213(6): The Simplest Commutator Argument for a Non-Zero Antisymmetric Connection.

The commutator argument produces the result:

$$[D_\mu, D_\nu] V^\rho = R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\lambda{}_{\mu\nu} D_\lambda V^\rho \quad -(1)$$

$$= -\left(\Gamma^\lambda_{\mu\nu} - \Gamma^\lambda_{\nu\mu}\right) D_\lambda V^\rho + R^\rho{}_{\sigma\mu\nu} V^\sigma$$

and $$[D_\nu, D_\mu] V^\rho = -\left(\Gamma^\lambda_{\nu\mu} - \Gamma^\lambda_{\mu\nu}\right) D_\lambda V^\rho + R^\rho{}_{\sigma\mu\nu} V^\sigma \quad -(2)$$

Let $$\Gamma^\lambda_{\mu\nu} = A, \quad \Gamma^\lambda_{\nu\mu} = B \quad -(3)$$

then:

$$[D_\mu, D_\nu] V^\rho = (B - A) D_\lambda V^\rho + \ldots \quad -(4)$$

$$[D_\nu, D_\mu] V^\rho = (A - B) D_\lambda V^\rho + \ldots \quad -(5)$$

Now let $$\mu = \nu. \quad -(6)$$

Eq (4) gives $$B - A = 0. \quad -(7)$$

Eq (5) gives $$A - B = 0 \quad -(8)$$

So $$B - A = A - B = 0 \quad -(9)$$

$$2A = 2B = 0 \quad -(10)$$

The symmetric connection is zero, Q.E.D.