225(8): Computer Check of GWS Theory

The check is of eq. (8.85) of L. H. Ryder, "Quantum Field Theory", page 303, eq. (8.85), second edition, Cambridge University Press, 1996:

$$i\bar{R}\gamma^\mu(\partial_\mu + ig'X_\mu)R + i\bar{L}\gamma^\mu\left(\partial_\mu + \frac{i}{2}(g'X_\mu - g\underline{\tau}\cdot\underline{W_\mu})\right)L$$

$$= i\bar{e}\gamma^\mu\partial_\mu e + i\bar{\nu}\gamma^\mu\partial_\mu\nu - g\sin\theta\,\bar{e}\gamma^\mu e A_\mu$$

$$+ \frac{g}{\cos\theta}\left(\sin^2\theta\,\bar{e}_R\gamma^\mu e_R - \frac{1}{2}\cos 2\theta\,\bar{e}_L\gamma^\mu e_L + \frac{1}{2}\bar{\nu}\gamma^\mu\nu\right)Z_\mu$$

$$+ \frac{g}{\sqrt{2}}\left(\left(\bar{\nu}\gamma^\mu e_L W_\mu^+\right) + \text{hermitian conjugate}\right):$$

$$= \mathcal{L}_1. \quad -(1)$$

Here:

$$e = e_L + e_R \quad -(2)$$

$$W_\mu = \frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2) \quad -(3)$$

$$\cos\theta = \frac{g}{(g^2+g'^2)^{1/2}} \quad -(4)$$

$$\sin\theta = \frac{g'}{(g^2+g'^2)^{1/2}} \quad -(5)$$

$$Z_\mu = W_\mu^3\cos\theta - X_\mu\sin\theta \quad -(6)$$

2)

$$A_\mu = W^3_\mu \sin\theta + X_\mu \cos\theta, \quad -(7)$$

$$R = e_R, \quad \overline{R} = \overline{e_R} \quad -(8)$$

$$L = \begin{bmatrix} \nu \\ e_L \end{bmatrix}, \quad \overline{L} = [\overline{\nu} \; \overline{e_L}], \quad -(9)$$

$$D_\mu R = \partial_\mu R + ig' X_\mu R, \quad -(10)$$

$$D_\mu L = \left(\partial_\mu + \frac{i}{2}\left(g' X_\mu - \underline{\tau} \cdot \underline{W_\mu}\right)\right) L, \quad -(11)$$

$$\underline{\tau} \cdot \underline{W}_\mu = \begin{bmatrix} W^3_\mu & W^1_\mu - iW^2_\mu \\ W^1_\mu + iW^2_\mu & -W^3_\mu \end{bmatrix}. \quad -(12)$$

Therefore: $-(13)$

$$i\overline{R}\gamma^\mu D_\mu R$$

$$= i\overline{e}_R \gamma^\mu (\partial_\mu + ig' X_\mu) e_R$$

$$= i\overline{e}_R \gamma^\mu \partial_\mu e_R - g' X_\mu \overline{e}_R \gamma^\mu e_R$$

giving the first two terms of $\mathcal{L}_1$.

The other two terms are given by:

3)

$$i\bar{L}\gamma^\mu D_\mu L$$

$$= i\begin{bmatrix}\bar{\nu} & \bar{e}_L\end{bmatrix}\gamma^\mu\left(\left(\partial_\mu + \frac{i}{2}g'X_\mu\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\right.\right. \quad -(14)$$

$$\left.\left. - \frac{ig}{2}\begin{bmatrix}W_\mu^3 & W_\mu^1 - iW_\mu^2\\ W_\mu^1 + iW_\mu^2 & -W_\mu^3\end{bmatrix}\right)\right)\begin{bmatrix}\nu\\ e_L\end{bmatrix}$$

$$= i\begin{bmatrix}\bar{\nu} & \bar{e}_L\end{bmatrix}\gamma^\mu\partial_\mu\begin{bmatrix}\nu\\ e_L\end{bmatrix}$$

$$- \frac{g'}{2}X_\mu\begin{bmatrix}\bar{\nu} & \bar{e}_L\end{bmatrix}\gamma^\mu\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}\begin{bmatrix}\nu\\ e_L\end{bmatrix}$$

$$+ \frac{g}{2}\begin{bmatrix}\bar{\nu} & \bar{e}_L\end{bmatrix}\gamma^\mu\begin{bmatrix}W_\mu^3 & W_\mu^1 - iW_\mu^2\\ W_\mu^1 + iW_\mu^2 & -W_\mu^3\end{bmatrix}\begin{bmatrix}\nu\\ e_L\end{bmatrix}$$

