232(5): Second Order Differential Equations of Higher Powers of u.

In recent work it has been found that the following two equations give similar results:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + x^2\left(\frac{1}{r} - \frac{1}{\alpha}\right) = 0 \quad -(1)$$

and:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} - \frac{1}{\alpha} = \frac{\delta}{r^2}. \quad -(2)$$

Eq. (1) is the true equation of the precessing ellipse:

$$\frac{1}{r} = \frac{1}{\alpha}\left(1 + \epsilon\cos\left(x\theta\right)\right) \quad -(3)$$

for all x and θ. Note carefully that the eccentricity ϵ does not appear in the equation of motion (1), which reduces to the Newtonian equation of motion when $x = 1$.

Eq. (2) is the equation of motion of Einsteinian general relativity (EGR), in which:

$$\frac{1}{\alpha} = \frac{G m^2 M}{L^2}, \quad \delta = \frac{3GM}{c^2} \quad -(4)$$

In eq. (4), an object of mass m orbits an object of mass M. The total angular momentum L is a constant of motion, G is Newton's

2) constant, c is the vacuum speed of light. The plane polar coordinate system is (r, θ).

Numerical integration of eq. (2) by Dr. Horst Eckardt shows that it is in general an unphysical equation, because in general it gives singularities and negative r. The solutions of eq. (2) depend critically on the initial conditions. The approximate method of solution of eq. (2) in a textbook such as that of Marion and Thornton give wildly incorrect results.

However, when:

$$\frac{\delta}{\alpha} << 1 \quad -(5)$$

eq. (2) gives the illusion of a precessing ellipse. The reason for this is not known mathematically but must be a consequence of the fact that under condition (5) eq. (2) is a very tiny perturbation of the Newtonian:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} - \frac{1}{\alpha} = 0 \quad -(6)$$

The conclusion is that EGR is meaningless. The easiest way to see this is that if EGR were to give an exactly precessing ellipse, eq. (2) would have to reduce exactly to

) eq. (1), in which case:

$$\frac{1}{r} - \frac{1}{\alpha} - \frac{\delta}{r^2} = x^2\left(\frac{1}{r} - \frac{1}{\alpha}\right) \quad -(7)$$

This can be true if and only if $\frac{1}{r}$ is given by the two roots of eq. (7).

Reductio ad absurdum.

To solve equation (7) denote:

$$u = \frac{1}{r} \quad -(8)$$

then

$$u - \frac{1}{\alpha} - \delta u^2 = x^2\left(u - \frac{1}{\alpha}\right) \quad -(9)$$

$$\delta u^2 + (x^2 - 1) u + \frac{1}{\alpha}(1 - x^2) = 0 \quad -(10)$$

Eq. (10) is a quadratic in u, with two roots, given by

$$u = \frac{1}{2\delta}\left(1 - x^2 \pm \left((x^2-1)^2 - \frac{4\delta}{\alpha}(1-x^2)\right)^{1/2}\right) \quad -(11)$$

$$\frac{1}{r} = u = \frac{(1-x^2)}{2\delta}\left(1 \pm \left(1 - \frac{4\delta}{\alpha(1-x^2)}\right)^{1/2}\right) \quad -(12)$$

4) Eq. (12) means that eq. (2) is a precessing ellipse (1) if and only if u is fixed at two points, given by eq. (12). For all other u (or 1/r) the curve of u versus θ given by eq. (2) is not a precessing ellipse.

The Newtonian eq. (6) gives a static ellipse, but the perturbation given by eq. (2) does not give the true precessing ellipse (1). Eq. (1) is analytically well behaved for all x and θ, but eq. (2) gives singularities and negative r in general. Therefore eq. (2) must be discarded as unphysical.

