214(5): Force Law of the Precessing Elliptical Orbit in the Coordinate System (r, θ).

In this case:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = -\frac{mr^2}{L^2} F(r) \quad -(1)$$

where

$$\frac{1}{r} = \frac{1}{\alpha}\left(1 + \epsilon \cos(x\theta)\right). \quad -(2)$$

The plane polar coordinate system (r, θ) is static. So:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = -\frac{\epsilon x^2}{\alpha} \cos(x\theta). \quad -(3)$$

and

$$\frac{1}{\alpha}\left(1 + \epsilon\left(1 - x^2\right)\cos(x\theta)\right) = -\frac{mr^2}{L^2} F(r); \quad -(4)$$

i.e.

$$F(r) = -\frac{k}{r^2}\left(1 + \epsilon\left(1 - x^2\right)\cos(x\theta)\right) \quad -(5)$$

where

$$\cos(x\theta) = \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right). \quad -(6)$$

Therefore:

$$\boxed{F(r) = -\frac{k}{r^2}\left(x^2 + \left(1 - x^2\right)\frac{\alpha}{r}\right)} \quad -(7)$$

which is a sum of inverse square and cube terms. The force law depends on the coordinate system.