225(5) : Algebraic Check of Standard GWS Electroweak Theory

Reference : L. H. Ryder, "Quantum Field Theory" (Cambridge Univ. Press, 2nd. ed., 1996)

On page 302 of this edition the Higgs field of the GWS mechanism is defined in eq. (8.79) as:

$$\phi = \begin{bmatrix} 0 \\ \eta + \frac{\sigma}{\sqrt{2}} \end{bmatrix} \quad -(1)$$

after a series of arbitrary assumptions. In the following equation the covariant derivative is defined as:

$$D_\mu \phi = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}}\sigma \end{bmatrix} - \left[\frac{ig}{2} \begin{bmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{bmatrix} + \frac{ig'}{2} X_\mu \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right] \begin{bmatrix} 0 \\ \eta + \frac{\sigma}{\sqrt{2}} \end{bmatrix} \quad -(2)$$

This is worked out as follows:

$$D_\mu \phi = \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}}\sigma \end{bmatrix} - \frac{ig}{2} \begin{bmatrix} W_\mu^3 & W_\mu^1 - iW_\mu^2 \\ W_\mu^1 + iW_\mu^2 & -W_\mu^3 \end{bmatrix} \begin{bmatrix} 0 \\ \eta + \frac{\sigma}{\sqrt{2}} \end{bmatrix} - \frac{ig'}{2} X_\mu \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \eta + \frac{\sigma}{\sqrt{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ \frac{1}{\sqrt{2}}\sigma \end{bmatrix} - \frac{ig}{2} \begin{bmatrix} (W_\mu^1 - iW_\mu^2)\left(\eta + \frac{\sigma}{\sqrt{2}}\right) \\ -W_\mu^3\left(\eta + \frac{\sigma}{\sqrt{2}}\right) \end{bmatrix} - \frac{ig'}{2} X_\mu \begin{bmatrix} 0 \\ \eta + \frac{\sigma}{\sqrt{2}} \end{bmatrix} \quad -(3)$$

2)

$$= \begin{bmatrix} -\frac{ig}{2}\left(\eta + \frac{\sigma}{\sqrt{2}}\right)\left(W^1_\mu - iW^2_\mu\right) \\ \frac{1}{\sqrt{2}}\partial_\mu \phi + \frac{ig}{2} W^3_\mu \left(\eta + \frac{\sigma}{\sqrt{2}}\right) - \frac{ig'}{2} X_\mu \left(\eta + \frac{\sigma}{\sqrt{2}}\right) \end{bmatrix}$$

$$= -\frac{i}{2}\begin{bmatrix} g\left(\eta + \frac{\sigma}{\sqrt{2}}\right)\left(W^1_\mu - iW^2_\mu\right) \\ + i\sqrt{2}\,\partial_\mu \phi - g\left(\eta + \frac{\sigma}{\sqrt{2}}\right) W^3_\mu + g' X_\mu \left(\eta + \frac{\sigma}{\sqrt{2}}\right) \end{bmatrix}$$

$$= -\frac{i}{2}\begin{bmatrix} g\left(\eta + \frac{\sigma}{\sqrt{2}}\right)\left(W^1_\mu - iW^2_\mu\right) \\ + i\sqrt{2}\,\partial_\mu \phi - \eta\left(g W^3_\mu - g' X_\mu\right) - \frac{\sigma}{\sqrt{2}}\left(g W^3_\mu - g' X_\mu\right) \end{bmatrix} \quad -(4)$$

The result claimed by Ryder is:

$$D_\mu \phi = -\frac{i}{2}\begin{bmatrix} g\left(\eta + \frac{\sigma}{\sqrt{2}}\right)\left(W^1_\mu - iW^2_\mu\right) \\ i\sqrt{2}\,\partial_\mu \phi + \eta\left(-g W^3_\mu + g' X_\mu\right) + \frac{\sigma}{\sqrt{2}}\left(-g W^3_\mu + g' X_\mu\right) \end{bmatrix}$$

~~and contains two errors. These negate the whole of electroweak theory, and the whole of Higgs Boson theory.~~