234(9): Self Consistency and Interpretation of the Minkowski Cosmology.

Any orbit (theoretical or experimental) is described by:

$$\frac{dr}{d\theta} = r^2 \left(\left(\frac{p}{L} \right)^2 - \frac{1}{r^2} \right)^{1/2} \quad -(1)$$

i.e. by the ratio of the relativistic momentum p to the relativistic angular momentum L. Here:

$$p = \gamma m v, \quad L = \gamma m r^2 \omega, \quad -(2)$$

$$\omega = d\theta / dt.$$

So

$$\frac{p}{L} = \frac{v}{\omega r^2} \quad -(3)$$

where the angular velocity is ω. So:

$$\left(\frac{dr}{d\theta} \right)^2 = r^4 \left(\frac{v^2}{\omega^2 r^4} - \frac{1}{r^2} \right) \quad -(4)$$

$$= \left(\frac{v}{\omega} \right)^2 - r^2$$

therefore

$$\boxed{\left(\frac{v}{\omega} \right)^2 = \left(\frac{dr}{d\theta} \right)^2 + r^2} \quad -(5)$$

This is a very useful equation that can be used for any observable orbit.

For a circular orbit there is no change in

2) r wrt θ, so:

$$\frac{dr}{d\theta} = 0 \quad -(6)$$

and eq. (5) gives:

$$v = \omega r \quad -(7)$$

the familiar equation of classical dynamics. For an elliptical orbit:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(8)$$

so

$$\frac{dr}{d\theta} = \frac{\epsilon r^2}{\alpha}\sin\theta \quad -(9)$$

and

$$\left(\frac{v}{\omega}\right)^2 = \left(\frac{\epsilon r^2}{\alpha}\right)^2 \sin^2\theta + r^2 \quad -(10)$$

or

$$\left(\frac{v}{\omega}\right)^2 - r^2 = \frac{\epsilon^2 r^4}{\alpha^2}\sin^2\theta \quad -(11)$$

Any elliptical orbit should obey this equation, and this can be tested to high accuracy wrt astronomical data.

For a precessing elliptical orbit:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(12)$$

and

$$\frac{dr}{d\theta} = \frac{x\epsilon r^2}{\alpha}\sin(x\theta) \quad -(13)$$

so a precessing elliptical orbit must obey the equation

3)
$$\left(\frac{v}{\omega}\right)^2 - r^2 = \left(\frac{x \in r^2}{d}\sin(x\theta)\right)^2 \quad -(14)$$

Again this can be tested with great accuracy. In general:

$$\frac{dr}{d\theta} = f(\theta) \quad -(15)$$

so
$$\left(\frac{v}{\omega}\right)^2 - r^2 = f^2(\theta). \quad -(16)$$

Galactic orbits are described by an equation of the type (16).

Rearranging eq. (1) gives:

$$\left(\frac{d\theta}{dr}\right)^2 = \frac{1}{r^4}\cdot\frac{1}{\left(\frac{p}{L}\right)^2 - \frac{1}{r^2}} \quad -(17)$$

$$= \frac{L^2}{r^2}\left(\frac{1}{r^2p^2 - L^2}\right).$$

Newtonian dynamics is valid for:

$$v \ll c \quad -(18)$$

and is defined by
$$E = \frac{p^2}{2m} + U \quad -(19)$$

where E is the constant total energy or hamiltonian and U is the potential energy. Here:

4)
$$p^2 = m^2\left(\left(\frac{dr}{dt}\right)^2 + r^2\left(\frac{d\theta}{dt}\right)^2\right) \quad -(20)$$

It follows from eqs. (19) and (20) that in Newtonian dynamics:
$$\left(\frac{d\theta}{dr}\right)^2 = \frac{L^2}{r^4}\left(\frac{1}{2m\left(E - U - \frac{L^2}{2mr^2}\right)}\right) \quad -(21)$$

using:
$$\frac{dr}{d\theta} = \frac{dr}{dt}\frac{dt}{d\theta} \quad -(22)$$

Eq. (21) is:
$$\left(\frac{d\theta}{dr}\right)^2 = \frac{L^2}{r^2}\left(\frac{1}{2mr^2(E-U) - L^2}\right) \quad -(23)$$

Eqs. (17) and (23) are the same if:
$$p^2 = 2m(E-U) \quad -(24)$$

which is eq. (19), Q.E.D.

Minkowski cosmology is a relativistic theory that defines the momentum p by:
$$E^2 = c^2p^2 + m^2c^4 \quad -(25)$$

and makes no assumption about p. Newtonian theory is a non-relativistic theory valid only for low v, and defines v by:
$$v^2 = \frac{2}{m}(E - U) \quad -(26)$$

Note carefully that the concept of U is not present in Minkowski cosmology, or any relativistic cosmology. In order to obtain an elliptical orbit, or any conical section, the potential U must be:

$$U = -\frac{mMG}{r}. \quad -(27)$$

At this point the Newtonian theory introduces the mass M and the constant G.

