1) 225(1): Development of Electroweak Theory from the Minimal Prescription and Fermion Equation.

In the notation of UFT172 ff the fermion equation is:

$$\pi_\mu \phi \sigma^\mu = mc\,\sigma^1 \phi \quad -(1)$$

and can be written as two simultaneous equations:

$$(\hat{E} + c\underline{\sigma}\cdot\hat{\underline{p}})\phi^L = mc^2\phi^R \quad -(2)$$

$$(\hat{E} - c\underline{\sigma}\cdot\hat{\underline{p}})\phi^R = mc^2\phi^L \quad -(3)$$

where

$$\phi^R = \begin{bmatrix} \phi_1^R \\ \phi_2^R \end{bmatrix}, \quad \phi^L = \begin{bmatrix} \phi_1^L \\ \phi_2^L \end{bmatrix} \quad -(4)$$

From eq. (3):

$$\phi^R = \left(\frac{mc^2}{\hat{E} - c\underline{\sigma}\cdot\hat{\underline{p}}}\right)\phi^L \quad -(5)$$

So

$$(\hat{E} + c\underline{\sigma}\cdot\hat{\underline{p}})(\hat{E} - c\underline{\sigma}\cdot\hat{\underline{p}})\phi^R = m^2c^4\phi^R \quad -(6)$$

$$(\hat{E} - c\underline{\sigma}\cdot\hat{\underline{p}})(\hat{E} + c\underline{\sigma}\cdot\hat{\underline{p}})\phi^L = m^2c^4\phi^L \quad -(7)$$

As in note 173(4) the fermion equation is:

2) $$\sigma^0 \hat{E} \phi \sigma^0 - c\sigma^3 \left(\hat{p}_x \phi \sigma^1 + \hat{p}_y \phi \sigma^2 + \hat{p}_z \phi \sigma^3 \right)$$
$$= mc^2 \sigma^1 \phi \quad -(8)$$

and
$$\phi = \begin{bmatrix} \phi_1^R & \phi_2^R \\ \phi_1^L & \phi_2^L \end{bmatrix} \quad -(9)$$
$$-(10)$$

The Pauli matrices are:

$$\sigma^0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \sigma^1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma^2 = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \quad \sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

The basic operators of quantum mechanics are:

$$\hat{E} = i\hbar \frac{\partial}{\partial t}, \quad \hat{\underline{p}} = -i\hbar \underline{\nabla} \quad -(11).$$

Eqs. (2) and (3) may be written as:

$$\left(E^2 - c^2 \underline{\sigma} \cdot \underline{p}\, \underline{\sigma} \cdot \underline{p} \right) \phi^R = m^2 c^4 \phi^R \quad -(12)$$
$$\left(E^2 - c^2 \underline{\sigma} \cdot \underline{p}\, \underline{\sigma} \cdot \underline{p} \right) \phi^L = m^2 c^4 \phi^L \quad -(13)$$

In note 173(4) the semi operator method of solving these equations was introduced:

$$E^2 = E\hat{E} = i\hbar E \frac{\partial}{\partial t} \quad -(14)$$

where
$$E = \gamma m c^2 \quad -(15)$$

Therefore eq. (12) becomes:

3)

$$\left(\gamma m c^2 \hat{E} - c^2 \underline{\sigma} \cdot \underline{p}\, \underline{\sigma} \cdot \underline{p}\right) \phi^R = m^2 c^4 \phi^R \quad -(16)$$

i.e

$$\hat{E} \phi^R = H \phi^R \quad -(17)$$

where

$$H = \frac{1}{E}\left(c^2 \underline{\sigma} \cdot \underline{p}\, \underline{\sigma} \cdot \underline{p} + m^2 c^4\right) \quad -(18)$$

is the hamiltonian, in which:

$$E = \gamma m c^2, \quad \underline{p} = \gamma m \underline{v}, \quad -(19)$$

so

$$H = \frac{\gamma}{m} \underline{\sigma} \cdot \underline{v}\, \underline{\sigma} \cdot \underline{v} + \frac{mc^2}{\gamma} \quad -(20)$$

So the fermion equation becomes:

$$i\hbar \frac{\partial \phi^R}{\partial t} = H \phi^R \quad -(21)$$

$$i\hbar \frac{\partial \phi^L}{\partial t} = H \phi^L \quad -(22)$$

with H given by eq. (20).

Eqs. (21) and (22) are equations for the right and left handed free electron.

Interaction with the electromagnetic field is developed with the minimal prescription:

$$p^\mu \rightarrow p^\mu - eA^\mu \quad -(23)$$

4) where $p^\mu = \left(\frac{E}{c}, \underline{p} \right)$ — (24)

so: $E \rightarrow E - e\phi,$ — (25)

$\underline{p} \rightarrow \underline{p} - e\underline{A}.$ — (26)

In ECE theory, these equations are true for each sense of polarization a.

