240(10): Collected Results of Relativistic Corrections to Some Formulae of Fitzpatrick

Correcting the formula:

$$V(r) = -\frac{mMG}{r}\left(1+\frac{\epsilon}{r^2}\right) \quad -(1)$$

for a symmetric top it is found that:

$$V(r) = -\frac{mMG}{r}\left(1+\frac{\epsilon}{r^2}\right) - \frac{L_0^2 MG}{mc^2 r^3} \quad -(2)$$

where

$$L_0^2 \sim r m^2 MG \quad -(3)$$

for a nearly circular orbit. Here:

$$\epsilon = R\Delta R \quad -(4)$$

where $R$ is the equatorial radius and $\Delta R$ the difference between the equatorial and polar radii. The perihelion precession from eq. (1) is:

$$\Delta\theta = 6\pi\left(\frac{\epsilon}{r^2} + \frac{MG}{c^2 r}\right) \quad -(5)$$

Denote

$$\Delta\theta(\text{oblate}) = \frac{6\pi\epsilon}{r^2} \quad -(6)$$

$$\Delta\theta(\text{EGR}) = \frac{6\pi MG}{c^2 r} \quad -(7)$$

Apply eqs. (6) and (7) to three systems:

2)

1) Earth / sun ; 2) earth / moon ; 3) earth / Gravity Probe B

1) Earth Sun

Equatorial radius $\Big[$ of sun $= 5.96 \times 10^{8}$ m

Mean radius $= 6.96 \times 10^{8}$ m

Polar radius $= 7.96 \times 10^{8}$ m

So $R = 5.96 \times 10^{8}$ m

$\Delta R = 2.0 \times 10^{8}$ m

and $R \Delta R = 1.39 \times 10^{17} \text{ m}^{2}$ — (8)

Earth sun distance $= 1.49 \times 10^{11}$ m

So

$$\boxed{\begin{aligned}\Delta\theta\,(\text{oblate}) &= 4.73 \times 10^{-5} \text{ radians per orbit}\\ \Delta\theta\,(\text{EGR}) &= 1.87 \times 10^{-7} \text{ radians per orbit}\end{aligned}}$$

— (9)

2) Earth Moon

In this case use:

$R \Delta R\,(\text{earth}) = 8.37 \times 10^{7} \text{ m}^{2}$ — (10)

Earth Moon distance $= 3.844 \times 10^{9}$ m — (11)

So

$$\boxed{\begin{aligned}\Delta\theta\,(\text{oblate}) &= 4.27 \times 10^{-11} \text{ radians per orbit}\\ \Delta\theta\,(\text{EGR}) &= 2.20 \times 10^{-11} \text{ radians per orbit.}\end{aligned}}$$

— (12)

3) ~~Earth~~ Gravity Probe B

From note 240(9):

$$\Delta\theta(\text{oblate}) = 2.56'' \text{ per orbit}$$
$$\Delta\theta(\text{EGR}) = 2.43 \times 10^{-3}'' \text{ per orbit.} \quad -(13)$$

Use:

$$1 \text{ radian} = 2.06265 \times 10^{5}'' \quad -(14)$$

So

$$\boxed{\begin{aligned}\Delta\theta(\text{oblate}) &= 1.24 \times 10^{-5} \text{ radians per orbit} \\ \Delta\theta(\text{EGR}) &= 1.18 \times 10^{-8} \text{ radians per orbit.}\end{aligned}} \quad -(15)$$

These results show already that a classical calculation would be changed greatly by EGR if the latter were applied logically to all phenomena. The largest effect is the earth moon system, where the classical result would be changed from $4.27 \times 10^{-11}$ radians per orbit to $6.47 \times 10^{-11}$ radians per orbit. The orbital period of the moon is 27 days. So in 27 days the moon advances by $4.27 \times 10^{-11}$ radian due to the oblate shape of the earth. In one year of 365 days it would advance by $5.77 \times 10^{-10}$ radians.

