219(3): Some Comments on 3-D Orbits

The helical like orbits are obtained with:

$$R^2 = \left(\frac{\alpha}{1+\epsilon\cos(x\theta)}\right)^2 + Z_0^2\theta^2 \quad -(1)$$

as in note 219(2). The graphics in the attached figures were made by Dr. Horst Eckardt.

Figure 1

This develops the familiar circular helix into the elliptical helix as drawn. This figure illustrates the Newtonian ellipse $\epsilon = 0.5$, $x = 1$. The conventional planar orbit occurs around a mass M. The latter is assumed to move in the Z axis out of the plane and perpendicular to it. The helical formula:

$$Z_1 = Z_0\theta \quad -(2)$$

is used for illustration. Some galaxies are observed to have such a structure. This is a static elliptical helix.

Figure 2 This is a precessing elliptical helix with $\epsilon = 0.5$, $x = 0.5$.

Figure 3 This is a precessing elliptical helix with $\epsilon = 0.5$, $x = 1.2$.

Figure 4 This is the result of Fig (3) with $Z = 0$.

2) Figure 5

This is a Newtonian hyperbolic orbit with $\epsilon = 1.2$, $x = 1.0$ around an object that moves in the Z axis according to eq. (2).

Figure 6

This is eq. (1) with $Z = 0$.

Figure 7

This is the movement upward in Z of the orbit in eq. Fig (6).

Figure 8

This is the precessing hyperbola projected on to the X-Z plane with $\epsilon = 1.2$, $x = 0.3$.

Figure 9

This is the three dimensional view of Fig (8), a completely new type of orbit emerges

Figure 10.

This figure is based on eq. ([illegible]) of note 219(2):

$$R^2 = r^2 + Z_0^2\theta^2 = \left(\frac{\alpha}{1+\epsilon\cos(x\theta)}\right)^2 + Z_0^2\theta^2 \quad -(3)$$

with $x = \theta$.

It produces a hitherto unknown elastic orbit projected on to the original plane in Fig 11.

3) The classic orbit in the plane XY is therefore given by:

$$r = \frac{\alpha}{1 + \epsilon \cos(\theta ^2)} \quad -(4)$$

with $\epsilon = 1.2$.

Eq. (1) correspond to the Euler Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(5)$$

and

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} \quad -(6)$$

with lagrangian:

$$\mathcal{L} = \frac{1}{2} m (\dot{r}^2 + r^2 \dot{\theta}^2 + \dot{z}^2) - U(r) \quad -(7)$$

in which

$$\dot{z} = z_0 \dot{\theta}. \quad -(8)$$

therefore:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{L} F(r) \quad -(9)$$

with

$$L = m(r^2 + z_0^2) \frac{d\theta}{dt}, \quad -(10)$$

$$F(r) = -\frac{mMGx^2}{r^2} + \frac{(x^2 - 1)L^2}{mr^3} \quad -(11)$$

4). The orbit in XY is:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(12)$$

where:

$$\alpha = \frac{L^2}{mk}, \quad \epsilon = \left(1+\frac{2EL^2}{mk^2}\right)^{1/2}, \quad -(13)$$

$$k = mMG. \quad -(14)$$

Finally:

$$R^2 = r^2 + r_0^2\theta^2. \quad -(15)$$

The force law (11) is that between m and M in the plane XY, separated by r. The additional eq. (8) produces the additional angular momentum:

$$L_1 = m r_0^2 \frac{d\theta}{dt} \quad -(16)$$

as in eq. (10), which is the result of the Euler Lagrange eq. (6).

5.)