The precession of the perihelia is given by the fact that the perihelion is displaced in each revolution of 2π (360°) by:

$$\Delta = 2\pi(1-x) \quad -(13)$$

For the planet Earth, Δ is 0.05 arc seconds per 2π in radians, i.e. ± 0.012 arc seconds per earth year. Now use:

$$1 \text{ arc second} = 4.848 \times 10^{-6} \text{ radians} \quad -(14)$$

so:

$$\Delta = 2.424 \times 10^{-7} \text{ radians} \quad -(15)$$

$$= 2\pi(1-x)$$

$$=$$

5) So:
$$1 - x = \frac{2.424}{2\pi} \times 10^{-7} \quad -(16)$$
$$= 3.548 \times 10^{-8} \quad -(17)$$
Therefore
$$x = 1 + 3.548 \times 10^{-8}$$
and:
$$1 - x^2 = (1+x)(1-x)$$
$$= 3.548 \times 10^{-8} \left(2 + 3.548 \times 10^{-8}\right)$$
$$= 7.096 \times 10^{-8} \quad -(18)$$
$$1 - x^2 \sim 7.096 \times 10^{-8}$$
to an excellent approximation. Therefore the two points in eq. (12) are given by:
$$u = \frac{1}{r} = \frac{7.096 \times 10^{-8}}{2\delta}\left(1 \pm \left(1 - \frac{4\delta}{7.096 \times 10^{-8}\, d}\right)^{1/2}\right)$$
$$= \frac{7.096 \times 10^{-8}}{2\delta}\left(1 \pm \left(1 - 5.637 \times 10^{7}\frac{\delta}{d}\right)^{1/2}\right) \quad -(19)$$
In the Einstein theory
$$\delta = \frac{3GM}{c^2} \quad -(20)$$

6) where:

$$G = 6.67385 \times 10^{-11}\ Nm^2kg^{-2}$$

$$M = 1.9891 \times 10^{30}\ kg$$

$$c = 2.998 \times 10^{8}\ ms^{-1}$$

so

$$\delta = 443.1 \quad -(21)$$

Therefore:

$$u = \frac{1}{r} = 8.007 \times 10^{-11}\left(1 \pm \left(1 - 5.637 \times 10^{7}\frac{\delta}{d}\right)^{1/2}\right) \quad -(22)$$

In the Einstein theory:

$$\frac{\delta}{d} = \frac{3GM}{ac^2(1-\epsilon^2)} \quad -(23)$$

where

$$a = \frac{r_{min}}{1-\epsilon} \quad -(24)$$

Here r_{min} is the distance of closest approach of the earth to the sun, and ϵ is the eccentricity of the earth's orbit. Here

$$r_{min} = 1.471 \times 10^{12}\ m \quad -(25)$$

$$\epsilon = 0.0167 \quad -(26)$$

7) So $a = 1.496 \times 10^{12}$ m $\quad -(27)$

and $1 - \epsilon^2 = 0.9997 \quad -(28)$

therefore $\frac{\delta}{d} = 2.962 \times 10^{-10} \quad -(29)$

This means that r from eq. (12):

$$\left.\begin{aligned} r &= 6.267 \times 10^{9} \text{ m} \\ \text{or} \quad r &= 1.561 \times 10^{11} \text{ m} \end{aligned}\right\} -(30)$$

However, the mean radius of the earth is:

$$r_{av} = 1.496 \times 10^{12} \text{ m} \quad -(31)$$

This result emphasizes once more the absurdity of the Einstein theory because it can give a precessing ellipse (3) if and only if there are two fixed points which are much smaller than the mean radius.

It is seen that the perturbation of the Newtonian orbit is:

$$\frac{\delta}{r^2} \sim \frac{443.1}{1.496^2 \times 10^{24}} \quad -(32)$$

$$\sim 10^{-22} \text{ m}^{-2}$$

8) So perihelion precession is an absurd way in which to test any deviation from Newton.

Continuing this argument one can test the effect of adding terms to higher order in u, for example:

$$\frac{d^2u}{d\theta^2} + u - \frac{1}{\alpha} = Au^2 + Bu^3 + Cu^4 + \dots \quad -(33)$$

Eq. (33) reduces to eq. (1) when:

$$x^2\left(u - \frac{1}{\alpha}\right) = u - \frac{1}{\alpha} - Au^2 - Bu^3 - Cu^4 \quad -(34)$$

This is a quartic in u with four roots. So there are only four points at which eq. (33) can give a true precessing ellipse (3).

Continuing this argument consider:

$$\frac{d^2u}{d\theta^2} + u - \frac{1}{\alpha} = A_1u^2 + A_2u^3 + A_3u^4 + \dots + A_{n-1}u^n + \dots \quad -(35)$$

and it is seen that in order to give a true precessing ellipse for all n,

$$\boxed{n \rightarrow \infty} \quad -(36).$$