213(3): Some Further Logical Contradictions in the Geometry of EGR.

The fundamental motivation for the construction of the Christoffel connection is the construction of the covariant derivative, and that the covariant derivative transform as a tensor:

$$D_{\mu'} V^{\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} D_{\mu} V^{\nu} \quad -(1)$$

The Christoffel connection is defined as:

$$D_{\mu} V^{\nu} = \partial_{\mu} V^{\nu} + \Gamma^{\nu}_{\mu\lambda} V^{\lambda}, \quad -(2)$$

so it is defined by:

$$\partial_{\mu'} V^{\nu'} + \Gamma^{\nu'}_{\mu'\lambda'} V^{\lambda'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} \left(\partial_{\mu} V^{\nu} + \Gamma^{\nu}_{\mu\lambda} V^{\lambda} \right). \quad -(3)$$

The Christoffel connection is:

$$(\Gamma_{\mu})^{\nu}{}_{\lambda} := \Gamma^{\nu}_{\mu\lambda}. \quad -(4)$$

For each index μ it is a matrix $\Gamma^{\nu}{}_{\lambda}$. The partial derivative in eq. (3) transforms as:

$$\partial_{\mu'} V^{\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu'}}{\partial x^{\nu}} \partial_{\mu} V^{\nu} + \frac{\partial x^{\mu}}{\partial x^{\mu'}} V^{\nu} \frac{\partial}{\partial x^{\mu}} \left(\frac{\partial x^{\nu'}}{\partial x^{\nu}} \right) \quad -(5)$$

because

2)

$$\partial_{\mu'} = \frac{\partial}{\partial x^{\mu'}} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial}{\partial x^{\mu}} \quad - (6)$$

and:

$$V^{\nu'} = \frac{\partial x^{\nu'}}{\partial x^{\nu}} V^{\nu} \quad - (7)$$

The Leibnitz theorem means that:

$$\partial_{\mu'} V^{\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \partial_{\mu} \left(\frac{\partial x^{\nu'}}{\partial x^{\nu}} V^{\nu} \right) \quad - (8)$$

$$= \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu'}}{\partial x^{\nu}} \partial_{\mu} V^{\nu} + \frac{\partial x^{\mu}}{\partial x^{\mu'}} V^{\nu'} \frac{\partial}{\partial x^{\mu}} \left(\frac{\partial x^{\nu'}}{\partial x^{\nu}} \right)$$

Q.E.D. The four derivative does not transform as a tensor. If it transformed as a tensor then the second term on the right hand side of eq. (8) would be zero. Christoffel devised the idea of the connection in order to arrive at eq. (1). He devised a derivative denoted D_{μ} that transforms as a tensor. This is called the covariant derivative.

Using eqns (3) and (8) it is possible to work out how $\Gamma^{\nu}_{\mu\lambda}$ transforms in general. This is known as the general coordinate transformation.

3) Therefore:

$$\frac{\partial x^\mu}{\partial x^{\mu'}}\frac{\partial x^{\nu'}}{\partial x^\nu}\partial_\mu V^\nu + \frac{\partial x^\mu}{\partial x^{\mu'}}\frac{\partial}{\partial x^\mu}\left(\frac{\partial x^{\nu'}}{\partial x^\nu}\right)V^\nu + \Gamma^{\nu'}_{\mu'\lambda'}V^{\lambda'} \quad -(9)$$

$$= \frac{\partial x^\mu}{\partial x^{\mu'}}\frac{\partial x^\nu}{\partial x^{\nu'}}\partial_\mu V^\nu + \frac{\partial x^\mu}{\partial x^{\mu'}}\frac{\partial x^\nu}{\partial x^{\nu'}}\Gamma^\nu_{\mu\lambda}V^\lambda$$

It follows that:

$$\Gamma^{\nu'}_{\mu'\lambda'}V^{\lambda'} + \frac{\partial x^\mu}{\partial x^{\mu'}}\frac{\partial}{\partial x^\mu}\left(\frac{\partial x^{\nu'}}{\partial x^\nu}\right)V^\nu = \frac{\partial x^\mu}{\partial x^{\mu'}}\frac{\partial x^\nu}{\partial x^{\nu'}}\Gamma^\nu_{\mu\lambda}V^\lambda \quad -(10)$$

where $$V^{\lambda'} = \frac{\partial x^{\lambda'}}{\partial x^\lambda}V^\lambda \quad -(11).$$