As shown in previous notes the acceleration due to any orbit in Minkowski cosmology is:

$$\underline{a} = \left(\frac{L}{mr}\right)^2 \left(\left(\frac{dr}{d\theta}\right) \frac{d}{dr}\left(\frac{1}{r^2}\frac{dr}{d\theta}\right) - \frac{1}{r}\right) \underline{e}_r \quad -(28)$$

$$= \frac{L^2}{2m^2} \frac{d}{dr}\left(\frac{p}{L}\right)^2 \underline{e}_r$$

and is directed along the radial unit vector $\underline{e}_r$ of the plane polar system (r, θ). Eq. (28) is a general result for any orbit. For the elliptical orbit (8) eq. (28) gives, as in previous notes:

$$\underline{a} = -\frac{L^2 \underline{e}_r}{m^2 r^2 d}, \quad -(29)$$

(note 234(6)). In eq. (29):

6)

$$L = \gamma m r^2 \omega \quad -(30)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(31)$$

so

$$\underline{a} = -\left(1 - \frac{v^2}{c^2}\right)^{-1} \omega^2 r \, \underline{e}_r \quad -(32)$$

In the limit:

$$v << c \quad -(33)$$

$$\underline{a} \rightarrow -\omega^2 r \, \underline{e}_r \quad -(34)$$

which is the centrifugal acceleration. The acceleration (29) in Newtonian dynamics is

$$\underline{a} = -\frac{MG}{r^2} \underline{e}_r \quad -(35)$$

which is found by using:

$$\underline{F} = m\underline{a} = -\frac{\partial U}{\partial r} \underline{e}_r \quad -(36)$$

and

$$\alpha = \frac{L^2}{m^2 M G}. \quad -(37)$$

The correct relativistic expression is however eq. (29). In the Newtonian system:

$$\underline{F} = m\underline{a} = -\frac{mMG}{r^2} \underline{e}_r \quad -(38)$$

7) is known as a "force of attraction", but the correct eq. (29) is the result of a rotating frame of reference, which is expressed as the infinitesimal line element:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 \quad -(39)$$

Eq. (39) produces eq. (1) and so produces an a.s.t.

The a.s.t is produced by geometry.

In Newtonian dynamics:

$$ds^2 = dr^2 + r^2 d\theta^2 \quad -(40)$$

and there is no concept of spacetime. Quantities such as E, p and L cannot be produced from the geometry. In the Minkowski cosmology:

$$p^2 = m^2\left(\left(\frac{dr}{d\tau}\right)^2 + r^2\left(\frac{d\theta}{d\tau}\right)^2\right) \quad -(41)$$

$$L^2 = m^2 r^4 \left(\frac{d\theta}{d\tau}\right)^2 \quad -(42)$$

so

$$\left(\frac{p}{L}\right)^2 = \frac{1}{r^4}\left(\frac{dr}{d\tau}\right)^2\left(\frac{d\tau}{d\theta}\right)^2 + \frac{1}{r^2}$$

$$= \frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2 + \frac{1}{r^2} \quad -(43)$$

which is eq. (1), QED. In other words all the orbital dynamics are produced from the metric.

8) The acceleration is defined from the metric as follows:

$$\underline{a} = \frac{L^2}{2m^2} \frac{d}{dr}\left(\frac{f}{L}\right)^2 \underline{e}_r \quad -(44)$$

$$= \frac{L^2}{2m^2} \frac{d}{dr}\left(\frac{1}{r^4}\left(\frac{dr}{d\theta}\right)^2 + \frac{1}{r^2}\right) \underline{e}_r.$$

This equation shows very clearly that the acceleration is a derivative property of the orbit. The acceleration is derived from the orbit.

In the Newtonian point of view there exists a gravitational force between m and M:

$$\underline{F} = -\frac{mMG}{r^2} \underline{e}_r \quad -(45)$$

and there exists an orbit:

$$r = \frac{\alpha}{1+\epsilon\cos\theta} \quad -(46)$$

Eq. (45) leads to eq. (46) if

$$E = \frac{f^2}{2m} + U \quad -(47)$$

and

$$U = -\frac{mMG}{r}, \quad -(48)$$

$$\underline{F} = -\frac{\partial U}{\partial r} \underline{e}_r. \quad -(49)$$

In the Minkowski cosmology there exists eq. (44), which is the direct result of the metric, and the existence of the orbit (46). The orbit itself produces the acceleration, because the orbit is a rotating frame of reference. These remarks are true for any metric, not just the Minkowski metric.

