208(11) : General Solution of the New Field Equation.

The field equation is:

$$\frac{d\omega}{d\theta}\frac{df}{d\theta} + \omega\frac{d^2 f}{d\theta^2} = 0 \qquad -(1)$$

where

$$f = \left(r\frac{d\theta}{dr}\right)^2. \qquad -(2)$$

Let

$$f_1 = \frac{df}{d\theta}, \qquad -(3)$$

then

$$\omega\frac{df_1}{d\theta} = -\left(\frac{d\omega}{d\theta}\right)f_1 \qquad -(4)$$

i.e.

$$\frac{df_1}{f_1} = -\frac{1}{\omega}\frac{d\omega}{d\theta}d\theta = -\frac{1}{\omega}d\omega \qquad -(5)$$

so

$$\boxed{\frac{df_1}{f_1} = -\frac{d\omega}{\omega}}. \qquad -(6)$$

A solution is:

$$\boxed{f_1\omega = \omega_0} \qquad -(7)$$

where

$$f_1 = \frac{df}{d\theta} = \frac{d}{d\theta}\left(r^2\left(\frac{d\theta}{dr}\right)^2\right)$$

$$= \left(\frac{dr^2}{d\theta}\right)\left(\frac{d\theta}{dr}\right)^2 + r^2\frac{d}{d\theta}\left(\frac{d\theta}{dr}\right)^2$$

$$= 2r\frac{dr}{d\theta}\frac{d\theta}{dr}\frac{d\theta}{dr} + 2r^2\frac{d\theta}{dr}\frac{d}{d\theta}\left(\frac{d\theta}{dr}\right)$$

$$= 2r\frac{d\theta}{dr}\left(1 + r\frac{d}{d\theta}\left(\frac{d\theta}{dr}\right)\right) \qquad -(8)$$

2)

So
$$\boxed{f_1 = 2r\frac{d\theta}{dr}\left(1 + r\frac{d}{d\theta}\left(\frac{d\theta}{dr}\right)\right)} \quad -(9)$$

and
$$\omega = \omega_0\left(2r\frac{d\theta}{dr}\left(1 + r\frac{d}{d\theta}\left(\frac{d\theta}{dr}\right)\right)\right)^{-1} \quad -(10)$$

The Newtonian solution is:
$$\omega_N = \frac{L}{m\alpha^2}\left(1 + \epsilon\cos\theta\right)^2 \quad -(11)$$

and the precessing elliptical solution is, to an excellent approximation:
$$\omega_p = \frac{L}{m\alpha^2}\left(1 + \epsilon\cos(x\theta)\right)^2 \quad -(12)$$

because
$$x - 1 \sim 10^{-6} \quad -(13)$$

in the solar system.

Now write the general solution (10) as:
$$\omega = \omega_p - A\omega_1 \quad -(14)$$

so
$$\boxed{\omega_1 = \frac{1}{A}\left(\omega_p - \omega\right)} \quad -(15)$$

the function ω_1 is a very small correction to ω_1, so A is very large.

3) Interpretation

The θ dependent correction ω_1 represents very small modulations of the precessing elliptical orbit. The latter is not a perfect precessing ellipse. The frequency ω_1 depends on the orbital function $d\theta/dt$ from eqs. (10) and (15). The new general relativity therefore predicts that the orbit "wobbles" a little, and this is a fact observed in astronomy.

To an excellent approximation:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \qquad -(16)$$

so ω may be found by computer algebra from eq. (10). Finally ω_1 may be expressed in terms of ω_p, ω_0 and A.