240(5): Precession due to an Average Perturbation.

Consider the well known equation:

$$\left\langle \frac{1}{|\underline{r}-\underline{a}|} \right\rangle = \frac{1}{r} \sum_{n=0}^{\infty} \left(\frac{a}{r}\right)^{n} P_n(\cos\theta) P_n(\cos\theta') \quad -(1)$$

where:

$$r > a \quad -(2)$$

In a plane:

$$\theta = \theta' = \frac{\pi}{2} \quad -(2)$$

so

$$\cos\theta = \cos\theta' = 0. \quad -(3)$$

Here $P_n(\cos\theta)$ are the Legendre polynomials. We have:

$$P_0(0) = 1, \quad P_1(0) = 0, \quad P_2(0) = -\frac{1}{2}, \quad P_3(0) = 0,$$
$$P_4(0) = \frac{3}{8} \quad -(4)$$

so

$$\left\langle \frac{1}{|\underline{r}-\underline{a}|} \right\rangle = \frac{1}{r}\left(1 + \frac{1}{4}\left(\frac{a}{r}\right)^{2} + \frac{9}{64}\left(\frac{a}{r}\right)^{4} + \ldots\right) \quad -(5)$$

for $r > a$.

Eq. (5) is a completely general result of mathematics.

It can be applied to calculate the average potential:

$$\boxed{\phi = -MG \left\langle \frac{1}{|\underline{r}-\underline{a}|} \right\rangle} \quad -(6)$$

2) generated between a mass $\underline{M}$ at the origin and a mass m separated from $\underline{M}$ by $|\underline{r} - \underline{a}|$, a mean distance vector $\underline{r}$ with fluctuations $\underline{a}$.

These fluctuations are enough to produce orbital precession as follows.

The potential is:

$$\phi = - \frac{MG}{r}\left(1 + \frac{1}{4}\left(\frac{a}{r}\right)^2 + \frac{9}{64}\left(\frac{a}{r}\right)^4 + \dots\right) \quad -(7)$$

and the force per unit mass is:

$$\frac{F}{m} = - \frac{d\phi}{dr} \quad -(8)$$

$$= - \frac{MG}{r^2}\left(1 + \frac{3}{4}\left(\frac{a}{r}\right)^2 + \frac{45}{64}\left(\frac{a}{r}\right)^4 + \dots\right) \quad -(8)$$

If:

$$a << r \quad -(9)$$

$$\frac{F}{m} \sim - \frac{MG}{r^2}\left(1 + \frac{3}{4}\left(\frac{a}{r}\right)^2\right) \quad -(10)$$

$$F = - \frac{mMG}{r^2}\left(1 + \frac{3}{4}\left(\frac{a}{r}\right)^2\right) \quad -(11)$$

This is the type of force encountered in EGR, in which:

3)

$$F = -\frac{mMG}{r^2}\left(1 + \frac{3L_0^2}{m^2c^2r^2}\right) \quad -(12)$$

Eqs. (11) and (12) are the same if:

$$\frac{L_0^2}{m^2c^2} = \frac{a^2}{4} \quad -(13)$$

i.e if

$$a = \frac{2L_0}{mc}. \quad -(14)$$

In the Newtonian approximation:

$$L_0^2 = \alpha m^2 MG \quad -(15)$$

so

$$\frac{a^2}{4} = \alpha \frac{MG}{c^2} = \frac{1}{2}\alpha r_0 \quad -(16)$$

where

$$r_0 = \frac{2MG}{c^2} \quad -(17)$$

is the old "Schwarzschild radius". So:

$$\boxed{a^2 = 2\alpha r_0} \quad -(18)$$

A small perturbation (18) of a Newtonian orbit will produce the same precession as EGR.

If the orbit is nearly circular the apsidal angle is:

$$\phi \sim \pi\left(3 + \frac{r}{F}\frac{dF}{dr}\right)^{-1/2} \quad -(19)$$

and for an EGR force of type (12) produces the perihelion precession calculated in note 240(2):

$$\Delta\theta = \frac{6\pi MG}{c^2}\frac{\alpha}{r^2} = 2\pi\frac{r_0\alpha}{r^2}$$

$$= \pi\left(\frac{a}{r}\right)^2 \quad -(20)$$

<u>Result</u>

For:

$$\phi = -MG\left\langle\frac{1}{|\underline{r}-\underline{a}|^2}\right\rangle$$

$$\sim -\frac{MG}{r^2}\left(1+\frac{3}{4}\left(\frac{a}{r}\right)^2\right) \quad -(21)$$

then

$$\Delta\theta = \pi\left(\frac{a}{r}\right)^2 \quad -(22)$$

<u>is</u> the <u>precession</u> of the <u>perihelion</u> <u>due</u> to <u>a</u>.

This is is a very useful and incisive result because it can be applied in the solar system to show that a precession of any kind is due to an average potential. This procedure leads to results such as eq. (16) of note 240(1). For example if there are perturbations a_1 and a_2 then:

5)
$$F = -\frac{mMG}{r^2}\left(1 + \frac{3}{4}\left(\frac{a_1^2}{r^2} + \frac{a_2^2}{r^2}\right)\right) \quad -(23)$$

and
$$\Delta\theta = \pi\left(\frac{a_1^2 + a_2^2}{r^2}\right) \quad -(24)$$

From eq. (8) it follows ~~that~~:
$$\left\langle \frac{1}{|\underline{r} - \underline{a}|^2} \right\rangle = \frac{1}{r^2}\left(1 + \frac{3}{4}\left(\frac{a}{r}\right)^2 + \frac{45}{64}\left(\frac{a}{r}\right)^4 + \dots\right) \quad -(25)$$

and:
$$\left\langle \frac{1}{|\underline{r} - \underline{a}|} \right\rangle = \frac{1}{r}\left(1 + \frac{1}{4}\left(\frac{a}{r}\right)^2 + \frac{9}{64}\left(\frac{a}{r}\right)^4 + \dots\right) \quad -(26)$$

and it is possible to consider a potential such as:
$$\phi = -\left(\frac{A}{\langle|\underline{r} - \underline{a}|\rangle} + \frac{B}{\langle|\underline{r} - \underline{a}|^2\rangle}\right) \quad -(27)$$

which is known to produce a true precessing ellipse.

Using computer algebra, the force due to a potential (27) can be calculated, and the perihelion precession recalculated.

6)

The Newtonian method of calculating precessions in the solar system relies on eq. (6). This method leads to a claimed anomaly of 43" per century. This anomaly is customarily treated with eq. (12).

This is self inconsistent. The correct method is to calculate all precessions with the same force law.

When this is done the anomaly is no longer 43" per century, and the entire basis of EGR collapses. A force law based on eq. (27) is as good as any, and this will be developed in the next note.