211(4): Cyclic Equation involving Torsion.

Consider the Riemann format of the Cartan identity with the correct antisymmetric connection:

$$\Gamma^{\lambda}_{\mu\nu} = -\Gamma^{\lambda}_{\nu\mu} \quad \text{—(1)}$$

In previous notes it was shown that possible solutions are:

$$R^{\lambda}{}_{\mu\nu\rho} = \partial_\mu T^{\lambda}{}_{\nu\rho} + \Gamma^{\lambda}_{\mu\sigma} T^{\sigma}{}_{\nu\rho} \quad \text{—(2)}$$

$$= \frac{1}{2}\left(\partial_\mu T^{\lambda}{}_{\nu\rho} + \Gamma^{\lambda}_{\mu\sigma} T^{\sigma}{}_{\nu\rho}\right) - \frac{1}{4}\left(\partial_\nu T^{\lambda}{}_{\mu\rho} + \Gamma^{\lambda}_{\nu\sigma} T^{\sigma}{}_{\mu\rho} - \left(\partial_\rho T^{\lambda}{}_{\mu\nu} + \Gamma^{\lambda}_{\rho\sigma} T^{\sigma}{}_{\mu\nu}\right)\right)$$

So it follows that the curvature can be eliminated entirely from the geometrical identity, which can be expressed purely in terms of torsion:

$$\boxed{2\left(\partial_\mu T^{\lambda}{}_{\nu\rho} + \Gamma^{\lambda}_{\mu\sigma} T^{\sigma}{}_{\nu\rho}\right) + \left(\partial_\nu T^{\lambda}{}_{\mu\rho} + \Gamma^{\lambda}_{\nu\sigma} T^{\sigma}{}_{\mu\rho}\right) - \left(\partial_\rho T^{\lambda}{}_{\mu\nu} + \Gamma^{\lambda}_{\rho\sigma} T^{\sigma}{}_{\mu\nu}\right) = 0 \quad \text{et cyclicum}}$$

—(3)

This identity shows that the most fundamental quantity is torsion. Usually the latter is defined as:

2)

$$T^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu} \quad -(4)$$

but it can be defined as the difference of any two connections:

$$T^{\lambda}_{\mu\nu} = R^{\lambda}_{\mu\nu} - S^{\lambda}_{\mu\nu} \quad -(5).$$

The two identities used in ECE theory are:

$$D \wedge T := R \wedge q \quad -(6)$$

and

$$D \wedge \tilde{T} := \tilde{R} \wedge q \quad -(7)$$

Written out in full these are:

$$d \wedge \tilde{T} = \tilde{R} \wedge q - \omega \wedge \tilde{T} \quad -(8).$$

and

$$d \wedge T = R \wedge q - \omega \wedge T \quad -(9)$$

being respectively the homogeneous and inhomogeneous field equations, both of dynamics and electrodynamics. In form notation eq. (3) is:

$$2\left(\partial_\mu T^a_{\nu\rho} + \omega^a_{\mu b} T^b_{\nu\rho}\right) + \left(\partial_\nu T^a_{\mu\rho} + \omega^a_{\nu b} T^b_{\mu\rho}\right) - \left(\partial_\rho T^a_{\mu\nu} + \omega^a_{\rho b} T^b_{\mu\nu}\right) = 0 \quad (10)$$

which gives a new equation both for the electromagnetic and gravitational field tensors.