239(1): Precessing Elliptical Orbit for the Minkowski Force Law.

The Minkowski force equation for any planar orbit was derived in UFT 238 and is:

$$\underline{F} = -\frac{L^2}{mr^2}\left(\gamma^2 \frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)+\frac{1}{r}\right)\underline{e}_r + \frac{L^4}{m^3 r^3 c^2}\frac{d}{d\theta}\left(\frac{1}{r}\right)\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)\underline{e}_\theta \quad -(1)$$

where

$$\underline{e}_r = \cos\theta\,\underline{i} + \sin\theta\,\underline{j} \quad -(2)$$

$$\underline{e}_\theta = -\sin\theta\,\underline{i} + \cos\theta\,\underline{j} \quad -(3)$$

For an elliptical orbit:

$$\frac{1}{r} = \frac{1}{\alpha}(1+\epsilon\cos\theta) \quad -(4)$$

so

$$\frac{d}{d\theta}\left(\frac{1}{r}\right) = -\frac{\epsilon}{\alpha}\sin\theta, \quad -(5)$$

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = -\frac{\epsilon}{\alpha}\cos\theta, \quad -(6)$$

so the second term on the RHS of eq. (1) becomes:

$$\underline{F}_\theta = \frac{L^4}{m^3 r^3 c^2}\left(\frac{\epsilon}{\alpha}\right)^2 \sin\theta\cos\theta\,\underline{e}_\theta. \quad -(7)$$

In the limit

$$\gamma \rightarrow 1 \quad -(8)$$

2) eq. (1) for an elliptical orbit reduces to:

$$\underline{F} \xrightarrow[\gamma \to 1]{} -\frac{L_0^2}{mr^2 \alpha}\underline{e}_r + \frac{L_0^2}{m^3 r^3 c^2}\left(\frac{\epsilon}{\alpha}\right)^2 \sin\theta\cos\theta\,\underline{e}_\theta \quad -(9)$$

$$= -\frac{L_0^2}{mr^2 \alpha}\left(\underline{e}_r + \frac{\epsilon^2}{m^2 c^2 \alpha r}\sin\theta\cos\theta\,\underline{e}_\theta\right) \quad -(10)$$

in which:

$$L = \gamma L_0 = \gamma m r^2 \omega. \quad -(11)$$

In the Newtonian theory:

$$\alpha = \frac{L_0^2}{m^2 M G} \quad -(12)$$

$$-(13)$$

so

$$\underline{F} \xrightarrow[\gamma \to 1]{} -\frac{mMG}{r^2}\left(\underline{e}_r - \left(\frac{MG}{c^2 r}\right)\sin\theta\cos\theta\,\underline{e}_\theta\right)$$

In Einsteinian general relativity the so called "Schwarzschild radius" is:

$$r_0 = \frac{2MG}{c^2} \quad -(14)$$

so:

$$\underline{F} \xrightarrow[\gamma \to 1]{} -\frac{mMG}{r^2}\left(\underline{e}_r - \frac{1}{2}\left(\frac{r_0}{r}\right)\sin\theta\cos\theta\,\underline{e}_\theta\right) \quad -(15)$$

3) It is seen that the relativistic correction is the limit that applies in the solar system because the orbital linear velocity magnitude $|\underline{v}|$ of the planets is much smaller than c. So in the limit $\gamma \rightarrow 1$ is very small. This is the limit that in the solar system eq. (1) is excellently approximated by:

$$\underline{F} = -\frac{L^2}{mr^2}\left(\gamma^2 \frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right)\underline{e}_r \quad -(16)$$

It can be shown as follows that the relativistic correction produces a precessing elliptical orbit from an elliptical orbit.

First assume that the elliptical orbit is produced by the non-relativistic force law:

$$\underline{F} = -\frac{L_0^2}{mr^2}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r}\right)\underline{e}_r, \quad -(17)$$

in which

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{1}{\alpha} \quad -(18)$$

for the ellipse:

$$r = \frac{\alpha}{1+\epsilon\cos\theta}. \quad -(19)$$

The relativistic correction to eq. (17) is:

4)
$$\underline{F} \rightarrow -\frac{L_0^2}{mr^2}\left(\gamma^4 \frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{\gamma^2}{r}\right)\underline{e}_r, \quad -(20)$$

and assume that this correction produces the precessing ellipse:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)}. \quad -(21)$$

The apparent force law of eq. (21) is given by eq. (17) with:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{1}{\alpha} + \left(1-x^2\right)\frac{\epsilon}{\alpha}\cos(x\theta) \quad -(22)$$

From eqs. (20) and (22):

$$\gamma^4 \frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{\gamma^2}{r} = \frac{1}{\alpha} + \left(1-x^2\right)\frac{\epsilon}{\alpha}\cos(x\theta) \quad -(23)$$

So the relativistic correction which means that the elliptical force law on the left hand side produces the precessing elliptical orbit on the right hand side.

For the ellipse:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = -\frac{\epsilon}{\alpha}\cos\theta \quad -(24)$$

so eq. (23) becomes:

5)
$$\gamma^2\left(1+\epsilon\cos\theta\left(1-\gamma^2\right)\right) = 1+\left(1-x^2\right)\epsilon\cos\left(x\theta\right). \quad -(25)$$

On the left hand side of this equation:
$$\cos\theta = \frac{1}{\epsilon}\left(\frac{\alpha}{r}-1\right) \quad -(26)$$

and on the right hand side:
$$\cos(x\theta) = \frac{1}{\epsilon}\left(\frac{\alpha}{r}-1\right). \quad -(27)$$

Denote:
$$f(r) = \frac{\alpha}{r}-1 \quad -(28)$$

then
$$\gamma^2\left(1+\left(1-\gamma^2\right)f(r)\right) = 1+\left(1-x^2\right)f(r). \quad -(29)$$

i.e
$$x^2 = 1+\left(1-\gamma^2\right)\left(\frac{1}{f(r)}-\gamma^2\right) \quad -(30)$$

$$\boxed{x^2 = 1+\left(1-\gamma^2\right)\left(\left(\frac{\alpha}{r}-1\right)^{-1}-\gamma^2\right)} \quad -(31)$$

where
$$\gamma^2 = \left(1-\frac{v^2}{c^2}\right)^{-1}; \quad -(32)$$

and
$$\alpha = (1+\epsilon)\,r_{min} = (1-\epsilon)\,r_{max}. \quad -(33).$$

Here r_{min} is the distance of closest approach

6) and r_{max} is the distance of furthest separation. The velocity v is defined as in previous work by:

$$v^2 = \left(\frac{L_0}{md}\right)^2 \left(1 + \epsilon^2 + 2\epsilon\cos\theta\right) \quad - (34)$$

or can be measured in astronomy.

So x can be calculated from data.

Summary

It has been shown that the Minkowski force equation produces a precessing ellipse using the relativistic conversion to the force equation of an elliptical orbit. The apparent force law of the precessing ellipse has been taken to be eq. (22), i.e.:

$$\underline{F} = -\frac{L_0^2}{mr^2}\left(\frac{1}{d} + \left(1 - x^2\right)\frac{\epsilon}{d}\cos(x\theta)\right)\underline{e}_r \quad - (35)$$

which is the effective force law assuming eq. (17). with x defined by eq. (31), this effective force law is the result of the relativistic conversion to the elliptical orbit. So precession is produced by the force laws of the relativistic conversion.