211(8): Proof of Connection Antisymmetry by General Coordinate Transformation.

As shown in previous notes, coordinate transformation of the antisymmetric connection leads to:

$$\frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\lambda}}{\partial x^{\lambda'}} \frac{\partial^2 x^{\nu'}}{\partial x^{\mu} \partial x^{\lambda}} = 0 . \quad -(1)$$

Consider the transformation of:

$$\Gamma^{a}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} q^{a}_{\lambda} \quad -(2)$$

then

$$\frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{a}}{\partial x^{a'}} \frac{\partial}{\partial x^{\mu}} \left(\frac{\partial x^{\nu'}}{\partial x^{a}} \right) = 0 \quad -(3).$$

which is consistent with:

$$\frac{\partial x^{\nu'}}{\partial x^{a}} = 0 \quad -(4)$$

Now use:

$$\frac{\partial x^{\nu'}}{\partial x^{a}} = \frac{\partial x^{\nu'}}{\partial x^{b}} \frac{\partial x^{b}}{\partial x^{a}} . \quad -(5)$$

The tangent spacetime of the Cartan geometry is defined by the Minkowski metric, so:

$$\frac{\partial x^{b}}{\partial x^{a}} = 0 \quad -(6)$$

and eq. (3) follows, QED.

2) Therefore $\Gamma^a_{\mu\nu}$ transforms as a tensor. The tetrad transforms as a tensor:

$$q^{a'}_{\nu'} = \frac{\partial x^{a'}}{\partial x^{a}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} q^a_\nu \quad -(7)$$

so from eq (2) $\Gamma^\lambda_{\mu\nu}$ transforms as a tensor, QED:

$$\Gamma^{\lambda'}_{\mu'\nu'} = \frac{\partial x^{\mu}}{\partial x^{\mu'}} \frac{\partial x^{\nu}}{\partial x^{\nu'}} \frac{\partial x^{\lambda'}}{\partial x^{\lambda}} \Gamma^\lambda_{\mu\nu} \quad -(8)$$

If the connection were symmetric then

$$\frac{\partial x^b}{\partial x^a} \neq 0 \quad -(9)$$

which is incorrect, reductio ad absurdum.