$$= i\bar{\nu}\gamma^\mu\partial_\mu\nu + i\bar{e}_L\gamma^\mu\partial_\mu e_L$$

$$- \frac{g'}{2}X_\mu\bar{\nu}\gamma^\mu\nu - \frac{g'}{2}X^\mu\bar{e}_L\gamma^\mu e_L$$

$$+ \frac{g}{2}\begin{bmatrix}\bar{\nu} & \bar{e}_L\end{bmatrix}\gamma^\mu\begin{bmatrix}W_\mu^3\nu + (W_\mu^1 - iW_\mu^2)e_L\\ (W_\mu^1 + iW_\mu^2)\nu - W_\mu^3 e_L\end{bmatrix}$$

$$
\begin{aligned}
&= i\bar{\nu}\gamma^\mu\partial_\mu\nu + i\bar{e}_L\gamma^\mu\partial_\mu e_L \\
&\quad - \frac{g'}{2}X_\mu\bar{\nu}\gamma^\mu\nu - \frac{g'}{2}X_\mu\bar{e}_L\gamma^\mu e_L \\
&\quad + \frac{g}{2}\bar{\nu}\gamma^\mu\left(W^3_\mu\nu + (W^1_\mu - iW^2_\mu)e_L\right) \\
&\quad + \frac{g}{2}\bar{e}_L\gamma^\mu\left((W^1_\mu + iW^2_\mu)\nu - W^3_\mu e_L\right)
\end{aligned}
$$

$$
\begin{aligned}
&= i\bar{\nu}\gamma^\mu\partial_\mu\nu + i\bar{e}_L\gamma^\mu\partial_\mu e_L \\
&\quad - \frac{g'}{2}X_\mu\bar{\nu}\gamma^\mu\nu - \frac{g'}{2}X_\mu\bar{e}_L\gamma^\mu e_L \\
&\quad + \frac{g}{2}W^3_\mu\bar{\nu}\gamma^\mu\nu + \frac{g}{2}(W^1_\mu - iW^2_\mu)\bar{\nu}\gamma^\mu e_L \\
&\quad + \frac{g}{2}(W^1_\mu + iW^2_\mu)\bar{e}_L\gamma^\mu\nu - \frac{g}{2}W^3_\mu\bar{e}_L\gamma^\mu e_L
\end{aligned}
\qquad (14)
$$

The complete lagrangian is the sum of terms in eqs. (13) and (14). According to Ryder this sum must be the same as that in eq. (1).

The sum of eq. (13) and eq. (14) gives:

5)

$$\mathcal{L}_1 = i\bar{e}_R\gamma^\mu\partial_\mu e_R - g' X_\mu \bar{e}_R\gamma^\mu e_R + i\bar{\nu}\gamma^\mu\partial_\mu\nu + i\bar{e}_L\gamma^\mu\partial_\mu e_L - \frac{g'}{2}X_\mu\bar{\nu}\gamma^\mu\nu - \frac{g'}{2}X_\mu\bar{e}_L\gamma^\mu e_L + \frac{g}{2}W_\mu^3\bar{\nu}\gamma^\mu\nu + \frac{g}{2}(W_\mu^1 - iW_\mu^2)\bar{\nu}\gamma^\mu e_L + \frac{g}{2}(W_\mu^1 + iW_\mu^2)\bar{e}_L\gamma^\mu\nu - \frac{g}{2}W_\mu^3\bar{e}_L\gamma^\mu e_L \quad -(15)$$

Computer algebra can now be used to find whether eq. (1) and eq. (15) can ever be the same.

It can be seen that only some terms are the same:

$$+i\bar{e}_R\gamma^\mu\partial_\mu e_L + i\bar{e}_L\gamma^\mu\partial_\mu e_R$$

1) $$i\bar{e}\gamma^\mu\partial_\mu e = \boxed{i\bar{e}_R\gamma^\mu\partial_\mu e_R + i\bar{e}_L\gamma^\mu\partial_\mu e_L} \quad -(16)$$

2) $$\frac{gW_\mu^+}{\sqrt{2}}\bar{\nu}\gamma^\mu e_L = \frac{g}{2}(W_\mu^1 - iW_\mu^2)\bar{\nu}\gamma^\mu e_L \quad -(17)$$

6) 3) $\frac{g}{\sqrt{2}} W_\mu \bar{\nu} \gamma^\mu e_L = \frac{g}{2}(W^1_\mu - i W^2_\mu) \bar{\nu} \gamma^\mu e_L$ — (18)

4) $i\bar{\nu}\gamma^\mu \partial_\mu \nu = i\bar{\nu}\gamma^\mu \partial_\mu \nu$ — (19)

