240(7) : Relativistic Correction to the Fitzpatrick Calculation.

In this case the potential is:

$$\phi = -\frac{mMG}{r} - \frac{MGL_0^2}{mc^2r^3} \qquad -(1)$$

$$- \frac{mm_1G}{|\underline{a_1} - \underline{r}|} - \frac{m_1GL_0^2}{mc^2|\underline{a_1} - \underline{r}|^3}$$

assuming that the Einsteinian general relativity applies. The fourth term in eq. (1) is very small so can be neglected. So:

$$F = -\frac{\partial \phi}{\partial r} = -\frac{mMG}{r^2} - \frac{3MGL_0^2}{mc^2r^4} \qquad -(2)$$

$$- \frac{mm_1G}{a_1^2}\left(\frac{1}{2}\left(\frac{r}{a_1}\right) + \frac{9}{16}\left(\frac{r}{a_1}\right)^3\right)$$

$$\frac{dF}{dr} = \frac{2mMG}{r^3} + \frac{12MGL_0^2}{mc^2r^5} + \frac{mm_1G}{a_1^3}\left(\frac{1}{2} + \frac{27}{16}\left(\frac{r}{a_1}\right)^2\right)$$

So in the notation of the previous note: $-(3)$

$$\phi = \pi\left(\frac{3 + \frac{2mMG}{r^2} + \frac{12MGL_0^2}{mc^2r^4} + xr}{-\frac{mMG}{r^2} - \frac{3MGL_0^2}{mc^2r^4} + y}\right)^{-1/2}$$

$-(4)$

$$= \pi \left[\frac{-\frac{mMG}{r^2} + \frac{3MGL_0^2}{mc^2r^4} + 3y + xr}{-\frac{mMG}{r^2} - 3\frac{MGL_0^2}{mc^2r^4} + y} \right]^{-1/2}$$

$$= \pi \left[\frac{1 - \frac{r^2}{mMG}(3y + xr) - \frac{3MGL_0^2}{mc^2r^4}\frac{r^2}{mMG}}{1 - \frac{m_1}{M}\left(\frac{r}{a_1}\right)^3\left(\frac{1}{2} + \frac{9}{16}\left(\frac{r}{a_1}\right)^2\right) + \frac{3MGL_0^2r^2}{mc^2r^4mMG}} \right]^{-1/2}$$

$$= \pi \left[\frac{1 - \frac{r^2(3y + xr)}{mMG} - 3\left(\frac{L_0}{mcr}\right)^2}{1 - \frac{m_1}{M}\left(\frac{r}{a_1}\right)^3\left(\frac{1}{2} + \frac{9}{16}\left(\frac{r}{a_1}\right)^2\right) + 3\left(\frac{L_0}{mcr}\right)^2} \right]^{-1/2}$$

So:

$$\phi \sim \pi \left[\frac{1 - \frac{r^2}{mMG}(3y + xr) - A}{1 - \frac{r^2 y}{mMG} + A} \right]^{-1/2} \quad -(5)$$

where

$$A = 3\left(\frac{L_0}{mcr}\right)^2 \quad -(6)$$

In the notation of the previous note:

$$\phi \sim \pi \left(1 + \frac{x_1}{2} + \frac{A}{2}\right)\left(1 - \frac{y_1}{2} + \frac{A}{2}\right) \quad -(7)$$

3) i.e:

$$\psi \sim \pi\left[\left(1+\frac{x_1}{2}\right)\left(1-\frac{y_1}{2}\right)+\frac{A}{2}\left(\left(1+\frac{x_1}{2}\right)+\left(1-\frac{y_1}{2}\right)\right)+\frac{A^2}{4}\right] \quad -(8)$$

So the perihelia advances by:

$$\Delta\theta = 2\pi\left[\frac{m_1}{M}\left(\frac{r}{a_1}\right)^3\left(\frac{3}{4}+\frac{45}{32}\left(\frac{r}{a_1}\right)^2\right)+A + \frac{A}{2}\left(\frac{x_1}{2}-\frac{y_1}{2}\right)+\frac{A^2}{4}\right] \quad -(9)$$

Using the largest terms:

$$\Delta\theta = 2\pi\left[\frac{m_1}{M}\left(\frac{r}{a_1}\right)^3\left(\frac{3}{4}+\frac{45}{32}\left(\frac{r}{a_1}\right)^2\right)+A\right]$$

$$\boxed{\Delta\theta \sim \frac{3\pi}{2}\cdot\frac{m_1}{M}\left(\frac{r}{a_1}\right)^3+6\pi\left(\frac{L_0}{mcr}\right)^2} \quad -(10)$$