Therefore:

$$\Gamma^{\nu'}_{\mu'\lambda'}\frac{\partial x^{\lambda'}}{\partial x^\lambda}V^\lambda + \frac{\partial x^\mu}{\partial x^{\mu'}}\frac{\partial}{\partial x^\mu}\left(\frac{\partial x^{\nu'}}{\partial x^\nu}\right)V^\nu = \frac{\partial x^\mu}{\partial x^{\mu'}}\frac{\partial x^\nu}{\partial x^{\nu'}}\Gamma^\nu_{\mu\lambda}V^\lambda \quad -(12)$$

At this point is the usual proof:

$$\frac{\partial x^{\nu'}}{\partial x^\nu}V^\nu \rightarrow \frac{\partial x^{\nu'}}{\partial x^\lambda}V^\lambda \quad -(13)$$

This can be done because there is summation over repeated indices. The label ν is changed to λ. Using eqs. (12) and (13) the vector

4) V^λ cancels out leaving:

$$\Gamma^{\nu'}_{\mu'\lambda'} = \frac{\partial x^\mu}{\partial x^{\mu'}}\frac{\partial x^\lambda}{\partial x^{\lambda'}}\frac{\partial x^{\nu'}}{\partial x^\nu}\Gamma^\nu_{\mu\lambda} - \frac{\partial x^\mu}{\partial x^{\mu'}}\frac{\partial x^\lambda}{\partial x^{\lambda'}}\frac{\partial}{\partial x^\mu}\left(\frac{\partial x^{\nu'}}{\partial x^\lambda}\right) \quad -(14)$$

The three index object $\Gamma^\nu_{\mu\lambda}$ does not transform as a tensor in Riemann / Christoffel geometry. In general this leads to strange results which are self contradictory as follows.

It is always possible that:

$$\frac{\partial x^{\nu'}}{\partial x^\nu}\Gamma^\nu_{\mu\lambda} = \frac{\partial}{\partial x^\mu}\left(\frac{\partial x^{\nu'}}{\partial x^\lambda}\right) \quad -(15)$$

in which case:

$$\Gamma^{\nu'}_{\mu'\lambda'} = ?\ 0 \quad -(16)$$

This would mean that a non Minkowski space could be transformed into a Minkowski space with zero connection as in eq. (16). Conversely, a Minkowski space could be transformed into a non Minkowski space. This is an absurd result because a Minkowski space by definition must be Lorentz transformed into another Minkowski space.

In eq. (15):

$$\frac{\partial}{\partial x^\mu}\left(\frac{\partial x^{\nu'}}{\partial x^\lambda}\right) = \frac{\partial}{\partial x^\lambda}\left(\frac{\partial x^{\nu'}}{\partial x^\mu}\right) \quad -(17)$$

because

$$[\partial_\mu, \partial_\lambda] = 0. \quad -(18)$$

It follows that for eq. (15) to be true:

$$\Gamma^\nu_{\mu\lambda} = \Gamma^\nu_{\lambda\mu} \quad -(19)$$

i.e the connection would have to be symmetric in its lower two indices.

Eq. (19) cannot be true because a flat or Minkowski space cannot be transformed into a space that is not a Minkowski space. In other words a zero connection cannot become non-zero by use of a coordinate transformation. Each component of a zero connection is zero by definition. For example:

$$\Gamma^3_{12} = 0 \quad -(20)$$

and so on.

In general, $\Gamma^\nu_{\mu\lambda}$ for each ν can be regarded as a matrix in μ and λ, so by the basic theorem of matrices:

$$\Gamma^\nu_{\mu\lambda} = \frac{1}{2}\left(\Gamma^\nu_{\mu\lambda} + \Gamma^\nu_{\lambda\mu}\right) + \frac{1}{2}\left(\Gamma^\nu_{\mu\lambda} - \Gamma^\nu_{\lambda\mu}\right)$$

$$-(21)$$

6) which means that $\Gamma^{\sim}_{\mu\lambda}$ has a symmetric component:

$$\Gamma^{\sim\,(S)}_{\mu\lambda} = \frac{1}{2}\left(\Gamma^{\sim}_{\mu\lambda} + \Gamma^{\sim}_{\lambda\mu}\right) \quad -(22)$$

and an antisymmetric component:

$$\Gamma^{\sim\,(A)}_{\mu\lambda} = \frac{1}{2}\left(\Gamma^{\sim}_{\mu\lambda} - \Gamma^{\sim}_{\lambda\mu}\right) \quad -(23)$$

as for any matrix.

