237(3): Completed Theory of the Whirlpool Galaxy

The trajectory of the whirlpool galaxy is:

$$r = -\frac{r_0}{\theta} \quad -(1)$$

and is a hyperbolic spiral. Therefore:

$$\frac{d}{d\theta}\left(\frac{1}{r}\right) = -\frac{1}{r_0} = -\frac{mv_\infty}{L} \quad -(2)$$

where

$$\underline{L} = \underline{r} \times \underline{p} \quad -(3)$$

Here v_∞ is the constant velocity:

$$v \xrightarrow[r \to \infty]{} v_\infty, \quad -(4)$$

and m is the mass of a star in orbit around the centre of the galaxy.

The total velocity is:

$$\underline{v} = \frac{d\underline{r}}{dt} = \left(\frac{L}{m}\right)\left(\frac{1}{r}\underline{e}_\theta - \frac{d}{d\theta}\left(\frac{1}{r}\right)\underline{e}_r\right)$$

$$\boxed{\underline{v} = \left(\frac{L}{m}\right)\left(\frac{1}{r}\underline{e}_\theta + \frac{1}{r_0}\underline{e}_r\right)} \quad -(5)$$

The kinetic energy is:

$$\boxed{T = \frac{1}{2}mv^2 = \frac{1}{2}\frac{L^2}{m}\left(\frac{1}{r^2} + \frac{1}{r_0^2}\right)} \quad -(6)$$

2) The force of attraction of the star to the galactic centre is:

$$\underline{F} = -\frac{L^2}{mr^3}\,\underline{e}_r \qquad -(7)$$

and is an inverse cube law.

The potential energy is:

$$U = -\frac{L^2}{2mr^2} \qquad -(8)$$

The total energy and Hamiltonian is:

$$H = E = T + U = \frac{L^2}{2mr_0^2} \qquad -(9)$$

$$= \text{constant}$$

Using: $$\frac{dr}{dt} = \frac{L}{mr^2}\frac{dr}{d\theta} \qquad -(10)$$

$$= -\frac{L}{m}\frac{d}{d\theta}\left(\frac{1}{r}\right)$$

it is found that:

$$r = \frac{L}{mr_0}\,t \qquad -(11)$$

which can be expressed as:

$$\boxed{r = v_\infty t} \quad -(12)$$

Using: $$\frac{d\theta}{dt} = \frac{L}{mr^2} \quad -(13)$$

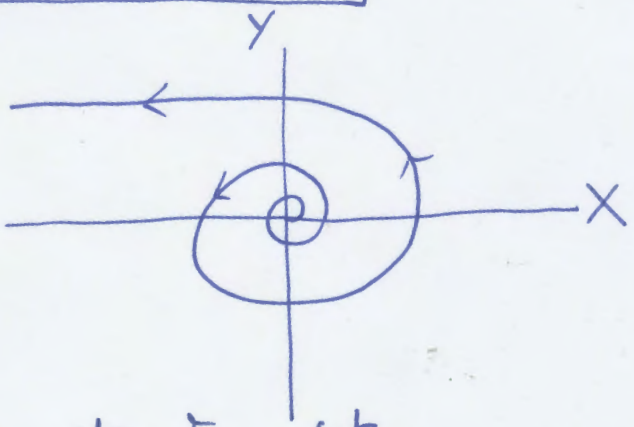
it is found that

$$\frac{d\theta}{dt} = \frac{L}{mv_\infty^2}\frac{1}{t^2} \quad -(14)$$

so

$$\theta(t) = \frac{L}{mv_\infty^2}\int \frac{1}{t^2}\,dt \quad -(15)$$

$$\boxed{\theta(t) = -\frac{L}{mv_\infty^2}\frac{1}{t}} \quad -(16)$$



From eqns. (1) and (16) ller at $t = 0$, $r = 0$, at $t \to \infty$, $r \to \infty$. The star enerys from the centre of the galaxy and the spiral connection puts it into orbit.