225(7): Inconsistencies in Accounts of Electroweak Theory

In Weinberg, "The Quantum Theory of Fields", volume 2, chapter 21 the electric charge is defined as

$$q = \frac{e}{g} t_3 - \frac{e}{g'} y \quad -(1)$$

and spontaneous symmetry breaking is said to lead to:

$$A^\mu_\alpha = C_\alpha A^\mu + \dots \quad -(2)$$

Weinberg defines:

$$Z^\mu = A^\mu_3 \cos\theta + B^\mu \sin\theta \quad -(3)$$

$$A^\mu = -A^\mu_3 \sin\theta + B^\mu \cos\theta \quad -(4)$$

but Ryder gives:

$$Z_\mu = W^3_\mu \cos\theta - X_\mu \sin\theta \quad -(5)$$

$$A_\mu = W^3_\mu \cos\theta + X_\mu \cos\theta \quad -(6)$$

These have different signs.

Weinberg claims "by inspection" to arrive at

$$q = -t_3 \sin\theta + y \cos\theta \quad -(7)$$

from eqs. (3) and (4), which have claimed to be examples of eq. (2). By comparison of eqs. (1) and (7) it is then claimed that:

$$g = -e/\sin\theta, \quad g' = -e/\cos\theta \quad -(8)$$

2) The actual results given by Weinberg are:

$$m_W = \frac{37.3}{|\sin\theta|} \text{ GeV}, \quad m_Z = \frac{74.6}{|\sin 2\theta|} \text{ GeV} \qquad -(9)$$

From comparison of Weinberg eq. (21.3.20), vol. 2 and Ryder eq. (8.85) it may be that:

$$e_L = \frac{1}{2}(1+\gamma_5)e \qquad -(10)$$

$$e_R = \frac{1}{2}(1-\gamma_5)e \qquad -(11)$$

but this is not at all clear. The Dirac γ_5 matrix is:

$$\gamma_5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}. \qquad -(12)$$

Adding eqs. (10) and (11):

$$\boxed{e = e_L + e_R} \qquad -(13)$$

It is now possible to check Ryder's claim in eq. (8.85).

Before proceeding to that check there is a site:

www.physics.buffalo.edu/gonsalves/phy522/

that explains that the Higgs mechanism allows fermions to acquire mass but the value of the mass is not fixed by the theory. It has one free

3) parameter for each type of fermion. So it is not a predictive theory.

By googling "Glashow Weinberg Salaam theory" second site, there appears an essay by Xian Hao Xin which appears a Higgs Lagrangian:

$$\mathcal{L} = (D_\mu \phi)^\dagger D^\mu \phi + \mu^2 \phi^\dagger \phi - \lambda (\phi^\dagger \phi)^2 \quad -(14)$$

but this has a different sign from Ryder's eq. (8.73):

$$\mathcal{L}_2 = (D_\mu \phi)^\dagger (D_\mu \phi) - \frac{m^2}{2} \phi^\dagger \phi - \frac{\lambda}{4} (\phi^\dagger \phi)^2 \quad -(15)$$

$$+ \dots$$

In this essay the covariant derivative is:

$$D_\mu \phi = (\partial_\mu + i g T^a A_\mu^a + i g' Y B_\mu) \phi \quad -(16)$$

but the covariant derivative in Ryder's eq (8.72) is:

$$D_\mu \phi = \left(\partial_\mu - \frac{i}{2} g \underline{\tau} \cdot \underline{W}_\mu - \frac{i}{2} g' X_\mu\right) \phi \quad -(17)$$

with two different signs.

The Higgs mechanism is used only because of gauge invariance of the Lagrangian. Any massive particle destroys this gauge invariance, as in the Proca equation for photon with mass. Finally the wikipedia article merely

4) asserts that the Weinberg angle is defined by:

$$\begin{bmatrix} A^\mu \\ Z^\mu \end{bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} B^\mu \\ W^{3\mu} \end{bmatrix} \quad -(18)$$

So we are now in a position to check Ryder's eq. (8.85), where he has used theory from Weinberg, not defined at all in his own book. The equation in question is:

$$i\bar{R}\gamma^\mu(\partial_\mu + ig' X_\mu)R + i\bar{L}\gamma^\mu\left(\partial_\mu + \frac{i}{2}g' X_\mu - \frac{ig\tau\cdot W_\mu}{2}\right)L$$

$$\stackrel{?}{=} i\bar{e}\gamma^\mu\partial_\mu e + i\bar{\nu}\gamma^\mu\partial_\mu\nu - g\sin\theta\,\bar{e}\gamma^\mu e A_\mu \quad -(19)$$

$$+ \frac{g}{\cos\theta}\left(\sin^2\theta\,\bar{e}_R\gamma^\mu e_R - \frac{1}{2}\cos 2\theta\,\bar{e}_L\gamma^\mu e_L + \frac{1}{2}\bar{\nu}\gamma^\mu\nu\right)Z_\mu$$

$$+ \frac{g}{\sqrt{2}}\left[(\bar{\nu}\gamma^\mu e_L W_\mu^+) + h.c.\right]$$

where

$$e = e_L + e_R, \quad -(20)$$

$$\cos\theta = \frac{g}{(g^2 + g'^2)^{1/2}}, \quad -(21)$$

$$\sin\theta = \frac{g'}{(g^2 + g'^2)^{1/2}} \quad -(22)$$

$$W_\mu = \frac{1}{\sqrt{2}}(W_\mu^1 + iW_\mu^2) \quad -(23)$$