3D orbits with $Z=Z_0*\theta$

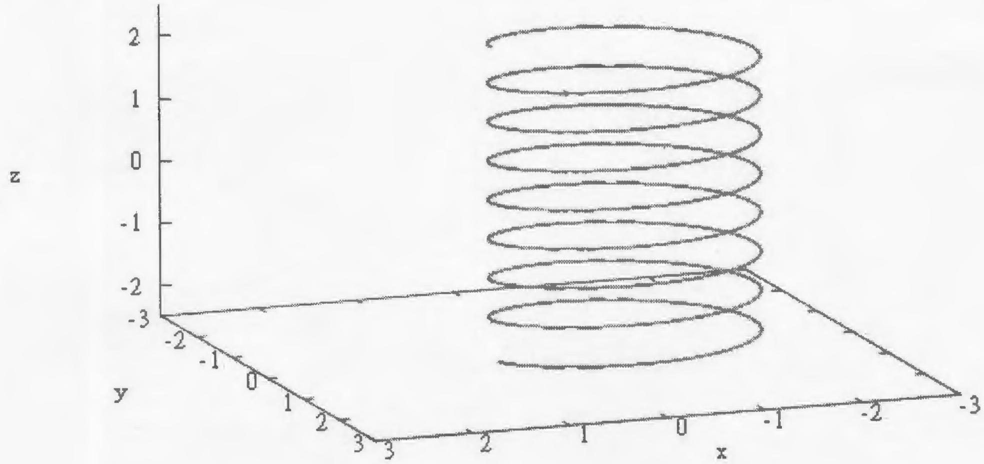

Fig. 1. Ellipse, epsilon=0.5, x=1

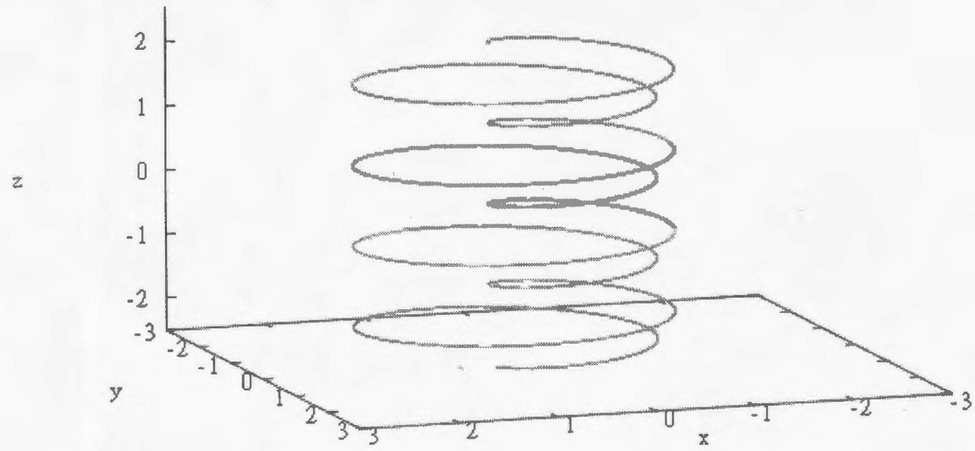

Fig. 2. Periodic orbit, epsilon=0.5, x=0.5

6.

