213(8) : Transformation of the Mixed Index Connection, Part 2.

As in note 213(7) :

$$\Gamma^{a'}_{\mu'\lambda'} = q^{\mu}_{\mu'} q^{\lambda}_{\lambda'} q^{a'}_{a} \left(\Gamma^{a}_{\mu\lambda} - q^{a}_{\nu'} \partial_\mu q^{\nu'}_{\lambda} \right) \quad -(1)$$

where

$$q^{a'}_{a} = \Lambda^{a'}_{a} \quad -(2)$$

the Lorentz transform.

The torsion transforms as:

$$T^{a'}_{\mu'\lambda'} = \Gamma^{a'}_{\mu'\lambda'} - \Gamma^{a'}_{\lambda'\mu'} = q^{\mu}_{\mu'} q^{\lambda}_{\lambda'} q^{a'}_{a} T^{a}_{\mu\lambda} \quad -(3).$$

in which:

$$q^{\mu}_{\mu'} \neq 0, \; q^{\lambda}_{\lambda'} \neq 0, \; q^{a'}_{a} \neq 0. \quad -(4)$$

The transformation law (3) of the torsion shows that there is no dependence of x^μ on x^a and no dependence of x^λ on x^a. If there were such a dependence then

$$\frac{\partial x^{\mu'}}{\partial x^{\mu}} \overset{?}{=} \frac{\partial x^{\mu'}}{\partial x^{a}} \frac{\partial x^{a}}{\partial x^{\mu}} = \delta^{\mu'}_{\mu} \quad -(5)$$

$$= 0$$

unless

$$\mu' \overset{?}{=} \mu \quad -(6)$$

and in general this is not true.

2) In general the torsion can be transformed in the tangent space:

$$T^{a'} = \Lambda^{a'}{}_{a} T^{a} \quad -(7)$$

for each μ and ν. The torsion is a vector valued two-form, in the tangent space it is a vector that transforms under the Lorentz transform.

Similarly, if:

$$\frac{\partial x^{\lambda'}}{\partial x^{\lambda}} =? \frac{\partial x^{\lambda'}}{\partial x^{a}} \frac{\partial x^{a}}{\partial x^{\lambda}} = \delta^{\lambda'}_{\lambda} \quad -(8)$$

$$= 0$$

$$\lambda' =? \lambda \quad -(9)$$

unless

another reduction to absurdity occurs.

So only x^{ν} depends on x^{a}, otherwise absurd results are obtained.

Self consistently, the Riemann torsion is:

$$T^{\nu}{}_{\mu\lambda} = T^{a}{}_{\mu\lambda} q^{\nu}{}_{a} \quad -(10)$$

is defined. Therefore:

in which only $q^{\nu}{}_{a}$

$$q^{a}{}_{\mu} = q^{a}{}_{\lambda} = 0, \quad -(11)$$

and

$$q^{a}{}_{\nu} \neq 0. \quad -(12)$$

3)

Now consider:

$$q^{a}_{\nu'}\partial_\mu q^{\nu'}_{\lambda} = q^{a}_{\nu'}\frac{\partial}{\partial x^\mu}\left(\frac{\partial}{\partial x^\lambda}x^{\nu'}\right). \quad -(13)$$

Using the Leibnitz Theorem:

$$\boxed{\begin{aligned} q^{a}_{\nu'}\partial_\mu q^{\nu'}_{\lambda} &= \partial_\mu\left(q^{a}_{\nu'}q^{\nu'}_{\lambda}\right) \\ &= \partial_\mu q^{a}_{\lambda} = 0 \end{aligned}} \quad -(14)$$

because:

$$\begin{aligned} &\partial_\mu\left(q^{a}_{\nu'}q^{\nu'}_{\lambda}\right) \\ &= q^{a}_{\nu'}\partial_\mu q^{\nu'}_{\lambda} + q^{\nu'}_{\lambda}\partial_\mu q^{a}_{\nu'} \end{aligned} \quad -(15)$$

and $$\partial_\mu q^{a}_{\nu'} = 0 \quad -(16)$$

because $$\frac{\partial}{x^\mu}\frac{\partial x^a}{\partial x^{\nu'}} = \frac{\partial}{\partial x^{\nu'}}\frac{\partial x^a}{\partial x^\mu}. \quad -(17)$$

