219(2): Derivation of Equation for Helical-Like Structures.

Start with the equation:

$$\underline{R} = R\,\underline{e}_R = r\,\underline{e}_r + Z\,\underline{k} \quad -(1)$$

so:

$$R^2 = r^2 + Z^2 \quad -(2)$$

When $Z = 0$:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(3)$$

so:

$$R^2 = \left(\frac{\alpha}{1+\epsilon\cos(x\theta)}\right)^2 + Z^2 \quad -(4)$$

For helical structures:

$$R^2 = \left(\frac{\alpha}{1+\epsilon\cos(x\theta)}\right)^2 + Z_0^2\,\theta^2 \quad -(5)$$

So R can be plotted against θ from eq. (5).

Eq. (5) is derived on the assumption that the dynamics in the original plane are defined by

$$\mathcal{L} = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - U(r) \quad -(6)$$

2) where

$$U(r) = -x^2 \frac{mMG}{r} + \frac{(x^2-1)L^2}{2mr^2}, \quad -(7)$$

$$L = mr^2 \frac{d\theta}{dt} \quad -(8)$$

and

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{L^2} F(r) \quad -(9)$$

Reserve the dynamics for example of the solar system when the sun moves in the Z axis according to:

$$Z = Z_0 \theta. \quad -(10)$$

The potential in eq. (7) is that between the mass m and the mass M, separated by the distance r.

If the whole solar system moves as the sun moves in space the distance between the sun and a planet in space remains the same, r. The curve drawn out by the combined movement is R. In general:

$$\mathcal{L} = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2 + \dot{Z}^2 \right) - U(r, Z) \quad -(11)$$