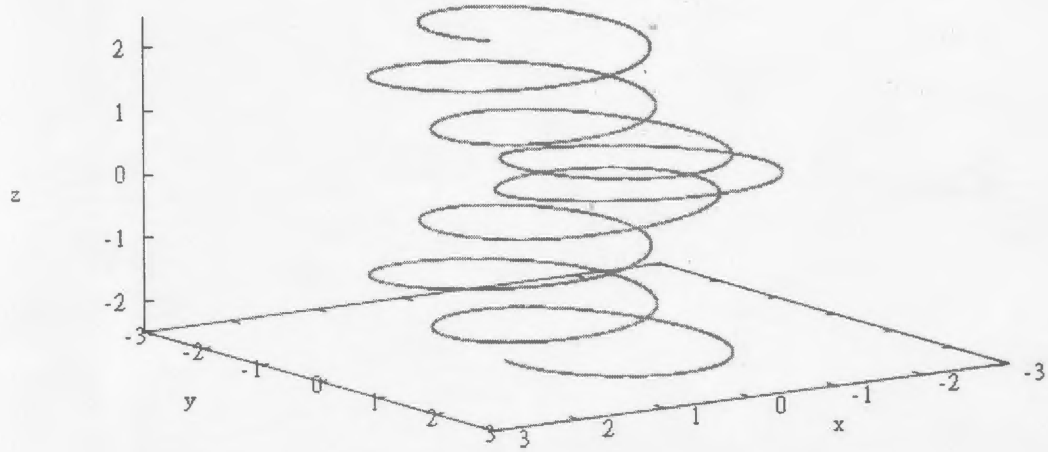

Fig. 3. Precessing ellipse, epsilon=0.5, x=1.2

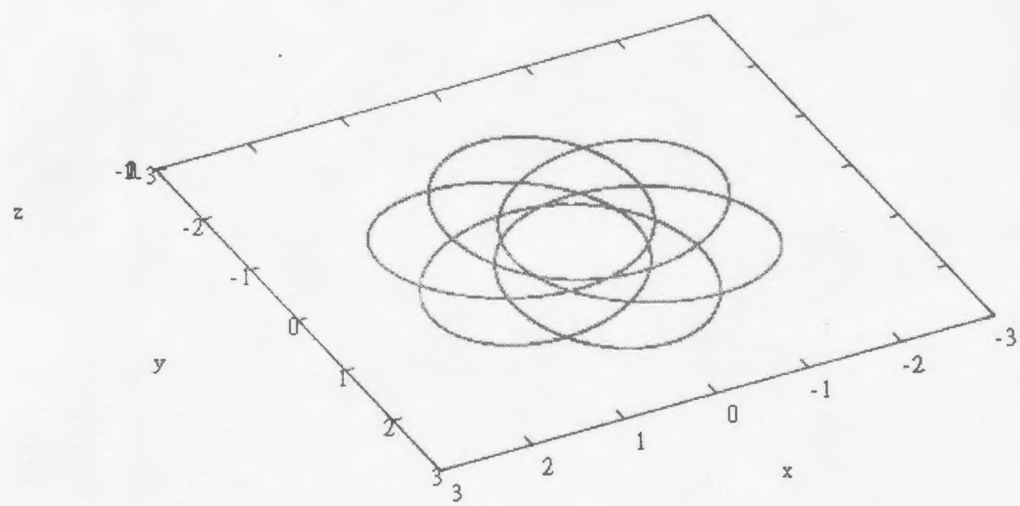

Fig. 4. Projection of ellipse of Fig. 3 to X-Y plane.

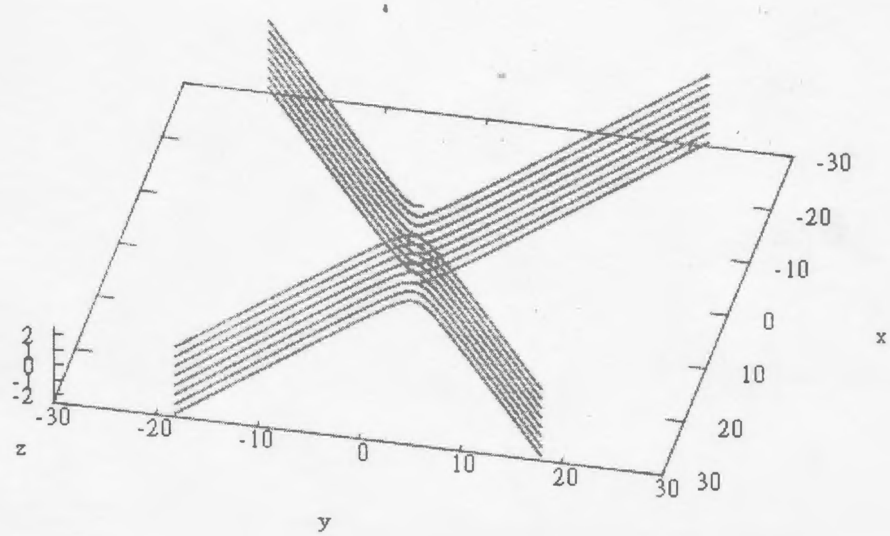

Fig. 5. Hyperbola, epsilon=1.2.

