240(4): Check on 240(3)

By computer algebra, eq. (7) was found to be:

$$\phi = \frac{\pi}{x\left(\frac{r}{(r-\alpha)x^2+\alpha}\right)^{1/2}} \quad -(1)$$

This can be shown to be eq. (7) as follows:

$$\phi = \pi\left(\frac{x^2 r}{(r-\alpha)x^2+\alpha}\right)^{-1/2} \quad -(2)$$

$$= \pi\left(\frac{r}{r-\alpha+\frac{\alpha}{x^2}}\right)^{-1/2}$$

$$= \pi\left(\frac{1}{1-\frac{\alpha}{r}+\frac{\alpha}{r}\frac{1}{x^2}}\right)^{-1/2}$$

$$= \pi\left(1+\frac{\alpha}{rx^2}\left(1-x^2\right)\right)^{1/2}$$

Therefore:

$$\phi \sim \pi\left(1+\frac{1}{2}\frac{\alpha}{rx^2}\left(1-x^2\right)\right) \quad -(3)$$

The perihelion advances by:

$$\Delta\theta = 2\Delta\phi = \frac{2\pi\left(1-x^2\right)\alpha}{2x^2 r} \quad -(4)$$

In previous work it was shown that:

2)

$$\Delta\theta = 2\pi(1-x) \quad -(5)$$

so $$1 - x = \frac{(1-x^2)}{2x^2}\left(\frac{\alpha}{r}\right) \quad -(6)$$

and $$2x^2 = \frac{\alpha}{r}(x+1) \quad -(7)$$

i.e. $$x = \frac{1}{4}\frac{\alpha}{r}\left(1 \pm \left(1 + \frac{8r}{\alpha}\right)^{1/2}\right) \quad -(8)$$

and as: $$\frac{\alpha}{r} \to 1, \; x \to 1 \quad -(9)$$

Q.E.D.

The equivalent result for the Einstein theory is:

$$2\pi(1-x) = 6\pi\frac{GM}{ac^2} \quad -(10)$$

as shown in note 240(2), i.e.

$$x\,(\text{Einstein}) = 1 - \frac{3GM}{ac^2} \quad -(11)$$

If the positive root is taken in eq. (8), then using:

$$\frac{r}{\alpha} \to 1 \quad -(12)$$

then $$x\,(\text{true precessing ellipse}) \doteq \frac{\alpha}{r} \quad -(13)$$

Finally use:

3)

$$r \sim a, \quad \alpha \sim \frac{L_0^2}{m^2 M G} \qquad -(14)$$

to find:

$$x \sim \frac{L_0^2}{m^2 M G a} \qquad -(15)$$

which is completely different from the Einstein result:

$$x(\text{Einstein}) = 1 - \frac{3L_0^2}{\alpha m^2 a c^2} \qquad -(16)$$

Therefore EGR never gives the true precessing ellipse.