205(12): Condition for Perturbation of the Orbit.

The general solution for the angular frequency of the orbit is:

$$\omega = \omega_0 \exp\left(-\left(F_1(\theta) + C\right)\right) \quad -(1)$$

where

$$\int F \, d\theta = F_1(\theta) + C \quad -(2)$$

To an excellent approximation the angular frequency of the precessing elliptical orbit is:

$$\omega_p = \frac{L}{m d^2}\left(1 + \epsilon \cos(x\theta)\right)^2 \quad -(3)$$

If:

$$\epsilon << 1 \quad -(4)$$

then

$$\omega_p \rightarrow \frac{L}{m d^2}. \quad -(5)$$

If

$$C >> F_1(\theta) \quad -(6)$$

then

$$\omega \rightarrow \omega_0 \exp(-C). \quad -(7)$$

In these limits the two solutions are the same if:

$$\omega_0 \exp(-C) = \frac{L}{m d^2} \quad -(8)$$

i.e.

$$C = \log_e\left(\frac{m \omega_0 d^2}{L}\right). \quad -(9)$$

To an excellent approximation the observed

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orbital angular frequency is given by eq. (3). However, the general solution is eq. (1), which can differ only very slightly from eq. (3). If:

$$\omega - \omega_p = \omega_1 \quad — (10)$$

then ω_1 is a very small quantity experimentally.

From eqs. (1) and (3):

$$\frac{\omega_1}{\omega_0} = \exp\left(-\left(F_1(\theta) + C\right)\right) - \frac{L}{m\omega_0 d^2}\left(1 + \epsilon\cos(x\theta)\right)^2$$

$$<< 1, \quad — (11)$$

i.e.

$$\boxed{\exp\left(-C\right)\exp\left(-F_1(\theta)\right) - \frac{L}{m\omega_0 d^2}\left(1 + \epsilon\cos(x\theta)\right)^2 << 1 = \omega_1 / \omega_0 \quad — (12)}$$

The function $F_1(\theta)$ is very complicated and is best worked out with the computer. The function ω_1 / ω_0 can then be graphed for various C and $L/(m\omega_0 d^2)$. For planets, $\epsilon << 1$ and x is essentially one.