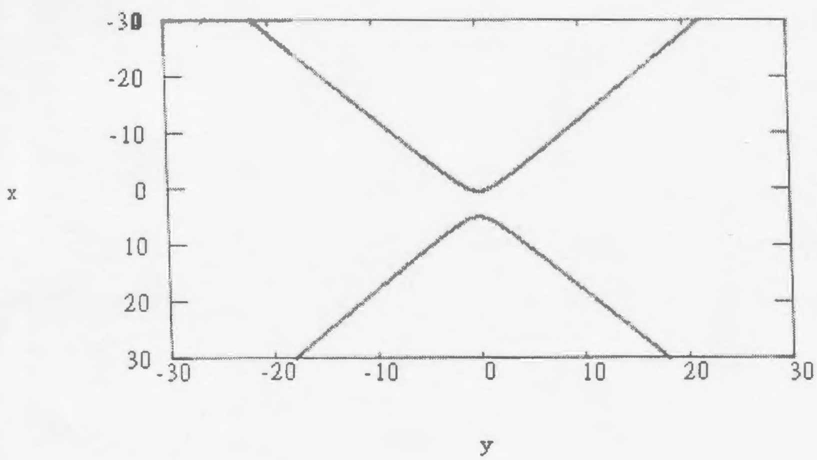

Fig. 6. Hyperbola, epsilon=1.2, projection to X-Y plane

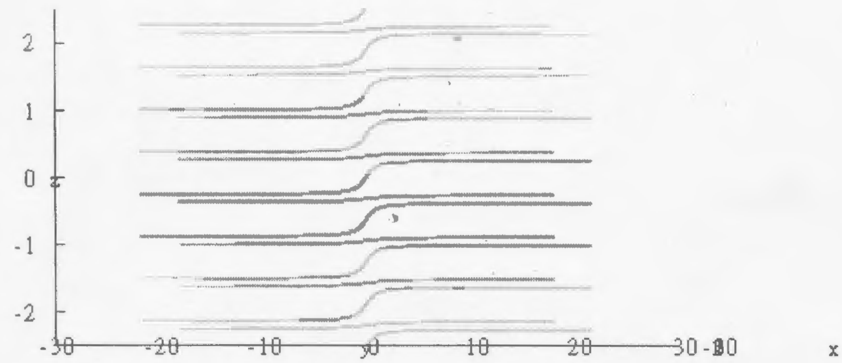

Fig. 7. Hyperbola, epsilon=1.2, projection to X-Z plane.

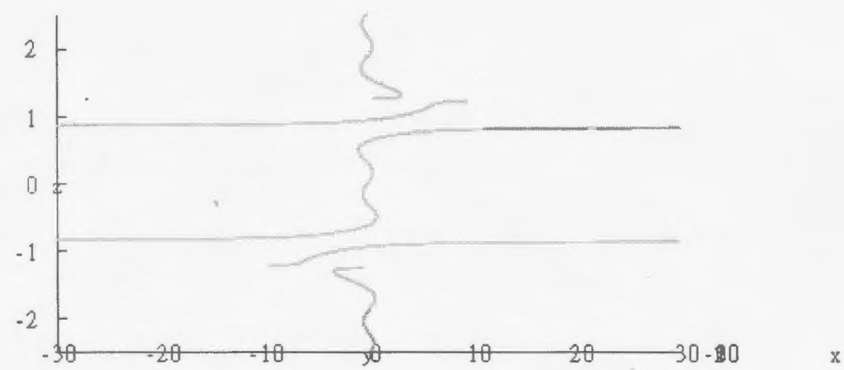

Fig. 8. Generalized hyperbola, epsilon=1.2, x=0.3, projection to X-Z plane.

