232(3): Further Refutations of EGR.

1) Consider the approximate solution given in eq. (13) of note 232(2):

$$u_t = u_1 + u_p = \frac{1}{\alpha}\left(1+\epsilon\cos\theta\right) + \frac{\delta\epsilon}{\alpha^2}\theta\sin\theta \quad -(1)$$

$$+ \frac{\delta}{\alpha^2}\left(1+\frac{\epsilon^2}{2}\right) - \frac{\delta\epsilon^2}{6\alpha^2}\cos 2\theta.$$

It is claimed that this gives a precessing ellipse:

$$\frac{1}{r} = \frac{1}{\alpha}\left(1+\epsilon\cos(x\theta)\right), \quad x = 1-\frac{\delta}{\alpha} \quad -(2)$$

If so:

$$1+\epsilon\cos(x\theta) = 1+\epsilon\cos\theta + \frac{\delta}{\alpha}\left(\epsilon\sin\theta + 1 + \frac{\epsilon^2}{2} - \frac{\epsilon^2}{6}\cos 2\theta\right)$$

i.e. $$\cos(x\theta) = \cos\theta + \frac{\delta}{\alpha}\left(\sin\theta + \frac{1}{\epsilon} + \frac{\epsilon}{2}\left(1-\frac{1}{3}\cos 2\theta\right)\right)$$

or $$x = \frac{1}{\theta}\cos^{-1}\left(\cos\theta + \frac{\delta}{\alpha}\left(\sin\theta + \frac{1}{\epsilon} + \frac{\epsilon}{2}\left(1-\frac{1}{3}\cos 2\theta\right)\right)\right)$$

$$\neq 1-\frac{\delta}{\alpha} \quad -(3)$$

Thus EGR is refuted, QED.

2) It is claimed that the function:

$$f = \frac{\delta}{\alpha^2}\left(1 + \frac{\epsilon^2}{2}\right) - \frac{\delta\epsilon^2}{6\alpha^2}\cos 2\theta \quad -(4)$$

does not produce precession and can be ignored.

However:

$$\cos\left(2\theta\left(1 - \frac{\delta}{\alpha}\right)\right) = \cos 2\theta \cos\left(2\frac{\delta}{\alpha}\theta\right) + \sin 2\theta \sin\left(\frac{2\delta\theta}{\alpha}\right) \quad -(5)$$

Using the same, albeit incorrect, approximation as used by Maria and Thornton, in order to refute their argument, then for small δ/α:

$$\cos\left(2\theta\left(1 - \frac{\delta}{\alpha}\right)\right) \sim \cos 2\theta + 2\frac{\delta\theta}{\alpha}\sin 2\theta \quad -(6)$$

$$= \cos(y\theta)$$

where

$$y = 2\left(1 - \frac{\delta}{\alpha}\right) \quad -(7)$$

So:

$$\cos 2\theta \sim \cos(y\theta) - 2\frac{\delta\theta}{\alpha}\sin 2\theta \quad -(8)$$

$$\sim \cos(y\theta)$$

for very small δ/α.

So:

$$f = \frac{\delta}{\alpha^2}\left(1 + \frac{\epsilon^2}{2} - \frac{\epsilon^2}{6}\cos(y\theta)\right) \quad -(9)$$

$$= \frac{\delta}{\alpha^2}\left(1 + \frac{\epsilon^2}{2} + \frac{\epsilon^2}{6}\cos(y\theta + \pi)\right)$$

This is a precessing conical section, QED.

Therefore the term (4) cannot be neglected and produces precession and EGR is refuted.

The precessing ellipse is:

$$\frac{1}{r} = \frac{1}{\alpha}\left(1 + \epsilon\cos(x\theta)\right) \quad -(10)$$

and its lagrangian equation of motion is:

$$\boxed{\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + x^2\left(\frac{1}{r} - \frac{1}{\alpha}\right) = 0} \quad -(11)$$

for all θ and for all x. Note carefully that there is no restriction on x.

The Einstein theory produces the

4) equation of motion:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} - \frac{1}{\alpha} - \frac{\delta}{r^2} = 0 \quad -(12)$$

and it is incorrectly claimed that this gives eq. (10) with:

$$x = 1 - \frac{\delta}{\alpha} \quad -(13)$$

and

$$\frac{\delta}{\alpha} \ll 1. \quad -(14)$$

In other words it is incorrectly claimed that the Einstein theory is valid for:

$$x \sim 1. \quad -(15)$$

However, it is seen from eq. (12) that in the limit

$$\delta \to 0, \quad -(16)$$

then the Einstein theory gives:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} - \frac{1}{\alpha} = 0 \quad -(17)$$

i. e.

$$\frac{1}{r} = \frac{1}{\alpha}\left(1 + \epsilon\cos\theta\right) \quad -(18)$$

which is the Newtonian a static ellipse.

5) From comparison of eqs. (11) and (17) it is seen that the Einstein theory has been forced to reduce incorrectly to the Newton theory.

Astronomical observation, however, produces the equation of motion (11), both in the solar system and in objects such as binary pulsars. The data never give eq. (12) of the Einstein theory. For the purpose of observation the apsidal distances are:

$$d = (1+\epsilon)\, r_{min} \quad — (19)$$

and

$$d = (1-\epsilon)\, r_{max}, \quad — (20)$$

so

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + x^2\left(\frac{1}{r} - \frac{1}{(1+\epsilon)\, r_{min}}\right) = 0 \quad — (21)$$

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + x^2\left(\frac{1}{r} - \frac{1}{(1-\epsilon)\, r_{max}}\right) = 0 \quad — (22)$$

i.e.

$$\frac{1}{r} = \frac{1}{(1+\epsilon)\, r_{min}}\left(1 + \epsilon \cos(x\theta)\right) \quad — (23)$$

$$= \frac{1}{(1-\epsilon)\, r_{max}}\left(1 + \epsilon \cos(x\theta)\right) \quad — (24)$$

Therefore:

b)

$$1+\epsilon\cos(x\theta) = \frac{(1+\epsilon)r_{min}}{r} = \frac{(1-\epsilon)r_{max}}{r} \quad -(25)$$

$$\cos(x\theta) = \frac{1}{\epsilon}\left(\frac{(1+\epsilon)r_{min}}{r} - 1\right) = \frac{1}{\epsilon}\left(\frac{(1-\epsilon)r_{max}}{r} - 1\right) \quad -(26)$$

So:

$$\boxed{\begin{aligned} x &= \frac{1}{\theta}\cos^{-1}\left(\frac{1}{\epsilon}\left(\frac{(1+\epsilon)r_{min}}{r} - 1\right)\right) \\ &= \frac{1}{\theta}\cos^{-1}\left(\frac{1}{\epsilon}\left(\frac{(1-\epsilon)r_{max}}{r} - 1\right)\right) \end{aligned}} \quad -(27)$$

for all θ and r.

These are the correct expressions for x. It is seen that x is never $1-\delta/\alpha$, QED.