240(9): Satellite Precession in the Gravitational Field of an Oblate Spheroid.

This is problem 5.8 of Fitzpatrick. The Newtonian potential energy is:

$$V(r) = -\frac{mMG}{r}\left(1+\frac{\epsilon}{r^2}\right) \quad -(1)$$

where m is the mass of a satellite, M is the mass of the earth, r is the distance from the centre of the earth to the satellite and:

$$\epsilon = \frac{2}{5} R \Delta R \quad -(2)$$

where R = 4000 miles is the earth's equatorial radius and ΔR = 13 miles is the difference between the equatorial and polar radius.

The force is:

$$F = -\frac{\partial V}{\partial r} = -\frac{A}{r^2} - \frac{B}{r^4}. \quad -(3)$$

The satellite is assumed to be in an equatorial orbit, and its perihelion precesses by:

$$\Delta\theta = \frac{2\pi B}{A r^2} = \frac{6\pi\epsilon}{r^2} \quad -(4)$$

$$= \frac{6\pi}{r^2}\left(\frac{2}{5} R \Delta R\right)$$

every orbit, i.e.

$$\Delta\theta = 6\pi\left(\frac{3.35\times10^7}{r^2}\right) \quad -(5)$$

2) where r is measured in metres.

However, the correct potential in EGR theory is:

$$V(r) = -\frac{mMG}{r}\left(1+\frac{\epsilon}{r^2}\right) - \frac{L_0^2 MG}{mc^2 r^3} \quad -(6)$$

where

$$L_0^2 = \alpha m^2 M G. \quad -(7)$$

For a nearly circular orbit:

$$\alpha \sim r \quad -(8)$$

so the correct perihelion precession is:

$$\boxed{\Delta\theta = 6\pi\left(\frac{\epsilon}{r^2} + \frac{MG}{c^2}\frac{1}{r}\right)} \quad -(9)$$

If EGR is correct this $\Delta\theta$ should be observable experimentally.

For the earth:

$$\frac{MG}{c^2} = 4.43 \times 10^{-3}\ \text{m} \quad -(10)$$

so the effect due to EGR is:

$$\Delta\theta = 3.89 \times 10^6 \times 4.43 \times 10^{-3} / r \quad -(11)$$

arc seconds per orbit

$$= 17.23 \times 10^3 / r$$

$$\boxed{\Delta\theta = 1.723 \times 10^4 / r \ \text{arc seconds per orbit}} \quad -(12)$$

An orbit such as that of Gravity Probe B is 650 km above the earth's surface. It is assumed that this is 4,000 miles + 650 km above the earth's centre.

So:

$$r = 4,000 \times 1.609 + 650 \text{ km} \\ = 7086 \text{ km} = 7.086 \times 10^{6} \text{ m} \quad -(13)$$

and

$$\Delta\theta = \frac{1.723 \times 10^{4}}{7.086 \times 10^{6}} \text{ arc second per orbit} \\ = 2.43 \times 10^{-3} \text{ arc second per orbit.} \quad -(14)$$

For a thousand orbits:

$$\boxed{\Delta\theta = 2.43 \text{ arc seconds per } 1,000 \text{ orbits}} \quad -(15)$$

<u>This is a very large effect</u> <u>and should be</u> <u>easily observable</u>. Gravity Probe B for example picked up a geodetic drift (or Thomas precession) of -6.60 arc seconds <u>a year</u> (many orbits).

At a height of 7.086×10^{6} m above the earth's centre the precession due to the oblate spheroidal effect is:

$$\Delta\theta = 6\pi \times \frac{3.35 \times 10^{7}}{r^{2}} \quad -(16)$$

radians per orbit

4) This is:

$$\Delta\theta = \frac{3.89 \times 10^{6} \times 3.35 \times 10^{7}}{7.086^{2} \times 10^{12}} \qquad -(17)$$

$$= 2.56 \text{ arc seconds per orbit.}$$

Results

At the height of Gravity Probe B:

$\Delta\theta$ (oblate spheroid) = 2.56 arc seconds per orbit

$\Delta\theta$ (EGR) $= 2.43 \times 10^{-3}$ arc seconds per orbit.

If $\Delta\theta$(EGR) existed it would have been observed easily by Gravity Probe B, but it was not observed. So EGR has been disproven experimentally.