\epsilon}{r^4} \quad -(5)$$

where:

2) $$k = mG, \quad -(6)$$

$$\epsilon = -\frac{3}{2} mG \left(I_{\parallel} - I_{\perp} \right) \left(\frac{3}{2} \cos^2\theta - 1 \right) \quad -(7)$$

From note 240(2), eq. (32), the perihelion precession is:

$$\Delta\theta = \frac{2\pi \epsilon}{k r^2} \quad -(8)$$

i.e.

$$\Delta\theta = -\frac{6\pi \left(I_{\parallel} - I_{\perp} \right)}{m r^2} \left(\frac{3}{2} \cos^2\theta - 1 \right). \quad -(9)$$

For every revolution of 2π (one year) the earth's orbit moves backward by this amount. i.e.:

$$\Delta\theta = -9.72 \times 10^{-12} \text{ radians per year} \quad -(10)$$

Now use:

$$1 \text{ radian} = 2.0626 5 \times 10^{5} \text{ arc seconds} \quad -(11)$$

so $$\Delta\theta = -2.0 \times 10^{-4} \text{ arc seconds per century} \quad -(12)$$

Every century the earth's orbit moves backwards by 2.0×10^{-4} arc seconds per century because it is not a perfect sphere.

3)

Note carefully that eqns. (1) and (8) are entirely standard equations found in a textbook such as Fitzpatrick. They represent a Newtonian calculation.

However, if Einsteinian general relativity (EGR) is to be taken seriously the starting equation (1) is changed to:

$$\phi = -\frac{mMG}{r} - \frac{L_0^2 MG}{m c^2 r^3} + \frac{mG(I_{\parallel} - I_{\perp})}{2r^3}\left(\frac{3}{2}\sin^2\theta - 1\right) \quad -(13)$$

+ relativistic corrections of the quadrupole term

For the sake of simplicity assume that the quadrupole corrections are small, so:

$$F = -\frac{\partial \phi}{\partial r} = -\frac{mMG}{r^2} - \frac{3L_0^2 MG}{m c^2 r^4} - \frac{3mG}{2}(I_{\parallel} - I_{\perp})\left(\frac{3}{2}\sin^2\theta - 1\right) \quad -(14)$$

where

$$L_0^2 = \alpha m^2 MG \quad -(15)$$

This means that eq. (9) is changed to:

$$\boxed{\Delta\theta = 6\pi\left(\frac{\alpha MG}{c^2 r^2} - \frac{1}{2}\left(\frac{I_{\parallel} - I_{\perp}}{m r^2}\right)\left(\frac{3}{2}\cos^2\theta - 1\right)\right)} \quad -(16)$$

4) The earth's orbit is nearly circular, so:

$$d \sim r. \quad -(17)$$

where d is the half right latitude. So the relativistic correction is:

$$\Delta\theta = \frac{6\pi MG}{c^2 r} \quad -(18)$$

$$= 3\pi \frac{r_0}{r}$$

where $r_0 = \frac{2MG}{c^2} = 2.950 \times 10^3$ metres

$r = 1.49 \times 10^{11}$ metres

so $$\Delta\theta = 3\pi \times \frac{2.95 \times 10^3}{1.49 \times 10^{11}} \quad -(19)$$

$= 1.87 \times 10^{-7}$ radians per year

$= 3.86$ arc seconds per century.

The relativistic correction is about four orders of magnitude greater than the effect due to the quadrupole term and in the opposite direction.

This is complete disaster for EGR

5) Because it gives a wildly erroneous result. As discussed by Fitzpatrick, eq. (1) give a precession period normal to the ecliptic plane of 79, 200 years, and when corrected for the Moon gives 27, 600 years, which compares to the experimental value of 25, 800 years. EGR would change this by roughly four orders of magnitude, and in the wrong direction. This is an absurd result which immediately refutes EGR. It is clear that standard astronomy has applied EGR only to one phenomenon, the precession of the perihelion caused by:

$$\phi = -\frac{mMG}{r} - \frac{L_0^2 MG}{m^2 c^2 r^3} \quad -(20)$$

but has not applied EGR to any other phenomenon. For all other phenomena it has always used the Newtonian:

$$\phi = -\frac{mMG}{r} \quad -(21)$$
