208(8) : Orbital Linear Velocity for Stars in the Hyperbolic and Archimedean Spirals

The orbital linear velocity is defined by:

$$v = \left(\left(\frac{dr}{dt} \right)^2 + r^2 \left(\frac{d\theta}{dt} \right)^2 \right)^{1/2}$$

$$= \left(\left(\frac{dr}{d\theta} \right)^2 + r^2 \right)^{1/2} \omega \quad \text{— (1)}$$

where

$$\omega = \frac{d\theta}{dt} . \quad \text{— (2)}$$

In the Archimedean spiral:

$$r = b\theta \quad \text{— (3)}$$

so

$$\frac{dr}{d\theta} = b . \quad \text{— (4)}$$

From previous work:

$$\omega = \omega_0 \exp\left(-\frac{r^2}{b^2} \right) \quad \text{— (5)}$$

so:

$$\boxed{v = \left(r^2 + b^2 \right)^{1/2} \omega_0 \exp\left(-\frac{r^2}{b^2} \right)} \quad \text{— (6)}$$

It is observed experimentally that:

$$v \xrightarrow[r \to \infty]{} \text{constant} \quad \text{— (7)}$$

By numerical analysis of eq. (6), ω_0 and b can be adjusted to give condition (7), or eq. (7) approximately.

2) In the hyperbolic spiral:

$$r = \frac{r_0}{\theta} \quad -(8)$$

$$\frac{dr}{d\theta} = -\frac{r_0}{\theta^2} \quad -(9)$$

From previous work it was found that:

$$\omega = \frac{\omega_0}{\theta}, \quad -(10)$$

so:

$$\boxed{v = \frac{\omega_0 r_0}{\theta^2}\left(1 + \frac{1}{\theta^2}\right)} \quad -(11)$$

Therefore if the spiral is such that:

$$\theta \rightarrow \text{constant} \quad -(12)$$

then the experimental results are given.

This analysis can be repeated for other types of spiral such as:

$$r = r_0 \theta^n \quad -(13)$$