224(2) : Limiting Value of x_1

In the limit:

$$\frac{a}{x} \gg 1 \quad -(1)$$

then:

$$\frac{V}{E} = \frac{a}{x} \gg 1 \quad -(2)$$

i.e

$$E \ll V. \quad -(3)$$

In this case:

$$x_1 \rightarrow 8.33 \times 10^6 \left(\frac{Z_1 Z_2}{a}\right)^{1/2} \int_a^0 \left(\frac{a}{x}\right)^{1/2} dx$$

$$= 8.33 \times 10^6 (Z_1 Z_2)^{1/2} \int_a^0 \frac{1}{x^{1/2}} dx \quad -(4)$$

$$\boxed{x_1 \doteq 8.33 \times 10^6 (Z_1 Z_2 a)^{1/2}} \quad -(5)$$

where

$$T = \frac{4}{\left(2e^{x_1} + \frac{1}{2}e^{-x_1}\right)^2} \quad -(6)$$

So if

$$a \sim 10^{-14} m, \quad -(7)$$

then

$$x_1 \rightarrow \sim 1 \quad -(8)$$

the energy corresponding to eq. (7) is:

2)

$$E = \frac{Z_1 Z_2 \times 1.60219^2 \times 10^{-38}}{1.11265 \times 10^{-10} \times 10^{-28}} \text{ J}$$

$$E = 2.31 Z_1 Z_2 \text{ joules} \quad -(9)$$

The Compton wavelength of the proton is :

$$\lambda_p = \frac{h}{mc} = \frac{6.62618 \times 10^{-34}}{1.67265 \times 10^{-27} \times 2.998 \times 10^{8}}$$

$$= 1.32 \times 10^{-14} \text{ m} \quad -(10)$$

so

$$\boxed{a \sim \lambda_p} \quad -(11)$$

to order of magnitude approximation, so substantial transmission occurs when :

$$V >> E \quad -(12)$$

and when a is tuned to the Compton wavelength of the proton.