208(7): Comparison and Interpretation of Results for the Hyperbolic and Archimedes Spirals.

Let the Archimedes spiral be defined in the simplest case by:

$$r = b\theta . \qquad -(1)$$

As θ increases, r increases. Here b is a constant. The angular velocity of a star in the whirlpool galaxy is:

$$\omega = \omega_0 \exp\left(-\frac{r^2}{b^2}\right) \qquad -(2)$$

and the torsion acting on the star is:

$$T^1{}_{01} = \frac{\omega_0}{c}\frac{r}{b}\left(1+\left(\frac{r}{b}\right)^2\right)^{-1}\exp\left(-\frac{r^2}{b^2}\right) \qquad -(3).$$

The angular velocity and torsion are maximum at the centre of the galaxy and dissipate to zero at the edges. This is intuitively acceptable and easy to understand.

On the other hand the hyperbolic spiral is defined by:

$$r = \frac{r_0}{\theta} . \qquad -(4)$$

As θ increases, r decreases. The angular velocity of a star in the whirlpool galaxy is:

$$\omega = \frac{\omega_0}{\theta} \qquad -(5)$$

$$= \frac{\omega_0 r}{r_0}$$

2) The torsion of the hyperbolic spiral is defined by:

$$T^{1}{}_{01} = \frac{1}{2}\frac{\omega}{c(1+f)}\frac{df}{d\theta} \quad -(6)$$

where

$$f = \theta^2. \quad -(7)$$

So

$$T^{1}{}_{01} = \frac{\omega\theta}{c(1+\theta^2)} = \frac{\omega_0}{c(1+\theta^2)}, \quad -(7)$$

i.e.

$$\boxed{\omega = \frac{\omega_0}{\theta}, \quad T^{1}{}_{01} = \frac{\omega_0}{c(1+\theta^2)}} \quad -(8)$$

At the centre of the galaxy:

$$\theta \to 0, \quad \omega \to \infty, \quad T^{1}{}_{01} \to \frac{\omega_0}{c}. \quad -(9)$$

At the edge of the galaxy:

$$\theta \to \infty, \quad \omega \to 0, \quad T^{1}{}_{01} \to 0 \quad -(10)$$

This means that at the centre of the galaxy:

$$r \to \infty \quad -(11)$$

and at the edge:

$$r \to 0 \quad -(12)$$

So in the hyperbolic spiral the distance is measured from the edge inward. In the Archimedes spiral it is measured from the centre outwards.