234(8): The Centrifugal Acceleration of the Minkowski Orbit expressed in terms of p / L.

The Minkowski orbit can be expressed as:

$$\frac{dr}{d\theta} = r^2 \left(\left(\frac{p}{L} \right)^2 - \frac{1}{r^2} \right)^{1/2} \quad -(1)$$

where p / L is a constant of motion, so:

$$\frac{d}{d\tau} \left(\frac{p}{L} \right) = 0. \quad -(2)$$

However,

$$\frac{d}{dr} \left(\frac{p}{L} \right) \neq 0 \quad -(3)$$

Here $\underline{p}$ is the relativistic linear momentum:

$$\underline{p} = \gamma m \underline{v} \quad -(4)$$

and $\underline{L}$ is the relativistic angular momentum:

$$\underline{L} = \gamma m r^2 \underline{\omega}. \quad -(5)$$

The Lorentz factor is:

$$\gamma = \frac{dt}{d\tau} = \left(1 - \frac{v^2}{c^2} \right)^{-1/2} \quad -(6)$$

and where τ is the proper time defined by:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 \quad -(7)$$

$$= \left(c^2 - v^2 \right) dt^2$$

$$=$$

2) Here: $\underline{dr} \cdot \underline{dr} = dr^2 + r^2 d\theta^2 = v^2 dt^2$ — (8)

so the velocity is defined by:

$$v^2 = \left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2 \quad -(9)$$

and appears in eq. (4). The three conserved quantities of the orbit are $\underline{p}$, $\underline{L}$ and E, where

$$E = \gamma m c^2 . \quad -(10)$$

Eq. (4) can be expressed as:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(11)$$

Self consistently, eq. (7) also produces eq. (11) as follows. Note that:

$$mc^2 = mc^2\left(\frac{dt}{d\tau}\right)^2 - m\left(\frac{dr}{d\tau}\right)^2 - mr^2\left(\frac{d\theta}{d\tau}\right)^2 \quad -(12)$$

this equation can be combined with eqs. (10), (4), (6) and (9) to give:

$$mc^2 = \frac{E^2}{mc^2} - \frac{p^2}{m} \quad -(13)$$

i.e.

3)

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(14)$$

which is eq. (11) or (4), Q.E.D.

Recent work has shown that the centrifugal acceleration of the orbit is:

$$\underline{a} = \left(\frac{L}{mr}\right)^2 \left(\left(\frac{dr}{d\theta}\right) \frac{d}{dr} \left(\frac{1}{r^2} \frac{dr}{d\theta} \right) - \frac{1}{r} \right) \underline{e}_r \quad -(15)$$

and in ECE theory this is due to spacetime torsion.

For ease of notation denote:

$$x = \left(\frac{p}{L}\right)^2 \quad -(16)$$

therefore:

$$\frac{1}{r^2}\frac{dr}{d\theta} = \left(x(r) - \frac{1}{r^2} \right)^{1/2} \quad -(17)$$

Denote

$$y = x(r) - \frac{1}{r^2} \quad -(18)$$

then if

$$f = \frac{1}{r^2}\frac{dr}{d\theta} \quad -(19)$$

$$\frac{df}{dr} = \frac{df}{dy}\frac{dy}{dr} \quad -(20)$$

where:

$$\frac{df}{dy} = \frac{1}{2} y^{-1/2}, \quad \frac{dy}{dr} = \frac{dx}{dr} + \frac{2}{r^3} \qquad -(21)$$

So:

$$\frac{dr}{d\theta} \frac{d}{dr}\left(\frac{1}{r^2} \frac{dr}{d\theta} \right) = \frac{r^2}{2}\left(\frac{dx}{dr} + \frac{2}{r^3} \right) \qquad -(22)$$

$$= \frac{r^2}{2} \frac{dx}{dr} + \frac{1}{r}$$

The centripetal acceleration from eqns. (15) and (22) is:

$$\boxed{\underline{a} = \frac{L^2}{2m^2} \frac{dx}{dr} \, \underline{e}_r} \qquad -(23)$$

i.e.

$$\boxed{\underline{a} = \frac{L^2}{2m^2} \frac{d}{dr} \left(\frac{p}{L} \right)^2 \underline{e}_r} \qquad -(24)$$

Any observable orbit can be analysed by the acceleration $\underline{a}$ without the use of any other theory. We denote this result by the appellation "Minkowski Cosmology", based on spacetime version in ECE theory.

) Elliptical Orbit

First note that:

$$\frac{P}{L} = \frac{v}{\omega r^2} \qquad -(25)$$

so:

$$\boxed{\frac{d}{d\tau}\left(\frac{v}{\omega r^2}\right) = 0} \qquad -(26)$$

In this case:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \qquad -(27)$$

so as in recent notes:

$$\left(\frac{dr}{d\theta}\right)^2 = \left(\frac{\epsilon}{\alpha}\right)^2 r^4 \sin^2\theta$$
$$= \left(\frac{\epsilon}{\alpha}\right)^2 r^4 \left(1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2\right) \qquad -(28)$$
$$= \left(\frac{r}{\alpha}\right)^2 \left(\epsilon^2 r^2 - (\alpha - r)^2\right)$$
$$= r^2 \left(\frac{1-\epsilon}{1+\epsilon}\right)\left(\frac{(r_{max}-r)(r-r_{min})}{r_{min}^2}\right)$$
$$= r^2 \left(\frac{1+\epsilon}{1-\epsilon}\right)\left(\frac{(r_{max}-r)(r-r_{min})}{r_{max}^2}\right)$$

where the nearest and furthest distances are r_{min} and

6) r_{max}.

