223(3): The Basic Error in EGR.

As in UFT150 the infinitesimal line element for EGR is claimed to be:

$$ds^2 = c^2 d\tau^2 = \left(1 - \frac{r_0}{r}\right) c^2 dt^2 - \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 - r^2 d\theta^2 \quad -(1)$$

where

$$r_0 = \frac{2GM}{c^2}. \quad -(2)$$

There is no concept of potential energy in EGR, so:

$$\mathcal{L} = H = \frac{1}{2} mc^2 = \frac{1}{2} m \left(\frac{ds}{dt}\right)^2 \quad -(3)$$

The EGR Euler Lagrange equation gives:

$$E = \left(1 - \frac{r_0}{r}\right) \gamma mc^2, \quad -(4)$$

$$L = \gamma m r^2 \left(\frac{d\theta}{dt}\right). \quad -(5)$$

$$= mr^2 \left(\frac{d\theta}{d\tau}\right)$$

From these equations:

$$\frac{1}{2} m \left(\frac{dr}{d\tau}\right)^2 = \frac{1}{2}\left(\frac{E^2}{mc^2} - mc^2\right) - \frac{L^2}{2mr^2} + \frac{r_0 mc^2}{2r} + \frac{r_0 L^2}{2mr^3}$$

$$= \frac{1}{2}\left(\frac{E^2}{mc^2} - \left(1 - \frac{r_0}{r}\right)\left(mc^2 + \frac{L^2}{mr^2}\right)\right). \quad -(6)$$

2)

Therefore:

$$\frac{p^2}{2m} = \frac{1}{2} m \left(\frac{dr}{d\tau}\right)^2 + \frac{L^2}{2mr^2} - \frac{r_0 mc^2}{2r} - \frac{r_0 L^2}{2mr^3} \quad -(7)$$

The effective potential of EGR is defined as:

$$V = -\frac{r_0 mc^2}{2r} - \frac{r_0 L^2}{2mr^3} \quad -(8)$$

$$= -\frac{mMG}{r} - \frac{MGL^2}{mc^2 r^3}$$

This does not produce a precessing elliptical orbit, as shown in previous UFT papers and notes.

Using:

$$L = mr^2 \frac{d\theta}{d\tau} \quad -(9)$$

it is seen that:

$$\frac{p^2}{2m} = \frac{1}{2} m \left(\left(\frac{dr}{d\tau}\right)^2 + r^2 \left(\frac{d\theta}{d\tau}\right)^2\right) - \frac{mMG}{r} - \frac{MGL^2}{mc^2 r^3} \quad -(10)$$

In the limit of special relativity:

$$r_0 \rightarrow 0 \quad -(11)$$

so:

3)
$$\frac{p^2}{2m} \rightarrow \frac{1}{2} m \left(\left(\frac{dr}{d\tau} \right)^2 + r^2 \left(\frac{d\theta}{d\tau} \right)^2 \right) \quad -(12)$$

In the limit of classical theory:

$$v^2 << c^2 \quad -(13)$$

so:

$$\frac{p^2}{2m} \rightarrow \frac{1}{2} m \left(\left(\frac{dr}{dt} \right)^2 + r^2 \left(\frac{d\theta}{dt} \right)^2 \right) \quad -(14)$$

$$= \frac{1}{2} m v^2 = T.$$

Therefore EGR is a strange and self-inconsistent theory. Its "effective potential" is:

$$V = -\frac{mMG}{r} - \frac{MGL^2}{mc^2 r^3} \quad -(15)$$

so its effective force is

$$F = -\frac{\partial V}{\partial r} = -\frac{mMG}{r^2} - \frac{3MGL^2}{mc^2 r^4} \quad -(16)$$

This force is used in the classical non-relativistic lagrangian theory with the equation:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{m r^2}{L^2} F(r) \quad -(17)$$

4) to give eq. (7.74) of Marion and Thornton:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{Gm^2M}{L^2} + \frac{3GM}{c^2}\frac{1}{r^2} \quad -(18)$$

It is claimed erroneously that this produces the precessing ellipse:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(19)$$

but eqs. (17) and (19) produce:

$$F = -\frac{mMGx^2}{r^2} + \frac{(x^2-1)L^2}{mr^3} \quad -(20)$$

and EGR cannot be correct. The correct potential for a precessing ellipse is:

$$V = -\frac{mMGx^2}{r} + \frac{(x^2-1)L^2}{2r^2} \quad -(21)$$

Comparing eqns (10) and (21) it is clear that EGR leads to an incorrect potential:

$$V = ? -\frac{mMG}{r} - \frac{MGL^2}{mc^2r^3} \quad -(22)$$

Eq. (10) should therefore be:

$$\frac{p^2}{2m} = \frac{1}{2} m \left(\left(\frac{dr}{d\tau} \right)^2 + r^2 \left(\frac{d\theta}{d\tau} \right)^2 \right) - \frac{mMGx^2}{r} + \frac{(x^2-1)L^2}{2r^2} \quad -(23)$$

In EGR the infinitesimal line element (1) is obviously incorrect. It is incorrectly attributed to Schwarzschild. Also in EGR, the concept of potential energy is abandoned but then re-introduced, and a classical eq. (17) used with an incorrect effective potential. EGR is also self inconsistent in that eq. (10) is an expression for kinetic energy only, the classical kinetic energy being

$$T = \frac{p^2}{2m} \quad -(24)$$

In the previous note it was shown that the correct expression for kinetic energy in special relativity is:

$$T = \frac{p^2}{(\gamma+1)m} \quad -(25)$$

where:

6) $$p^2 = m^2\left(\left(\frac{dr}{d\tau}\right)^2 + r^2\left(\frac{d\theta}{d\tau}\right)^2\right) \quad -(26)$$

Obviously, from eq. (10) in the limit:

$$r_0 \to 0 \quad -(27)$$

the correct expression (25) is not obtained.

The constrained Minkowski theory is based on

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\theta^2 \quad -(28)$$

which leads to:

$$\frac{p^2}{2m} = \frac{1}{2} m\left(\left(\frac{dr}{d\tau}\right)^2 + r^2\left(\frac{d\theta}{d\tau}\right)^2\right) \quad -(29)$$

with

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(30)$$

so

$$p^2 = \frac{E^2 - m^2 c^4}{c^2} \quad -(31)$$

$$= m^2\left(\left(\frac{dr}{d\tau}\right)^2 + r^2\left(\frac{d\theta}{d\tau}\right)^2\right).$$

and the effect of the correct potential (31) is incorporated into the metric through:

$$\frac{dr}{d\theta} = \frac{x\epsilon}{\alpha} r^2 \sin(x\theta) \quad -(32)$$