239(2). Consideration of the Limit $r \rightarrow \alpha$.

The expression for x derived by computer algebra is:

$$x^2 = \gamma^4 + \frac{\alpha(\gamma^2 - 1)}{r - \alpha} \quad -(1)$$

in which

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(2)$$

and

$$v^2 = \left(\frac{dr}{dt}\right)^2 + \omega^2 r^2. \quad -(3)$$

By definition:

$$\alpha = a(1-\epsilon^2) = r_{min}(1+\epsilon) \quad -(4)$$

$$= r_{max}(1-\epsilon).$$

In the limit:

$$r \rightarrow \alpha \quad -(5)$$

the orbit approaches that of the circle, so:

$$\epsilon \rightarrow 0. \quad -(6)$$

From previous work, for an elliptical orbit:

$$v^2 = \left(\frac{L_0}{m\alpha}\right)^2 \left(1 + \epsilon^2 + 2\epsilon\cos\theta\right) \quad -(7)$$

$$= \left(\frac{L_0}{m\alpha}\right)^2 \left(1 + \epsilon^2 + 2\left(\frac{\alpha}{r} - 1\right)\right),$$

2) Therefore for a circle:

$$v^2 = \left(\frac{L_0}{md}\right)^2 = \left(\frac{L_0}{mr}\right)^2 \quad -(8)$$

$$= \omega^2 r^2 .$$

However, for a circle there is no effect of precession. A precessing circle is another circle indistinguishable from the original circle. So there is no effect of γ.

This means that

$$\gamma \to 1 \quad -(9)$$

$$r \to d , \quad -(10)$$

as

$$x \to 1 \quad -(11)$$

i.e.

$$r \to d . \quad -(12)$$

as

In the classical limit:

$$E = T + U \quad -(13)$$

and

$$T = \frac{1}{2} m \left(\left(\frac{dr}{dt}\right)^2 + \omega^2 r^2 \right) \quad -(14)$$

Therefore for the circle:

$$E = \frac{1}{2} m \omega^2 r^2 + U \quad -(15)$$

because

$$dr / dt = 0 \quad -(16)$$

3) If it is assumed that:

$$U = -\frac{mMG}{r} \quad -(17)$$

then:

$$E = \text{constant} = \frac{1}{2} m \omega^2 r^2 - \frac{mMG}{r} \quad -(18)$$

It is only possible for an orbit to be a circle if:

$$r \rightarrow \infty \quad -(19)$$

and in order to keep E finite and constant:

$$\omega \rightarrow 0 \quad -(20)$$

In this theory the orbital velocity v is given by:

$$v^2 = MG\left(\frac{2}{r} - \frac{1}{a}\right) \quad -(21)$$

where

$$a = \frac{d}{1-\epsilon^2} \quad -(22)$$

so as $r \rightarrow \infty$, $d \rightarrow \infty$, $\epsilon \rightarrow 0$ and

$$\boxed{v \rightarrow 0} \quad -(23)$$

Therefore:

$$v \xrightarrow[r \rightarrow \infty]{} 0 \quad -(24)$$

and

$$\gamma \rightarrow 1 \quad -(25)$$

Therefore it is only possible for $d \rightarrow r$

4) when $r \to d \to \infty$ — (26)

and $v \xrightarrow[r \to \infty]{} 0$, — (27)

i.e $\boxed{\gamma \xrightarrow[r \to d]{} 1}$ — (28)

From eqns. (1) and (21) — (29)

$$x^2 = \left(1 - \frac{v^2}{c^2}\right)^{-2} + \left(\frac{d}{r-d}\right)\left(\left(1 - \frac{v^2}{c^2}\right)^{-1} - 1\right)$$

with: $v^2 = mG\left(\frac{2}{r} - \frac{(1-\epsilon^2)}{d}\right)$ — (30)

and as $r \to d \to \infty$ — (31)

then $v \to 0$ — (32)

and $\boxed{x \to 1}$ — (33)