Therefore from eqns. (1), (25) and (28):

$$r^4\left(\left(\frac{v}{\omega r^2}\right)^2-\frac{1}{r^2}\right)=r^2\left(\left(\frac{v}{\omega r}\right)-1\right) \quad -(29)$$

$$= r^2\left(\frac{1+\epsilon}{1-\epsilon}\right)\left(\frac{(r_{max}-r)(r-r_{min})}{r_{max}^2}\right)$$

i.e

$$\boxed{\begin{aligned}\frac{v}{\omega r} &= 1+\left(\frac{1+\epsilon}{1-\epsilon}\right)\left(\frac{(r_{max}-r)(r-r_{min})}{r_{max}^2}\right) \quad -(30)\\ &= r^2\left(\frac{p}{L}\right)^2\end{aligned}}$$

Therefore $(p/L)^2$ can be measured experimentally from eq. (30).

Circular Orbit

In this case:

$$\epsilon = 0, \quad -(31)$$

and

$$r_{max} = r_{min} = r \quad -(32)$$

so

$$v = \omega r \quad -(33)$$

and:

7)

$$\frac{p}{L} = \frac{1}{r} \quad -(34)$$

So:

$$\underline{a} = \frac{L^2}{2m^2} \frac{d}{dr}\left(\frac{1}{r^2}\right) \underline{e}_r \quad -(35)$$

$$\boxed{\underline{a} = -\frac{L^2}{m^2 r^3} \underline{e}_r} \quad -(36)$$

This has a negative sign and is directed outwards. It is due to spacetime itself.

As in note 234(6):

$$\underline{a} = -\frac{L^2}{m^2 r^2 d} \underline{e}_r \quad -(37)$$

Comparing eqs. (36) and (37):

$$d = r \quad -(38)$$

which is true for a circular orbit, QED.

Using:

$$L = \gamma m r^2 \omega \quad -(39)$$

then from eqs. (36) and (39):

$$\underline{a} = -\gamma^2 r \omega^2 \underline{e}_r \quad -(40)$$

8) For $v << c$ — (41)

then $\gamma \rightarrow 1$ — (42)

and
$$\boxed{\underline{a} \rightarrow -\omega^2 r \underline{e}_r} \quad -(43)$$

which is the classical centrifugal acceleration, QED.

For the elliptical orbit, eqns. (37) and (24) give:

$$\frac{L^2}{2m^2}\frac{d}{dr}\left(\frac{p}{L}\right)^2 = -\frac{L^2}{m^2 r^2 \alpha} \quad -(44)$$

so:

$$\frac{d}{dr}\left(\frac{p}{L}\right)^2 = -\frac{2}{m^2 \alpha} \quad -(45)$$

i.e.

$$\frac{d}{dr}\left(\frac{p}{L}\right)^2 = -\frac{2}{m^2 r}\left(1+\epsilon\cos\theta\right) \quad -(46)$$

and

$$\left(\frac{p}{L}\right)^2 = -\frac{2}{m^2}\int \frac{1+\epsilon\cos\theta}{r}\, dr \quad -(47)$$

giving the elegant result:

9)

$$\boxed{\left(\frac{p}{L}\right)^2 = -\frac{2}{m^2}\int \frac{1}{\alpha}\,dr} \quad -(48)$$

This is a self consistent theory of all orbits, and for the "Newtonian" ellipse gives a simple result (48).

The Question of Why an Orbit is Stable

The acceleration for a circular orbit gives the answer in its simplest form, it is:

$$\underline{a} = -\frac{L^2}{m^2 r^3}\,\underline{e}_r = -\frac{L^2}{m^2 r^2 \alpha}\,\underline{e}_r$$

$$\rightarrow -\omega^2 r\,\underline{e}_r \quad -(49)$$

for $v \ll c$. It can be viewed as an outward centrifugal acceleration equal to an inward negative valued acceleration proportional to $1/r^2$. In the older Newtonian view the latter is:

$$a = -\frac{L^2}{m^2 r^2 \alpha} = -\frac{MG}{r^2} \quad -(50)$$

giving a force:

10)

$$\underline{F} = -\frac{mMG\underline{e}_r}{r^2} = m\underline{a} \quad -(51)$$

In the older Newtonian view, the idea is that m is attracted to a mass $\underline{M}$ by a force known as "gravitation". The problem with that is that the attractive force is not counterbalanced by a "centrifugal force".

In the Minkowski cosmology the "attractive force and the centrifugal force are equal, as in eq. (49). The negative sign of the centrifugal force means that it is outwardly directed, the negative sign of the "attractive" force is by convention indicative of an inward force. So the orbit is stable.

More elegantly however, the orbit is a consequence of the metric (7), and this is true for all orbits, which can all be expressed as $(p/L)^2$.