208(1): Development of the Second Set of Equations of the Constrained Minkowski Method:

Consider the Minkowski metric in the plane:

$$d\tau^2 = 0 \quad -(1)$$

i.e.

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu = c^2 dt^2 - dr^2 - r^2 d\theta^2 \quad -(2)$$

$$= g_{00}\, dx^0 dx^0 + g_{11}\, dx^1 dx^1 + g_{22}\, dx^2 dx^2 .$$

So:

$$dx^0 = c\,dt, \quad dx^1 = dr, \quad dx^2 = r\,d\theta \quad -(3)$$

$$g_{00} = 1, \quad g_{11} = -1, \quad g_{22} = -1 .$$

The observed orbit is defined by:

$$g = \frac{dr}{d\theta} . \quad -(4)$$

Write eq. (2) as:

$$ds^2 = c^2 dt^2 - \left(\frac{dr}{d\theta}\right)^2 d\theta^2 - r^2 d\theta^2 . \quad -(5)$$

From eq. (4):

$$dr^2 = g^2 d\theta^2 \quad -(6)$$

so

$$\left(dx^1\right)^2 = \frac{g^2}{r^2}\left(dx^2\right)^2 \quad -(7)$$

and

$$ds^2 = g_{00}\, dx^0 dx^0 + \frac{g_{11}}{f}\, dx^2 dx^2 + g_{22}\, dx^2 dx^2$$

$$-(8)$$

2) Define:

$$g_{22}' = \frac{g_{11}}{f} + g_{22} \quad -(9)$$

then

$$ds^2 = g_{00}\, dx^0 dx^0 + g_{22}'\, dx^2 dx^2 \quad -(10)$$

This infinitesimal line element is that of a two dimensional space labelled 0 and 2. The metric matrix is:

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 \\ 0 & -\left(1 + \frac{1}{f}\right) \end{bmatrix} \quad -(11)$$

The metric compatibility condition is:

$$D_\rho\, g_{\mu\nu} = 0 \quad -(12)$$

i.e.:

$$D_\rho\, g_{00} = D_\rho\, g_{22}' = 0. \quad -(13)$$

where:

$$g_{00} = 1, \quad g_{22}' = -\left(1 + \frac{1}{f}\right). \quad -(14)$$

Therefore:

$$D_\rho \left(1 + \frac{1}{f}\right) = 0 \quad -(15)$$

Eq. (12) can be expanded as:

3)

$$\partial_\rho g_{\mu\nu} - \Gamma^\lambda_{\rho\mu} g_{\lambda\nu} - \Gamma^\lambda_{\rho\nu} g_{\mu\lambda} = 0. \quad -(16)$$

The only non-trivial case is:

$$\partial_\rho g_{22} - \Gamma^\lambda_{\rho 2} g_{\lambda 2} - \Gamma^\lambda_{\rho 2} g_{2\lambda} = 0 \quad -(17)$$

The metric is diagonal so the only possibility is:

$$\partial_\rho g_{22} = 2\Gamma^2_{\rho 2} g_{22}. \quad -(18)$$

The connection is antisymmetric so the only possibility is:

$$\partial_0 g_{22} = 2\Gamma^2_{02} g_{22} \quad -(19)$$

i.e.

$$\Gamma^2_{02} = \frac{1}{2c g_{22}} \frac{\partial g_{22}}{\partial t}. \quad -(20)$$

The only torsion element is:

$$\boxed{T^2_{02} = -T^2_{20} = \frac{1}{c\left(1+\frac{1}{f}\right)} \frac{\partial}{\partial t}\left(1+\frac{1}{f}\right)} \quad -(21)$$

The non-vanishing curvature elements and identities can now be evaluated by computer.