Denoting $\underline{\pi} = \underline{p} - e\underline{A}$ — (27)

eq. (16) becomes:

$$\left((E - e\phi)(\hat{E} - e\phi) - c^2 \underline{\sigma} \cdot \underline{\pi} \, \underline{\sigma} \cdot \underline{\pi} \right) \phi^R = m^2 c^4 \phi^R \quad -(28)$$

so $\hat{E} \phi^R = H \phi^R$ — (29)

— (30)

where
$$H = \frac{m^2 c^4}{E - e\phi} + e\phi + c^2 \frac{\underline{\sigma} \cdot \underline{\pi} \, \underline{\sigma} \cdot \underline{\pi}}{E - e\phi}$$

$$= \frac{m^2 c^4}{E - e\phi} + e\phi + H_3$$

5) The correctly relativistic Landé and Thomas factors are obtained from:

$$H_3 = c^2 \underline{\sigma} \cdot \underline{\Pi} \left(E - e\phi \right)^{-1} \underline{\sigma} \cdot \underline{\Pi} \quad -(31)$$

$$= \frac{c^2}{E} \underline{\sigma} \cdot \underline{\Pi} \left(1 - \frac{e\phi}{E} \right)^{-1} \underline{\sigma} \cdot \underline{\Pi}$$

At this point an approximation is made:

$$\left(1 - \frac{e\phi}{E} \right)^{-1} \sim 1 + \frac{e\phi}{E} \quad -(32)$$

so it is assumed that:

$$e\phi << E \quad -(33)$$

where

$$E = \gamma m c^2. \quad -(34)$$

So:

$$H_3 = \frac{1}{\gamma m} \left(\underline{\sigma} \cdot \underline{\Pi} \right) \left(\underline{\sigma} \cdot \underline{\Pi} \right) + \frac{e}{\gamma^2 m^2 c^2} \underline{\sigma} \cdot \underline{\Pi} \phi \underline{\sigma} \cdot \underline{\Pi} \quad -(35)$$

The first term gives the relativistic Landé factor and the second term the relativistic Thomas factor.

6) Eq. (28) can be written as:

$$\left(E - e\phi - c^2\frac{\underline{\sigma}\cdot\underline{\Pi}\;\underline{\sigma}\cdot\underline{\Pi}}{\hat{E} - e\phi}\right)\phi^R = \left(\frac{m^2c^4}{E - e\phi}\right)\phi^R \qquad -(36)$$

Add mc^2 to each side:

$$\left(E + mc^2 - e\phi - c^2\frac{\underline{\sigma}\cdot\underline{\Pi}\;\underline{\sigma}\cdot\underline{\Pi}}{\hat{E} - e\phi}\right)\phi^R = \left(\frac{m^2c^4}{E - e\phi} + mc^2\right)\phi^R \qquad -(37)$$

In the non-relativistic approximation:

$$E \rightarrow mc^2 \qquad -(38)$$

so eq. (37) becomes:

$$\left((2mc^2 - e\phi)(\hat{E} - e\phi) - c^2(\underline{\sigma}\cdot\underline{\Pi})(\underline{\sigma}\cdot\underline{\Pi})\right)\phi^R = \left(mc^2(2mc^2 - e\phi)\right)\phi^R \qquad -(39)$$

i.e.

$$\hat{E}\,\phi^R = H\,\phi^R \qquad -(40)$$

where:

$$H = mc^2 + e\phi + c^2(\underline{\sigma}\cdot\underline{\Pi})(2mc^2 - e\phi)^{-1}(\underline{\sigma}\cdot\underline{\Pi}) \qquad -(41)$$

7) The hamiltonian is approximated by:

$$H \sim mc^2 + e\phi + \frac{1}{2m}\underline{\sigma}\cdot\underline{\pi}\left(1 - \frac{e\phi}{2mc^2}\right)\underline{\sigma}\cdot\underline{\pi}$$

$$= mc^2 + e\phi + \frac{1}{2m}\underline{\sigma}\cdot\underline{\pi}\,\underline{\sigma}\cdot\underline{\pi} + \frac{1}{4m^2c^2}\underline{\sigma}\cdot\underline{\pi}\,\phi\,\underline{\sigma}\cdot\underline{\pi} \quad -(42)$$

where

$$\underline{\pi} = \underline{p} - e\underline{A} \quad -(43)$$

The Landé factor, Zeeman effect, ESR, NMR, MRI and FSR are obtained from:

$$H_3 = \frac{1}{2m}\underline{\sigma}\cdot\underline{\pi}\,\underline{\sigma}\cdot\underline{\pi} \quad -(44)$$

and the Thomas factor, spin-orbit coupling and Darwin term from:

$$H_4 = \frac{1}{4m^2c^2}\underline{\sigma}\cdot\underline{\pi}\,\phi\,\underline{\sigma}\cdot\underline{\pi} \quad -(45)$$

In the next note these terms will be developed for electromagnetic and weak field interaction