Finally use the Newtonian approximation:

$$L_0^2 = d\,m^2 M G \quad -(11)$$

to express eq. (10) as:

4)

$$\Delta\theta \sim \frac{3\pi}{2}\frac{m_1}{M}\left(\frac{r}{a_1}\right)^3 + \frac{6\pi\, d\, M G}{c^2 r^2} \quad -(12)$$

For an approximately circular orbit:

$$d \sim r \quad -(13)$$

so

$$\Delta\theta \sim \frac{3\pi}{2}\frac{m_1}{M}\left(\frac{r}{a_1}\right)^2 + \frac{6\pi M G}{c^2 r} \quad -(14)$$

For the mercury / sun system, the second term is about 43" per century.

However, the system being considered is one consisting of two planets of mass m and m_1 interacting with the sun of mass M. Here m is the mass of Mercury and m_1 the mass of some other planet, such as Venus. If we now consider another planet of mass m_2, such as the earth there are two potential functions:

$$\phi_1 = -\frac{m M G}{r} - \frac{M G L^2}{m c^2 r^3} - \frac{m m_1 G}{|\underline{a}_1 - \underline{r}|} \quad -(15)$$

and

$$\phi_2 = -\frac{m M G}{r} - \frac{M G L^2}{m c^2 r^3} - \frac{m m_2 G}{|\underline{a}_2 - \underline{r}|} \quad -(16)$$

The first one ϕ_1 is the potential energy

5) of Mercury interacting simultaneously with the sun and Venus. The second one ϕ_2 is the potential energy of Mercury interacting simultaneously with the sun and the earth.

The total potential energy is:

$$\phi = \phi_1 + \phi_2 \quad -(17)$$

of Mercury interacting with the sun and Venus added to that of Mercury interacting with the sun and the earth.

In the Newtonian case:

$$\phi_1 (\text{Newton}) = -\frac{mMG}{r} - \frac{mm_1G}{|\underline{a}_1 - \underline{r}|} \quad -(18)$$

and

$$\phi_2 (\text{Newton}) = -\frac{mMG}{r} - \frac{mm_2G}{|\underline{a}_2 - \underline{r}|} \quad -(19)$$

and

$$\phi = \phi_1 + \phi_2 \quad -(20)$$

Eq. (18) gives:

$$\Delta\theta_1 = \frac{3\pi}{2}\frac{m_1}{M}\left(\frac{r}{a_1}\right)^2 \quad -(21)$$

and Eq. (19) gives:

$$\Delta\theta_2 = \frac{3\pi}{2}\frac{m_2}{M}\left(\frac{r}{a_2}\right)^2 \quad -(22)$$

6) and the total perihelion precession is:

$$\Delta\theta = \Delta\theta_1 + \Delta\theta_2 . \quad -(23)$$

In order to calculate $\Delta\theta_1$ and $\Delta\theta_2$, the term $-mMG/r$ has to be present twice, in the definition of ϕ_1 and of ϕ_2.

In the relativistic calculation, the relativistic term $MGL_0^2/(mc^2r^3)$ also has to be present twice, giving:

$$\Delta\theta_1 = \frac{3\pi}{2}\frac{m_1}{M}\left(\frac{r}{a_1}\right)^2 + \frac{6\pi MG}{c^2 r} \quad -(24)$$

and

$$\Delta\theta_2 = \frac{3\pi}{2}\frac{m_2}{M}\left(\frac{r}{a_2}\right)^2 + \frac{6\pi MG}{c^2 r} \quad -(25)$$

so

$$\boxed{\Delta\theta \rightarrow \Delta\theta(\text{Newton}) + 2A} \quad -(26)$$

So each of the perihelion precessions due to the Newtonian calculation is supplemented by the factor A.

Therefore EGR is incorrect because it considers A only once, for the Mercury sun system only, i.e. for:

$$\phi = -\frac{mMG}{r} - \frac{MGL_0^2}{mc^2r^3} \quad -(27).$$

without consideration of any planetary perturbations.