4) It would take $1.09 \times 10^{10}$ years for the moon to advance by $2\pi$ radians. The relativistic correction would change this to $1.65 \times 10^{10}$ years.

For the earth/sun system the classical precession of the earth due to the oblate nature of the sun is $4.73 \times 10^{-5}$ radians per orbit, and the EGR effect changes this to $(4.73 + 0.0187) \times 10^{-5}$ $= 4.749 \times 10^{-5}$ radians per orbit. In the classical calculation it would take $2\pi / 4.73 \times 10^{5}$ $= 132,837$ years for the earth to precess by $2\pi$ radians, but in the relativistic calculation it would take 133,371 years.

To make matters worse Fitzgerald gives a different formula to eq. (1) for the precession of the moon around the earth:

$$V(r) = -\frac{G m M}{r} + \frac{G m (I_{\parallel} - I_{\perp}) P_2 \cos(\gamma_m)}{r^3} \quad -(16)$$

where $m$ is the mass of the moon, $M$ is the mass of the earth and $\gamma_m$ is the angle subtended by the earth's angular velocity vector and $\underline{r}_m$, the position of the moon relative to the earth.

5) Eq. (16) gives the oblate precession:

$$\Delta\theta = \frac{6\pi\left(I_\perp - I_\parallel\right) P_2\left(\cos\gamma_m\right)}{M r^2} \quad -(17)$$

where:

$$I_\parallel = 8.034 \times 10^{37}\ kg\ m^2 \quad -(18)$$
$$I_\perp = 8.008 \times 10^{37}\ kg\ m^2 \quad -(19)$$
$$M = 6.05 \times 10^{24}\ kg \quad -(20)$$
$$r = 3.844 \times 10^{9}\ m \quad -(21)$$

So

$$\boxed{\Delta\theta\,(\text{oblate}) = -5.48 \times 10^{-8} \text{ rad per orbit}}$$

This is nearly three orders of magnitude greater than formula (1), although both formulae are claimed to describe the same phenomenon. It has been assumed for the sake of simplicity that:

$$P_2(\cos\gamma_m) \sim 1 \quad -(22)$$

As in note 240(8), Fitzpatrick gives a formula analogous to eq. (16) for the precession of the earth around the sun due to nutation:

$$\phi = -\frac{mMG}{r} + \frac{MG\left(I_\parallel - I_\perp\right)}{2r^3}\left(\frac{3}{2}\sin^2\theta - 1\right) \quad -(23)$$

Eq. (23) results in:

b)

$$\Delta\theta(\text{oblate}) = -3.65 \times 10^{-11} \text{ radian per orbit} \qquad -(24)$$

which is seven orders of magnitude smaller than the result (a).

This means that the oblate shape of the sun is by far the greater effect. This causes the earth to precess by $4.73 \times 10^{-5}$ radians per orbit, i.e. this is 9.76" per orbit. 976 arc seconds per century. The relativistic effect is 3.85 arc seconds per century.

Finally the equivalent effects on the planet Mercury of the oblate nature of the sun are determined by the distance of Mercury from the sun, which is $0.579 \times 10^{11}$ m, compared with $1.49 \times 10^{11}$ m for the earth. So for Mercury:

$$\boxed{\begin{aligned} \Delta\theta(\text{oblate}) &= 6{,}463 \text{ arc second per century} \\ \Delta\theta(\text{EGR}) &= 43 \text{ '' ''} \end{aligned}} \qquad -(25)$$

due to the oblate nature of the sun

## 7) Conclusion

In every classical calculation the EGR correction must be applied self consistently. In these examples it has been shown that the EGR correction sometimes has a very large effect. In standard physics it is applied only to the potential:

$$V(EGR) = -\frac{mMG}{r} - \frac{L_0^2 MG}{mc^2 r^3} \quad -(26)$$

and to no other situation at all. This is obviously irrational and self inconsistent.