225(2): Development of the H_3 Hamiltonian.

The H_3 Hamiltonian of note 225(1) is:

$$H_3 = \frac{1}{2m} \underline{\sigma} \cdot \underline{\pi} \, \underline{\sigma} \cdot \underline{\pi} \quad -(1)$$

so

$$\hat{E}_3 \phi^R = H_3 \phi^R \quad -(2)$$

and

$$\hat{E}_3 \phi^L = H_3 \phi^L \quad -(3)$$

The algebra of Pauli matrices gives:

$$\underline{\sigma} \cdot \underline{\pi} \, \underline{\sigma} \cdot \underline{\pi} = \underline{\pi} \cdot \underline{\pi} + i \underline{\sigma} \cdot \underline{\pi} \times \underline{\pi} \quad -(4)$$

For real valued $\underline{\pi}$:

$$\underline{\sigma} \cdot \underline{\pi} \, \underline{\sigma} \cdot \underline{\pi} = \underline{\sigma} \cdot (\underline{p} - e\underline{A}) \, \underline{\sigma} \cdot (\underline{p} - e\underline{A})$$

$$= \underline{\sigma} \cdot \underline{p} \, \underline{\sigma} \cdot \underline{p} - e \underline{\sigma} \cdot \underline{A} \, \underline{\sigma} \cdot \underline{p} - e \underline{\sigma} \cdot \underline{p} \, \underline{\sigma} \cdot \underline{A} + e^2 \underline{\sigma} \cdot \underline{A} \, \underline{\sigma} \cdot \underline{A}$$

$$= p^2 + e^2 A^2 - e(\underline{A} \cdot \underline{p} + \underline{p} \cdot \underline{A}) - ie(\underline{\sigma} \cdot \underline{A} \times \underline{p} + \underline{\sigma} \cdot \underline{p} \times \underline{A}) \quad -(5)$$

The phenomena known as the Zeeman effect, ESR, MRI, NMR and so on can be understood by using:

$$\underline{p} \rightarrow -i\hbar \underline{\nabla} := \hat{p} \quad -(6)$$

This produces the Schroedinger equation

2) in which:

$$H_3 \rightarrow \hat{H}_3, \quad -(7)$$

the hamiltonian operator. This acts on a wave function to give eigenvalues of energy:

$$\hat{H}_3 \phi^R = E \phi^R \quad -(8)$$

$$\hat{H}_3 \phi^L = E \phi^L. \quad -(9)$$

From eq. (6): $\quad \hat{p}^2 = -\hbar^2 \nabla^2. \quad -(10)$

In the absence of interaction:

$$\hat{H}_3 = -\frac{\hbar^2}{2m} \nabla^2 \quad -(11)$$

and the Schroedinger equation is:

$$-\frac{\hbar^2}{2m} \nabla^2 \phi^R = E \phi^R \quad -(12)$$

$$-\frac{\hbar^2}{2m} \nabla^2 \phi^L = E \phi^L. \quad -(13)$$

In the presence of interaction there are terms such as:

$$\hat{H}_B \phi^R = -\frac{ie}{2m} \left(\underline{\sigma} \cdot \underline{A} \times \underline{\hat{p}} + \underline{\sigma} \cdot \underline{\hat{p}} \times A \right) \phi^R \quad -(14)$$

and similarly for ϕ^R.

3) Therefore:

$$\hat{H}_B \phi^R = -\frac{\hbar e}{2m}\left(\underline{\sigma}\cdot\left(\underline{A}\times\underline{\nabla} + \underline{\nabla}\times\underline{A}\right)\right)\phi^R$$

$$= -\frac{\hbar e}{2m}\left(\underline{\sigma}\cdot\left(\underline{A}\times\underline{\nabla}\phi^R + \underline{\nabla}\times\left(\underline{A}\phi^R\right)\right)\right)$$

$$= -\frac{\hbar e}{2m}\underline{\sigma}\cdot\left(\underline{A}\times\underline{\nabla}\phi^R + \left(\underline{\nabla}\times\underline{A}\right)\phi^R + \left(\underline{\nabla}\phi^R\right)\times\underline{A}\right)$$

$$= -\frac{e\hbar}{2m}\left(\underline{\sigma}\cdot\underline{\nabla}\times\underline{A}\right)\phi^R \quad -(15)$$

$$= E\phi^R$$

So

$$\boxed{E = -\frac{e\hbar}{2m}\underline{\sigma}\cdot\underline{\nabla}\times\underline{A}} \quad -(16)$$

In the usual $u(1)$ theory of electromagnetism the magnetic flux density is:

$$\underline{B} = \underline{\nabla}\times\underline{A} \quad -(17)$$

so the energy eigenvalue of the Zeeman effect, ESR, NMR, MRI and so on are:

4)

$$E = -\frac{e\hbar}{2m} \underline{\sigma} \cdot \underline{B} \quad -(18)$$

In ECE theory the spin connection enters into the analysis, together with antisymmetry.

For a magnetic field aligned in the Z axis, the relevant Pauli matrix is:

$$\sigma^3 = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad -(19)$$

so there are two energy levels:

$$E_+ = \frac{e\hbar}{2m} B_Z, \quad -(20)$$

$$E_- = -\frac{e\hbar}{2m} B_Z. \quad -(21)$$

Electron spin resonance occurs when:

$$\hbar\omega = E_+ - E_- = \frac{e\hbar}{m} B_Z \quad -(22)$$

i.e.

$$\boxed{\omega = \frac{eB_Z}{m}} \quad -(23)$$

for a free electron.

5) In extending this theory to the electroweak field the right handed wave function remains the same:

$$R = \phi^R \quad -(24)$$

but the left handed wave function becomes:

$$L = \begin{bmatrix} \phi^L \\ \nu_e \end{bmatrix} \quad -(25)$$

where ν is the electron neutrino.

So:

$$\hat{H}_3 L = E L \quad -(26)$$

The minimal prescription is extended to:

$$\underline{p} \rightarrow \underline{p} - e\underline{A} - g\underline{W} \quad -(27)$$

where $\underline{W}$ is the weak field vector potential and g plays a role similar to the charge e on the proton. This means that the free electron will interact with the weak field and will cause a Zeeman effect, ESR and so forth. The next note will develop this theory.