1) This glaring error of logic by Einstein has also been pointed out by Miles Mathis and others. The easiest way to see this error by Einstein is to realize that the perihelion precession $\Delta\theta_1$ requires $-mMG/r$ to be present in eq. (18). Similarly $\Delta\theta_2$ needs this term to be present in eq. (19). So $-m\underline{M}G/r$ is present twice in the total potential:

$$\phi = \phi_1 + \phi_2 = -2\frac{mMG}{r} - \frac{mm_1G}{|\underline{a}_1 - \underline{r}|} - \frac{mm_2G}{|\underline{a}_2 - \underline{r}|} \quad -(27)$$

i.e.:

$$\phi = \underbrace{\left(-\frac{mMG}{r} - \frac{mm_1G}{|\underline{a}_1 - \underline{r}|}\right)}_{\substack{\downarrow \\ \Delta\theta_1}} + \underbrace{\left(-\frac{mMG}{r} - \frac{mm_2G}{|\underline{a}_2 - \underline{r}|}\right)}_{\substack{\downarrow \\ \Delta\theta_2}} \quad -(28)$$

$$\Delta\theta(\text{observed}) = \Delta\theta_1 + \Delta\theta_2 \quad -(29)$$

If:

$$\phi = \underbrace{\left(-\frac{m\underline{M}G}{r} - \frac{mm_1G}{|\underline{a}_1 - \underline{r}|}\right)}_{\substack{\downarrow \\ \Delta\theta_1}} \underbrace{- \frac{mm_2G}{|\underline{a}_2 - \underline{r}|}}_{\substack{\downarrow \\ ?}} \quad -(30)$$

8) then $\Delta\theta_2$ is undefined because $-mMG/r$ has already been used for $\Delta\theta_1$.

In order to see this in another way, note that $\Delta\theta_1$ is produced by m_1 interacting with an orbit. The orbit is produced by m interacting with M. The force for this system is F_1 and:

$$\phi_1 = \pi \left(3 + \frac{r}{F_1}\frac{dF_1}{dr}\right)^{-1/2} \quad -(31)$$

Similarly, $\Delta\theta_2$ is produced by m_2 interacting with the orbit produced by the interaction of M and m. The force for this system is F_2 and:

$$\phi_1 = \pi \left(3 + \frac{r}{F_2}\frac{dF_2}{dr}\right)^{-1/2} \quad -(32)$$

This procedure assumes that the interaction of m_1 and m_2 is negligible. The total result is:

$$\phi = \phi_1 + \phi_2 \quad -(33)$$

as observed experimentally.

So to produce ϕ_1, m_1 must interact with m interacting with M. In order to produce ϕ_2, m_2 must interact with m interacting with M.

The older procedure is to assume:

$$\phi = \pi \left(3 + \frac{r}{F}\frac{dF}{dr} \right)^{-1/2} \quad -(34)$$

where:

$$F = -\frac{mMG}{r^2} - \frac{mm_1G}{|\underline{a}_1 - \underline{r}|^2} - \frac{mm_2G}{|\underline{a}_2 - \underline{r}|^2} \quad -(35)$$

In the Newtonian approximation eqs. (33) and (34) give the same result and the experimental data cannot distinguish between the two methods.

However, the two methods give different results once the relativistic term is added.

In eq. (33):

$$F_1 = -\frac{mMG}{r^2} - \frac{mm_1G}{|\underline{a}_1 - \underline{r}|^2} \quad -(36)$$

$$F_2 = -\frac{mMG}{r^2} - \frac{mm_2G}{|\underline{a}_2 - \underline{r}|^2}. \quad -(37)$$

In the relativistic case:

$$F = -\frac{mMG}{r^2} - \frac{3MGL_0^2}{mc^2r^4} - \frac{mm_1G}{|\underline{a}_1 - \underline{r}|^2} - \frac{mm_2G}{|\underline{a}_2 - \underline{r}|^2} \quad -(38)$$

$$F_1 = -\frac{mMG}{r^2} - \frac{3MGL_0^2}{mc^2r^4} - \frac{mm_1G}{|\underline{a}_1 - \underline{r}|^2} \quad -(39)$$

$$F_2 = -\frac{mMG}{r^2} - \frac{3MGL_0^2}{mc^2r^4} - \frac{mm_2G}{|\underline{a}_2 - \underline{r}|^2} \quad -(40)$$

and the relativistic precession for eq. (33) is

10) twice that from eq. (34).

For the reasons stated in this note it is considered that the correct calculation is based on eq. (33). If the calculation is not based on eq. (33) it would not be possible to calculate the perihelion precession of a given planet of mass m due to another planet of mass m_1. It would only be possible to calculate the precession due to a collection of masses $m_1, m_2, \ldots, m_n$.

This is a reduction to absurdity argument against EGR because it must always be possible to calculate the effect of a given mass such as m_1 on the interacting system m and M.

EGR adds the relativistic correction eq. (38). As only one, by assuming eq. (38). As soon as eqs. (39) and (40) are used, the relativistic correction appears twice, and EGR fails.