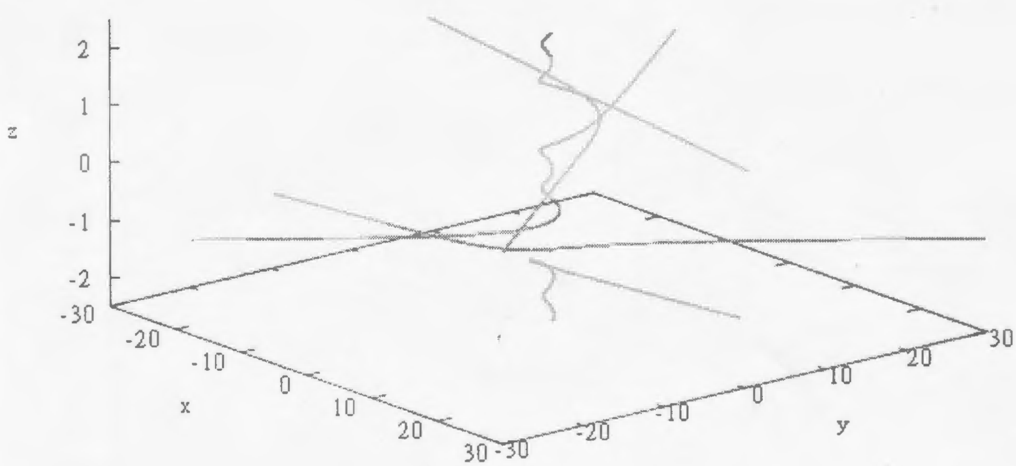

Fig. 9 Generalized hyperbola, epsilon=1.2, x=0.3.

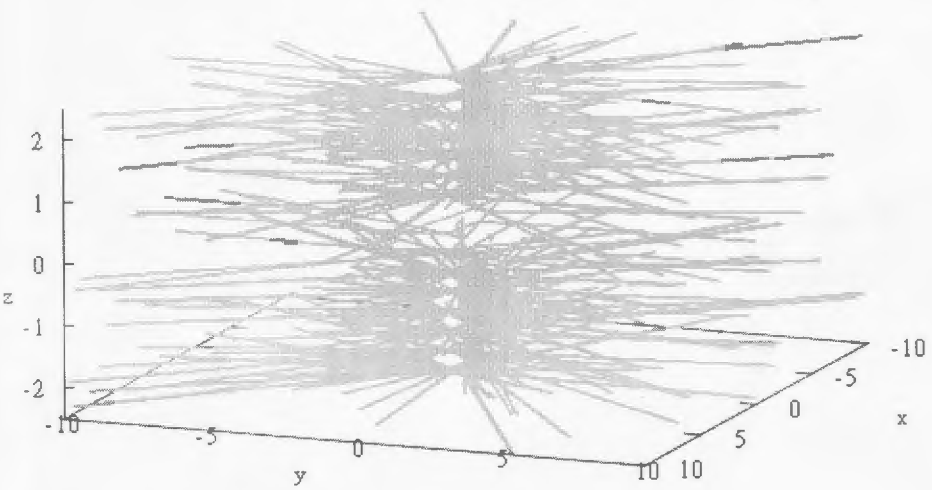

Fig. 10. Chaotic orbit with epsilon=1.2, x(theta) = theta.

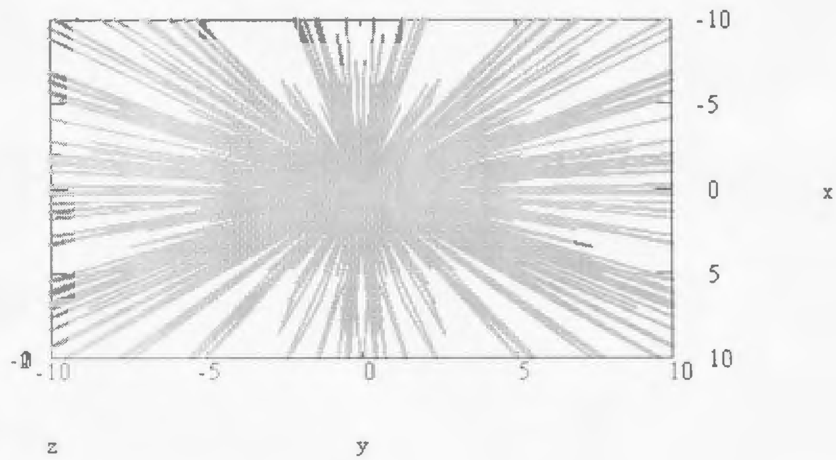

Fig. 11. Projection of Fig. 10 to X-Y plane.