220(3): Initial Conditions and Constraints

The three Kepler equations are:

$$v_1^2 = 2(m_1+m_2)\left(\frac{2}{R_1} + \frac{E_1}{m_1 m_2}\right) \quad -(1)$$

$$v_2^2 = 2(m_1+m_3)\left(\frac{2}{R_2} + \frac{E_2}{m_1 m_3}\right) \quad -(2)$$

$$v_3^2 = 2(m_2+m_3)\left(\frac{2}{R_3} + \frac{E_3}{m_2 m_3}\right) \quad -(3).$$

where:

$$E_1 = -\frac{k_1}{2a_1}, \quad E_2 = -\frac{k_2}{2a_2}, \quad E_3 = -\frac{k_3}{2a_3}, \quad -(4)$$

$$k_1 = 2m_1m_2, \quad k_2 = 2m_1m_3, \quad k_3 = 2m_2m_3, \quad -(5)$$

so

$$\boxed{v_i^2 = 2(m_1+m_2)\left(\frac{2}{R_i} - \frac{1}{a_i}\right), \quad i = 1, 2, 3} \quad -(6)$$

These are constrained by:

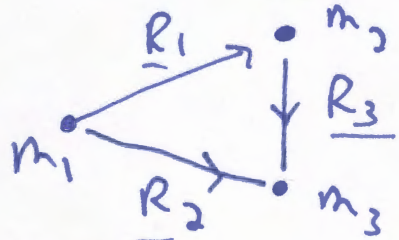

Fig (1)

$$\boxed{\underline{R_2} = \underline{R_1} + \underline{R_3}} \quad -(7)$$

The initial condition is:

$$\underline{R}_2(0) = \underline{R}_1(0) + \underline{R}_3(0) \quad -(8)$$

The velocity is:

$$\underline{v}_i = \frac{d\underline{R}_i}{dt}, \quad -(9)$$

so

$$\boxed{\underline{v}_2 = \underline{v}_1 + \underline{v}_3} \quad -(10)$$

The initial condition is:

$$\underline{v}_2(0) = \underline{v}_1(0) + \underline{v}_3(0). \quad -(11)$$

Therefore:

$$v_{2x} = v_{1x} + v_{3x} \quad -(12)$$

$$v_{2y} = v_{1y} + v_{3y} \quad -(13)$$

and

$$v_2^2 = v_{2x}^2 + v_{2y}^2 \quad -(14)$$

$$v_2^2 = (v_{1x} + v_{3x})^2 + (v_{1y} + v_{3y})^2 .$$

Initially:

$$v_i^2(0) = 2(m_1 + m_2)\left(\frac{2}{R_i^2(0)} - \frac{1}{a_i}\right) \quad -(15)$$

In cylindrical polar coordinates:

$$\underline{R_1} = R_1 \, \underline{e}_r$$
$$\underline{R_2} = R_2 \, \underline{e}_r \qquad -(16)$$
$$\underline{R_3} = R_3 \, \underline{e}_r$$

so:

$$\boxed{R_2 = R_1 + R_3} \qquad -(17)$$

from eq. (7). This means that:

$$R_2 = \frac{\alpha_2}{1 + \epsilon_2 \cos\theta_2} = \frac{\alpha_1}{1 + \epsilon_1 \cos\theta_1} + \frac{\alpha_3}{1 + \epsilon_3 \cos\theta_3} \qquad -(18)$$

and

$$R_3 = \frac{\alpha_2}{1 + \epsilon_2 \cos\theta_2} - \frac{\alpha_1}{1 + \epsilon_1 \cos\theta_1} \qquad -(19)$$

The velocities are:

$$\underline{v_1} = \frac{d}{dt}\left(R_1 \, \underline{e}_r\right)$$
$$\underline{v_2} = \frac{d}{dt}\left(R_2 \, \underline{e}_r\right) \qquad -(20)$$
$$\underline{v_3} = \frac{d}{dt}\left(R_3 \, \underline{e}_r\right)$$

where

$$\frac{d}{dt}\left(R_i \, \underline{e}_r\right) = \dot{R}_i \, \underline{e}_r + R_i \, \dot{\underline{e}}_r \qquad -(21)$$
$$i = 1, 2, 3$$

4) From eq. (17):

$$\dot{R}_2 = \dot{R}_1 + \dot{R}_3 \quad -(22)$$

$$R_2 = R_1 + R_3 \quad -(23)$$

So:

$$\boxed{\underline{v}_2 = \left(\dot{R}_1 + \dot{R}_3\right)\underline{e}_r + \left(R_1 + R_3\right)\dot{\underline{e}}_r} \quad -(24)$$

$$\boxed{\underline{R}_2 = \left(R_1 + R_3\right)\underline{e}_r} \quad -(25)$$

The simulation result should obey eqns. (24) and (25).

Note on Eq. (19)

This can be plotted simply as:

$$R_3 = \frac{\alpha_2}{1+\epsilon_2 \cos\theta} - \frac{\alpha_1}{1+\epsilon_1 \cos\theta} \quad -(26)$$

and should give very interesting results.

The actual $\theta(t)$ can also be worked out analytically.