So it must be proven that:

$$-g \sin\theta \, \bar{e} \gamma^\mu e A_\mu$$
$$+ \frac{g}{\cos\theta}\left(\sin^2\theta \, \bar{e}_R \gamma^\mu e_R - \frac{1}{2}\cos 2\theta \, \bar{e}_L \gamma^\mu e_L + \frac{1}{2}\bar{\nu}\gamma^\mu \nu\right) Z_\mu$$

$$= -g' X_\mu \bar{e}_R \gamma^\mu e_R - \frac{g'}{2} X_\mu \bar{e}_L \gamma^\mu e_L$$
$$- \frac{g}{2} W^3_\mu \bar{e}_L \gamma^\mu e_L - \frac{g'}{2} X_\mu \bar{\nu}\gamma^\mu \nu$$
$$+ \frac{g}{2} W^3_\mu \bar{\nu}\gamma^\mu \nu$$

$$= -g' X_\mu \bar{e}_R \gamma^\mu e_R$$
$$- \frac{1}{2}\left(g' X_\mu + g W^3_\mu\right) \bar{e}_L \gamma^\mu e_L$$
$$- \frac{1}{2}\left(g' X_\mu - g W^3_\mu\right) \bar{\nu}\gamma^\mu \nu$$
— (20)

7)

The last term is:

$$\frac{1}{2}\bar{\nu}\gamma^\mu\nu\left(gW^3_\mu - g'X_\mu\right)$$

$$= \frac{1}{2}\left(g^2+g'^2\right)^{1/2} Z_\mu \bar{\nu}\gamma^\mu\nu$$

$$= \frac{1}{2}\frac{g}{\cos\theta} Z_\mu \bar{\nu}\gamma^\mu\nu \qquad -(21)$$

and this is correct. So it must be proven ~~that~~:

$$-g'X_\mu \bar{e}_R\gamma^\mu e_R - \frac{1}{2}\left(g'X_\mu + gW^3_\mu\right)\bar{e}_L\gamma^\mu e_L$$

$$= -g\sin\theta A_\mu \bar{e}\gamma^\mu e$$

$$+ gZ_\mu \frac{\sin^2\theta}{\cos\theta}\bar{e}_R\gamma^\mu e_R \qquad -(22)$$

$$- \frac{1}{2} g Z_\mu \frac{\cos 2\theta}{\cos\theta}\bar{e}_L\gamma^\mu e_L$$

where

$$e = e_L + e_R \qquad -(23)$$

$$A_\mu = W^3_\mu \sin\theta + X_\mu\cos\theta$$

$$= \frac{g'W^3_\mu + gX_\mu}{\left(g^2+g'^2\right)^{1/2}} \qquad -(24)$$

and

$$\cos 2\theta = 2\cos^2\theta - 1 . \quad -(25)$$

1) First note that the left hand side of eq. (22) is

$$\boxed{\mathcal{L}_{em} = -g' X_\mu \bar{e}_R \gamma^\mu e_R - \frac{1}{2}\left(g^2 + g'^2\right)^{1/2} A_\mu \bar{e}_L \gamma e_L} \quad -(26)$$

So the electromagnetic potential A_μ interacts only with the left handed electron. This is the actual result of the GWS theory and is clearly unphysical because A_μ must interact with both electrons

2) Secondly the right hand side of eq. (22) is:

$$\begin{aligned}\mathcal{L}_{em}(RHS) = & -g \sin\theta A_\mu (\bar{e}_R + \bar{e}_L)\gamma^\mu (e_R + e_L) \\ & + g Z_\mu \frac{\sin^2\theta}{\cos\theta} \bar{e}_R \gamma^\mu e_R \\ & - \frac{1}{2} g Z_\mu \frac{\cos 2\theta}{\cos\theta} \bar{e}_L \gamma^\mu e_L \end{aligned} \quad -(27)$$

and contains mixed terms such as:

$$-g \sin\theta A_\mu \bar{e}_R \gamma^\mu e_L \quad -(28)$$

2) and these do not appear in the correct eq. (26)

3) Thirdly when:

$$\cos\theta = 1, \quad \sin\theta = 0 \quad -(29)$$

then

$$A_\mu = X_\mu, \quad -(30)$$

$$g' = 0, \quad -(31)$$

and

$$\mathcal{L}_{em} = -\frac{g}{2} A_\mu \bar{e}_L \gamma e_L, \quad -(32)$$

in which case

$$e = \frac{g}{2}. \quad -(33)$$

when:

$$\cos\theta = 0, \quad \sin\theta = 1, \quad -(34)$$

$$A_\mu = W_\mu^3, \quad -(35)$$

$$g = 0 \quad -(36)$$

and

$$\mathcal{L}_{em} = -g' X_\mu \bar{e}_R \gamma^\mu e_R - \frac{g'}{2} A_\mu \bar{e}_L \gamma^\mu e_L \quad -(37)$$

and the field interacts with e_R and e_L.

therefore these considerations comprehensively refute the GWS and Higgs theory.