From eqs. (44) and (46):

$$\underline{a} = \frac{L^2}{2m^2}\frac{d}{dr}\left[\left(\frac{\epsilon}{\alpha}\right)^2 \sin^2\theta + \frac{1}{r^2}\right]\underline{e}_r \qquad -(50)$$

$$= \left(\frac{\epsilon^2 L^2}{2m^2\alpha^2}\frac{d}{dr}\sin^2\theta - \frac{L^2}{m^2 r^3}\right)\underline{e}_r$$

$$= \left(\frac{\epsilon^2 L^2}{m^2\alpha^2}\cdot\sin\theta\cos\theta\frac{d\theta}{dr} - \frac{L^2}{m^2 r^3}\right)\underline{e}_r$$

where

$$\frac{d\theta}{dr} = \frac{\alpha}{\epsilon r^2 \sin\theta} \qquad -(51)$$

So:

$$\underline{a} = \left(\frac{\epsilon L^2}{m^2\alpha r^2}\cos\theta - \frac{L^2}{m^2 r^3}\right)\underline{e}_r$$

$$\underline{a} = \left(\frac{L}{mr}\right)^2\left(\frac{\epsilon}{\alpha}\cos\theta - \frac{1}{r}\right)\underline{e}_r. \qquad -(52)$$

However, for an ellipse:

$$\frac{1}{r} = \frac{1}{\alpha}\left(1+\epsilon\cos\theta\right) \qquad -(53)$$

10)

So
$$\underline{a} = -\left(\frac{L}{mr}\right)^2 \cdot \frac{1}{\alpha} \underline{e}_r . \quad -(54)$$

Using
$$\alpha = \frac{L^2}{m^2 M G} \quad -(55)$$

then:
$$\underline{a} = -\frac{MG}{r^2} \underline{e}_r \quad -(56)$$

However, eq. (54) is a property of the metric or infinitesimal line element, whereas eq. (56) is the anthropomorphic assumption of an inverse square force of attraction between m and M in space, not spacetime. In Newtonian dynamics the force

$$\underline{F} = m\underline{a} = -\frac{mMG}{r^2} \underline{e}_r \quad -(57)$$

must be counterbalanced in order that the orbit be stable. In Newtonian dynamics this is done by defining the "potential energy"

$$U_c = \frac{L^2}{2mr^2} \quad -(58)$$

and the "force"
$$\underline{F}_c = -\frac{\partial U_c}{\partial r} \underline{e}_r \quad -(59)$$
$$= \frac{L^2}{mr^3} \underline{e}_r$$

11) As discussed in many textbooks this is incorrect, because the "potential energy" (58) is the rotational part of the kinetic energy:

$$T = \frac{1}{2} m v^2 = \frac{1}{2} m \left(\left(\frac{dr}{dt} \right)^2 + r^2 \left(\frac{d\theta}{dt} \right)^2 \right) \quad -(60)$$

The rotational kinetic energy is:

$$T_{rot} = \frac{1}{2} m r^2 \left(\frac{d\theta}{dt} \right)^2 \quad -(61)$$

where

$$L = m r^2 \frac{d\theta}{dt} \quad -(62)$$

So

$$T_{rot} = \frac{1}{2} m r^2 \left(\frac{L}{m r^2} \right)^2 = \frac{L^2}{2 m r^2} \quad -(63)$$

This is not a potential energy, and a force cannot be defined as in eq. (59). That definition applies only to potential energy, not kinetic energy.
In the Minkowski cosmology, there is no need for any "correct to balance" eq. (54) because eq. (54) is derived from a stable orbit, the line element or metric. These remarks are true for any metric.

12) In the "Schwarzschild" metric for example:

$$\left(\frac{dr}{d\theta}\right)^2 = r^4\left(\frac{1}{a^2} - \left(1-\frac{r_0}{r}\right)\left(\frac{1}{b^2}+\frac{1}{r^2}\right)\right) \quad -(64)$$

and for an infinitesimal line element of type:

$$ds^2 = c^2 d\tau^2 = A c^2 dt^2 - B dr^2 - r^2 d\theta^2 \quad -(65)$$

then

$$\left(\frac{dr}{d\theta}\right)^2 = \frac{r^4}{B}\left(\frac{1}{Aa^2} - \frac{1}{b^2} - \frac{1}{r^2}\right) \quad -(66)$$

the acceleration due to eq. (66) is:

$$\underline{a} = \left(\frac{L^2}{m^2 r^2}\right)\left(\left(\frac{dr}{d\theta}\right)\frac{d}{dr}\left(\frac{1}{r^2}\frac{dr}{d\theta}\right) - \frac{1}{r}\right)\underline{e}_r \quad -(67)$$

$$= \left[\frac{L^2}{m^2 B^{1/2}}\left(\frac{1}{Aa^2} - \frac{1}{b^2} - \frac{1}{r^2}\right)^{1/2}\frac{d}{dr}\left(\frac{r^2}{B^{1/2}}\left(\frac{1}{Aa^2} - \frac{1}{b^2} - \frac{1}{r^2}\right)^{1/2}\right) - \frac{L^2}{m^2 r^3}\right]\underline{e}_r \quad -(68)$$

where

$$a = \frac{cL}{E}, \quad b = \frac{L}{mc} \quad -(69)$$

and for the "Schwarzschild" metric:

$$A = B^{-1} = 1 - \frac{r_0}{r} \quad -(70)$$

Eq. (68) produces a very complicated force law which does not produce a precessing ellipse.