218(1): Transformation of Fractal Conical Section to Logarithmic Spiral.

The force law for the fractal conical section is:

$$F(r) = -\frac{kx^2}{r^2} - \frac{k\alpha}{r^3}\left(1-x^2\right). \quad -(1)$$

As:

$$x \to 0 \quad -(2)$$

$$F(r) \to -\frac{k\alpha\left(1-x^2\right)}{r^3} \quad (3)$$

Now use the lagrangian equation:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{L^2} F(r) \quad -(4)$$

$$k\alpha = L^2 / m. \quad -(5)$$

and

The force law: $$F(r) = -\frac{L^2\left(1-x^2\right)}{mr^3} \quad -(6)$$

corresponds to the logarithmic spiral orbit:

$$r = r_0 \exp\left(-\xi\theta\right) \quad -(7)$$

with force law:

$$F(r) = -\frac{L^2}{mr^3}\left(1+\xi^2\right) \quad -(8)$$

if

$$x^2 = -\xi^2 \quad -(9)$$

i.e.

$$\boxed{x = \pm i\xi} \quad -(10)$$

2) If
$$x = i\xi \quad -(11)$$
then
$$\cos(i\xi\theta) = \cosh(\xi\theta) \quad -(12)$$
$$= \frac{1}{2}\left(e^{\xi\theta} + e^{-\xi\theta}\right) \quad -(13)$$
The fractal conical section in this case becomes:
$$r = \frac{\alpha}{1+\epsilon\cosh(\xi\theta)} \quad -(14)$$
for
$$x^2 = -\xi^2 \ll 1 \quad -(15)$$
As:
$$\theta \rightarrow \infty \quad -(16)$$
eq. (14) becomes:
$$r \rightarrow 2\alpha \exp(-\xi\theta) \quad -(17)$$
which is eq. (7) if:
$$r_0 = 2\alpha. \quad -(18)$$
The usual claim for a binary pulsar orbit is that it decreases over time. This can be understood from eq. (1) by allowing x to decrease with time. This procedure gradually changes the precessing ellipse into an inward spiralling orbit.

The angular velocity is described by:
$$\frac{d\theta}{dt} = \frac{L}{mr^2} \quad -(19)$$