Here $$q^{a}_{\mu} = \frac{\partial x^a}{\partial x^\mu} = 0. \quad -(18)$$

So:

$$\boxed{\Gamma^{a'}_{\mu'\lambda'} = q^{\mu}_{\mu'}q^{\lambda}_{\lambda'}q^{a'}_{a}\Gamma^{a}_{\mu\lambda}} \quad -(19)$$

It follows that:

$$\Gamma^{\nu'}_{\mu'\lambda'} = q^{\nu'}_{a'} \Gamma^{a'}_{\mu'\lambda'}$$

$$= q^{\nu'}_{\nu} q^{a}_{a'} q^{\nu}_{a} q^{\mu}_{\mu'} q^{\lambda}_{\lambda'} q^{a'}_{a} \Gamma^{a}_{\mu\lambda}$$

$$\boxed{\Gamma^{\nu'}_{\mu'\lambda'} = q^{\nu'}_{\nu} q^{\mu}_{\mu'} q^{\lambda}_{\lambda'} \Gamma^{\nu}_{\mu\lambda}} \quad -(20)$$

and:

$$\Gamma^{\nu'}_{\mu'\lambda'} = - \Gamma^{\nu'}_{\lambda'\mu'} \quad -(21)$$

$$\Gamma^{\nu}_{\mu\lambda} = - \Gamma^{\nu}_{\lambda\mu}. \quad -(22)$$

The Christoffel connection is antisymmetric in any frame of reference.

This proof refutes Einsteinian general relativity.

Notes

The transformation law means that:

$$q^{\mu'}_{a} = \frac{\partial x^{\mu'}}{\partial x^{\mu}} q^{\mu}_{a} \quad -(23)$$

and

$$q^{\mu'}_{\mu} = \frac{\partial x^{\mu'}}{\partial x^{\mu}} \frac{\partial x^{a'}}{\partial x^{a}} q^{\mu}_{a} \quad -(24)$$

Therefore:

5) $$q^{\mu'}{}_a = q^{\mu'}{}_\mu q^\mu{}_a \quad -(25)$$

The dimensions of μ' are the same as those of μ so both $q^{\mu'}{}_\mu$ and $q^\mu{}_a$ are invertible matrices by definition. Therefore $q^{\mu'}{}_a$ is also an invertible matrix and

$$q^{\mu'}{}_a = q^{\mu'}{}_a q^b{}_{\mu'} = \delta^b_a \quad -(26)$$

and

$$q^{\mu'}{}_a q^a{}_\mu = \delta^{\mu'}_\mu \quad -(27)$$

The error in the old theory was to assert that

$$\Gamma^{a'}_{\mu'\lambda'} =? \Gamma^{a'}_{\lambda'\mu'} \quad -(28)$$

because:

$$\partial_\mu q^{\nu'}{}_\lambda = \partial_\lambda q^{\nu'}{}_\mu \quad -(29)$$

i.e.

$$\frac{\partial}{\partial x^\mu}\frac{dx^{\nu'}}{\partial x^\lambda} = \frac{\partial}{\partial x^\lambda}\frac{\partial x^{\nu'}}{\partial x^\mu} \quad -(30)$$

Cartan geometry was never considered and this proof shows that:

$$\boxed{q^a{}_{\nu'}\partial_\mu q^{\nu'}{}_\lambda - q^a{}_{\nu'}\partial_\lambda q^{\nu'}{}_\mu = 0} \quad -(31)$$