1) Note 234(7): Centrifugal Acceleration of Any Orbit.

In general the centrifugal acceleration is:

$$\underline{a} = \left(\frac{L}{mr^2}\right)\left(\left(\frac{dr}{d\theta}\right)\frac{d}{dr}\left(\frac{1}{r^2}\frac{dr}{d\theta}\right) - \frac{1}{r}\right)\underline{e}_r \quad -(1)$$

and in ECE theory is represented by spacetime torsion. The orbit can be represented experimentally by $dr / d\theta$ provided the dependence of r on θ can be measured.

Assuming that the orbit can be represented by:

$$ds^2 = c^2 d\tau^2 = Ac^2dt^2 - Bdr^2 - r^2 d\theta^2 \quad -(2)$$

then

$$mc^2 = Amc^2\left(\frac{dt}{d\tau}\right)^2 - Bm\left(\frac{dr}{d\tau}\right)^2 - mr^2\left(\frac{d\theta}{d\tau}\right)^2 \quad -(3)$$

in which the constants of motion are:

$$E = Amc^2\frac{dt}{d\tau}, \quad L = mr^2\frac{d\theta}{d\tau} \quad -(4)$$

so

$$Bm\left(\frac{dr}{d\tau}\right)^2 = Amc^2\left(\frac{dt}{d\tau}\right)^2 - mc^2 - mr^2\left(\frac{d\theta}{d\tau}\right)^2 \quad -(5)$$

$$= \frac{E^2}{Amc^2} - mc^2 - \frac{L^2}{mr^2}$$

Therefore:

2) $$\left(\frac{dr}{d\tau}\right)^2 = \frac{1}{mB}\left(\frac{E^2}{Amc^2} - mc^2 - \frac{L^2}{mr^2}\right), \quad -(6)$$

in which: $$\frac{dr}{d\tau} = \frac{d\theta}{d\tau}\frac{dr}{d\theta} = \frac{L}{mr^2}\frac{dr}{d\theta} \quad -(7)$$

$-(8)$

So: $$\frac{L^2}{m^2 r^4}\left(\frac{dr}{d\theta}\right)^2 = \frac{1}{mB}\left(\frac{E^2}{Amc^2} - mc^2 - \frac{L^2}{mr^2}\right)$$

and $$\left(\frac{dr}{d\theta}\right)^2 = \frac{r^4}{B}\left(\frac{1}{A}\left(\frac{E}{cL}\right)^2 - \left(\frac{mc}{L}\right)^2 - \frac{1}{r^2}\right)$$

$-(9)$

If $$a := \frac{cL}{E}, \quad b := \frac{L}{mc} \quad -(10)$$

then $$\left(\frac{dr}{d\theta}\right)^2 = \frac{r^4}{B}\left(\frac{1}{A}\cdot\frac{1}{a^2} - \frac{1}{b^2} - \frac{1}{r^2}\right) \quad -(11)$$

where a and b are <u>constants.</u>

For the Minkowski metric:

$$A = B = 1 \quad -(12)$$

For the wrongly named "Schwarzschild" metric:

$$A = \frac{1}{B} = 1 - \frac{r_0}{r} \quad -(13)$$

3) Therefore for the Minkowski metric:

$$\left(\frac{dr}{d\theta}\right)^2 = r^4\left(\left(\frac{E}{cL}\right)^2 - \left(\frac{mc}{L}\right)^2 - \frac{1}{r^2}\right) \quad -(14)$$

$$= r^4\left(\frac{E^2 - m^2c^4}{c^2L^2} - \frac{1}{r^2}\right)$$

$$= r^4\left(\left(\frac{p}{L}\right)^2 - \frac{1}{r^2}\right)$$

here

$$E^2 - m^2c^4 = c^2p^2 \quad -(15)$$

Therefore

$$\boxed{\frac{dr}{d\theta} = r^2\left(x - \frac{1}{r^2}\right)^{1/2}} \quad -(16)$$

where

$$x = p/L \quad -(17)$$

is a constant.

For the "Schwarzschild" metric: $\quad -(18)$

$$\left(\frac{dr}{d\theta}\right)^2 = r^4\left(\frac{1}{a^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{b^2} + \frac{1}{r^2}\right)\right)$$

so

$$\boxed{\frac{dr}{d\theta} = r^2\left(\frac{1}{a^2} - \left(1 - \frac{r_0}{r}\right)\left(\frac{1}{b^2} + \frac{1}{r^2}\right)\right)^{1/2} \quad -(19)}$$

4) Therefore the centrifugal acceleration (1) can be worked out from eqs. (1) and (16) for the Minkowski metric or eqs. (1) and (19) for the "Schwarzschild" metric.

This is best done by computer algebra to eliminate human error. The calculation can be done by hand and is analytical. By Ockham's Razor the simplest representation is the Minkowski metric. Finally the centrifugal acceleration is related to the spacetime torsion as in UFT 55. These hand calculations will be done in the next note.
