234(6): Calculation of the Acceleration for Any Metric.

As in previous notes the acceleration of an orbit can be written in general as:

$$\underline{a} = \left(\frac{d^2 r}{d\tau^2} - r\left(\frac{d\theta}{d\tau}\right)^2\right)\underline{e}_r + \left(r\frac{d^2\theta}{d\tau^2} + 2\left(\frac{dr}{d\tau}\right)\left(\frac{d\theta}{d\tau}\right)\right)\underline{e}_\theta . \quad -(1)$$

For a metric of the type:

$$ds^2 = c^2 d\tau^2 = Ac^2 dt^2 - B dr^2 - r^2 d\theta^2 \quad -(2)$$

the angular momentum is a constant of motion:

$$L = mr^2 \frac{d\theta}{d\tau} \quad -(3)$$

and:

$$r\frac{d^2\theta}{d\tau^2} + 2\left(\frac{dr}{d\tau}\right)\left(\frac{d\theta}{d\tau}\right) = 0 \quad -(4)$$

so:

$$\underline{a} = \left(\frac{d^2 r}{d\tau^2} - \frac{L^2}{m^2 r^3}\right)\underline{e}_r . \quad -(5)$$

Any metric of type (2) produces the acceleration (5).

This can be rewritten as:

$$\underline{a} = \left(\frac{L}{mr}\right)^2\left(\left(\frac{dr}{d\theta}\right)\frac{d}{dr}\left(\frac{1}{r^2}\frac{dr}{d\theta}\right) - \frac{1}{r}\right)\underline{e}_r \quad -(6)$$

which is a generalization of the Newtonian theory and a valid new cosmology based on the Minkowski metric, or any metric of type (2). For the Minkowski metric:

$$\left(\frac{dr}{d\theta}\right)^2 = r^4\left(\left(\frac{p}{L}\right)^2 - \frac{1}{r^2}\right) \quad -(7)$$

) where

$$p = \gamma m \underline{v}, \quad -(8)$$

$$\underline{L} = \gamma m r^2 \underline{\omega} \, . \quad -(9)$$

are the linear and angular momenta. Therefore the acceleration in this case is found from a combination of eqs (6) and (7).
The orbit is therefore characterized by the acceleration (6) and by:

$$\left(\frac{p}{L}\right)^2 = \frac{1}{r^2}\left(1 + \frac{1}{r^2}\left(\frac{dr}{d\theta}\right)^2\right). \quad -(10)$$

For an elliptical orbit:

$$\frac{dr}{d\theta} = \frac{\epsilon r^2}{d} \sin\theta \quad -(11)$$

so

$$\left(\frac{p}{L}\right)^2 = \frac{1}{r^2}\left(1 + \left(\frac{\epsilon}{d}\right)^2 r^2 \sin^2\theta\right) \quad -(12)$$

i.e

$$\left(\frac{pr}{L}\right)^2 = 1 + \left(\frac{\epsilon r \sin\theta}{d}\right)^2 \quad -(13)$$

and for a circular orbit:

$$\left.\begin{aligned} L &= pr, \\ \epsilon &= 0. \end{aligned}\right\} \quad -(14)$$

The acceleration for an elliptical orbit is given by:

$$\underline{a} = \left(\frac{L}{mr}\right)^2 \left(\left(\frac{dr}{d\theta}\right) \frac{d}{dr} \sin\theta - \frac{1}{r}\right) \underline{e}_r \quad -(15)$$

3)

$$= \left(\frac{L}{mr}\right)^2 \left(\frac{\epsilon r^2}{\alpha} \sin\theta \frac{d\sin\theta}{dr} - \frac{1}{r}\right) \underline{e}_r$$

$$= \left(\frac{L^2 \epsilon \cos\theta}{m^2 \alpha r^2} - \frac{L^2}{m^2 r^3}\right) \underline{e}_r$$

$$\underline{a} = -\frac{L^2}{m^2 r^2 \alpha} \underline{e}_r \quad -(16)$$

as in the previous note. The precise result is:

$$\underline{a} = \left(\frac{L^2}{m^2 r^3} - \frac{L^2}{m^2 r^2 \alpha} - \frac{L^2}{m^2 r^3}\right) \underline{e}_r, \quad -(17)$$

$$\left(\frac{f}{L}\right)^2 = \frac{1}{r^2}\left(1 + \left(\frac{\epsilon r}{\alpha}\right)^2 \sin^2\theta\right), \quad -(18)$$

where

$$\cos\theta = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right) \quad -(19)$$

so

$$\left(\frac{fr}{L}\right)^2 = 1 + \frac{1}{\alpha^2}\left(\epsilon^2 r^2 - (\alpha - r)^2\right) \quad -(20)$$

For other orbits and metrics the result is different. In this theory:

$$L = \gamma m r^2 \omega \quad -(21)$$

so

$$\underline{a} = -\frac{\gamma^2 r^2}{\alpha} \omega^2 \underline{e}_r \quad -(22)$$

4)

i.e.

$$\underline{a} = -\left(1-\frac{v^2}{c^2}\right)^{-1} \frac{r^2\omega^2}{\alpha} \underline{e}_r \quad -(23)$$

$$\underline{a} = -\frac{1}{\alpha} \frac{(\omega r)^2}{\left(1-\frac{v^2}{c^2}\right)} \underline{e}_r \quad -(24)$$

In the limit:

$$v \ll c \quad -(25)$$

and for a circular orbit:

$$\alpha = r \quad -(26)$$

then

$$\boxed{\underline{a} = -\omega^2 r \, \underline{e}_r} \quad -(27)$$

which is the centripetal acceleration due to the elliptical orbit. The orbit is caused by the metric itself, i.e. by a rotating frame of reference.