There is nothing in Riemann Christoffel geometry to lead to a conclusion that the connection has any symmetry. The contradiction represented by eq. (16) leads to the conclusion that the connection cannot be symmetric, i.e.

$$\Gamma^{\sim}_{\mu\lambda} = -\Gamma^{\sim}_{\lambda\mu}. \quad -(24)$$

This contradiction has just been inferred in this note. Previous notes also lead to eq. (24) in at least three separate categories of proof.

The use of Cartan's geometry shows immediately that there exists a tetrad postulate:

$$D_\mu q^a_\nu = \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu - \Gamma^\lambda_{\mu\nu} q^a_\lambda$$
$$= 0 \quad -(25)$$

7) in which the Cartan connection or spin connection is defined by:

$$D_\mu V^a = \partial_\mu V^a + \omega^a_{\mu b} V^b \quad -(26)$$

and in which the Cartan tetrad is defined by:

$$V^a = q^a_\mu V^\mu \quad -(27)$$

In Cartan's geometry a Minkowski space with Latin indices is used at point P in the base manifold, a general space labelled by Greek indices. The Christoffel connection is related to the Cartan connection by eq. (25) and is very well known. By the definition of Cartan geometry there exist mixed index connections:

$$\omega^a_{\mu\nu} = \omega^a_{\mu b} q^b_\nu \quad -(28)$$

$$\Gamma^a_{\mu\nu} = \Gamma^\lambda_{\mu\nu} q^a_\lambda \quad -(29)$$

$$\Gamma^\lambda_{\mu\nu} = q^\lambda_a \Gamma^a_{\mu\nu}. \quad -(30)$$

By definition the tetrads obey the rules:

$$q^a_\mu q^\mu_b = \delta^a_b \quad -(31)$$

$$q^a_\mu q^\nu_a = \delta^\nu_\mu \quad -(32)$$

The tetrad is a tensor that transforms as:

8) $$q^{a'}_{\mu'} = \frac{\partial x^{a'}}{\partial x^{a}} \frac{\partial x^{\mu}}{\partial x^{\mu'}} q^{a}_{\mu} \quad -(32)$$

By definition: $$\Lambda^{a'}_{a} = \frac{\partial x^{a'}}{\partial x^{a}} \quad -(33)$$

is a Lorentz transform.

Therefore:

$$\Gamma^{a'}_{\mu'\lambda'} = q^{a'}_{\nu'} \Gamma^{\nu'}_{\mu'\lambda'} = \frac{\partial x^{a'}}{\partial x^{a}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} q^{a}_{\nu} \Gamma^{\nu'}_{\mu'\lambda'}$$

$$= \frac{\partial x^{a'}}{\partial x^{a}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} q^{a}_{\nu} \left(\frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\lambda}}{\partial x^{\lambda'}} \frac{\partial x^{\nu'}}{\partial x^{\nu}} \Gamma^{\nu}_{\mu\lambda} - \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\lambda}}{\partial x^{\lambda'}} \frac{\partial}{\partial x^{\mu}} \left(\frac{\partial x^{\nu'}}{\partial x^{\lambda}} \right) \right) \quad -(34)$$

$$= \frac{\partial x^{a'}}{\partial x^{a}} \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\lambda}}{\partial x^{\lambda'}} \Gamma^{a}_{\mu\lambda} - \frac{\partial x^{a'}}{\partial x^{a}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\lambda}}{\partial x^{\lambda'}} q^{a}_{\nu} \frac{\partial}{\partial x^{\mu}} \left(\frac{\partial x^{\nu'}}{\partial x^{\lambda}} \right)$$

where $$q^{a}_{\nu} = \frac{\partial x^{a}}{\partial x^{\nu}} . \quad -(35)$$

So:

$$\Gamma^{a'}_{\mu'\lambda'} = \frac{\partial x^{a'}}{\partial x^{a}} \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\lambda}}{\partial x^{\lambda'}} \Gamma^{a}_{\mu\lambda} - \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\lambda}}{\partial x^{\lambda'}} \frac{\partial x^{a'}}{\partial x^{\nu'}} \frac{\partial}{\partial x^{\mu}} \left(\frac{\partial x^{\nu'}}{\partial x^{\lambda}} \right) \quad -(36)$$

In this expression the indices ν' and a' in the second term run over the same number of dimensions, e.g. 0, 1, 2, 3 in four dimensional spacetime.

So we arrive at:

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$$\frac{\partial x^{a'}}{\partial x^{\nu'}}\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial x^{\nu'}}{\partial x^{\lambda}}\right) = \frac{\partial x^{a'}}{\partial x^{a'}}\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial x^{a'}}{\partial x^{\lambda}}\right)$$

$$= \frac{\partial}{\partial x^{\mu}}\left(\frac{\partial x^{a'}}{\partial x^{\lambda}}\right) \quad -(37)$$

So:

$$\boxed{\Gamma^{a'}_{\mu'\lambda'} = \frac{\partial x^{a'}}{\partial x^{a}}\frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\Gamma^{a}_{\mu\lambda} - \frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial x^{a'}}{\partial x^{\lambda}}\right)}$$

$$-(38)$$

as in previous notes and papers.

However:

$$q^{a'}_{\lambda} = \frac{\partial x^{a'}}{\partial x^{\lambda}} = q^{a'}_{a}\frac{\partial x^{a}}{\partial x^{\lambda}}$$

$$= q^{a'}_{a}\,q^{a}_{\lambda} = \delta^{a'}_{\lambda}. \quad -(39)$$

$$= 0$$

unless

$$a' = \lambda. \quad -(40)$$

In which case:

$$\frac{\partial}{\partial x^{\mu}}\left(\frac{\partial x^{a'}}{\partial x^{\lambda}}\right) = 0. \quad -(41)$$

So:

$$\boxed{\Gamma^{a'}_{\mu'\lambda'} = \frac{\partial x^{a'}}{\partial x^{a}}\frac{\partial x^{\mu}}{\partial x^{\mu'}}\frac{\partial x^{\lambda}}{\partial x^{\lambda'}}\Gamma^{a}_{\mu\lambda}} \quad -(42)$$

and

$$\boxed{\Gamma^{\nu'}_{\mu'\lambda'} = \frac{\partial x^{\nu'}}{\partial x^{\nu}} \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\lambda}}{\partial x^{\lambda'}} \Gamma^{\nu}_{\mu\lambda}} \quad -(43)$$

The Christoffel connection transforms as a tensor. Eq. (15) therefore means that the symmetric part of the connection must be zero in all frames of reference.

The geometry used in Einsteinian general relativity is therefore incorrect, and the theory is refuted in yet another way.

The tetrad postulate is:

$$\partial_\mu q^a_{\ \nu} = \Gamma^a_{\mu\nu} - \omega^a_{\mu\nu} \quad -(44)$$

$$= \left(\Gamma^\lambda_{\mu\nu} - \omega^\lambda_{\mu\nu}\right) q^a_{\ \lambda}$$

so

$$\boxed{\begin{aligned} \Gamma^\lambda_{\mu\nu} &= q^\lambda_{\ a}\, \partial_\mu q^a_{\ \nu} + \omega^\lambda_{\mu\nu} \\ \Gamma^a_{\mu\nu} &= \partial_\mu q^a_{\ \nu} + \omega^a_{\mu\nu} \end{aligned}}$$

$-(45)$

By symmetry:

$$\Gamma^a_{\mu\nu} = - \Gamma^a_{\nu\mu} \quad -(46)$$

So:

$$\boxed{\partial_\mu q^a_\nu + \omega^a_{\mu\nu} = - \left(\partial_\nu q^a_\mu + \omega^a_{\nu\mu} \right)} \quad -(47)$$

which is the antisymmetry law of ECE, QED.

If

$$\mu = \nu \quad -(48)$$

then

$$\partial_\mu q^a_\mu + \omega^a_{\mu\mu} = 0 \quad -(49)$$

i.e

$$\begin{aligned} \partial_0 q^a_0 + \omega^a_{00} &= 0 \\ \partial_1 q^a_1 + \omega^a_{11} &= 0 \\ \partial_2 q^a_2 + \omega^a_{22} &= 0 \\ \partial_3 q^a_3 + \omega^a_{33} &= 0 \end{aligned} \quad -(50)$$

and

$$\Gamma^a_{00} = \Gamma^a_{11} = \Gamma^a_{22} = \Gamma^a_{33} = 0 \quad -(51)$$