208(2): Development of the First Equation of Motion of the Constrained Minkowski Method.

As in UFT 207 the first equation of motion is:

$$\frac{\partial}{\partial t}\left(\frac{\partial f}{\partial t}\right) = 0 \quad -(1)$$

where

$$f = r^2\left(\frac{d\theta}{dr}\right)^2. \quad -(2)$$

Here:

$$\frac{\partial f}{\partial t} = \frac{1}{2}\omega\frac{df}{d\theta} \quad -(3)$$

Now define:

$$F = \frac{\partial f}{\partial t}. \quad -(4)$$

so eq. (1) is

$$\frac{\partial F}{\partial t} = \frac{1}{2}\omega\frac{dF}{d\theta} = 0. \quad -(5)$$

Therefore:

$$\frac{1}{2}\omega\frac{d}{d\theta}\left(\frac{1}{2}\omega\frac{df}{d\theta}\right) = 0. \quad -(6)$$

For:

$$\omega \neq 0, \quad -(7)$$

eq. (6) is:

$$\boxed{\frac{d}{d\theta}\left(\omega\frac{df}{d\theta}\right) = 0.} \quad -(8)$$

2) Using the Leibniz rule:

$$\frac{d\omega}{d\theta}\frac{df}{d\theta} + \omega \frac{d^2 f}{d\theta^2} = 0 \quad \text{— (9)}$$

in which:

$$f = \left(r \frac{d\theta}{dr} \right)^2 \quad \text{— (10)}$$

Eq. (9) is:

$$\boxed{\frac{d\omega}{d\theta} = -\left(\frac{d^2 f}{d\theta^2} \Big/ \frac{df}{d\theta} \right) \omega} \quad \text{— (11)}$$

<u>Spiral Galaxy</u>

If the stars are arranged in a hyperbolic spiral then:

$$r = \frac{r_0}{\theta}, \quad \frac{d\theta}{dr} = -\frac{\theta^2}{r_0} \quad \text{— (12)}$$

$$r^2 \left(\frac{d\theta}{dr} \right)^2 = \left(\frac{r}{r_0} \right)^2 \theta^4 = \theta^2 \quad \text{— (13)}$$

So

$$f = \theta^2. \quad \text{— (14)}$$

Therefore

$$\boxed{\frac{d\omega}{d\theta} = -\frac{\omega}{\theta}} \quad \text{— (15)}$$

and

$$-\frac{d\theta}{\theta} = \frac{d\omega}{\omega} \quad \text{— (16)}$$

3)

A solution of eq. (15) is:

$$\boxed{\theta = \frac{\omega_0}{\omega}} \quad — (17)$$

Check

$$\frac{d\theta}{d\omega} = -\frac{\omega_0}{\omega^2} \quad — (18)$$

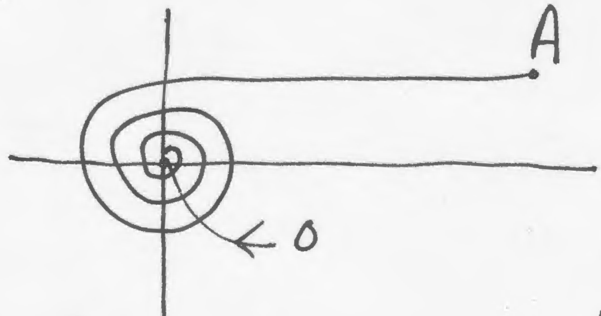
$$d\theta = -\frac{\omega_0}{\omega^2}\, d\omega \quad — (19)$$

$$= -\frac{\theta}{\omega}\, d\omega$$

so

$$\frac{d\omega}{d\theta} = -\frac{\omega}{\theta}. \quad — (20)$$

Fig (1)



For θ starting from zero, $r = r_0/\theta$ starts from infinity and the spiral winds to infinity as it approaches the pole. The solution (17) can also be written as:

$$\boxed{\frac{\omega}{\omega_0} = \frac{r}{r_0} = \frac{1}{\theta}} \quad -(21)$$

In this case an object of mass m at point A begins to be attracted by an object of mass

4)

m at point O. At point A it has travelled a distance $r = 0$ along the spiral toward O. So its angular velocity ω at point A is zero. As it approaches O it spins faster and faster, and move further and further along the spiral, so both ω and r increase to infinity. For infinite r and ω the angle θ is zero.

In this case the equation of motion (11) makes perfect sense.

Precessing Ellipse

In this case:

$$r = \frac{\alpha}{1 + \epsilon \cos(x\theta)} \quad -(22)$$

and

$$\frac{dr}{d\theta} = \left(\frac{x\epsilon}{\alpha}\right) r^2 \sin(x\theta) \quad -(23)$$

so

$$f = \left(\frac{\alpha}{x\epsilon}\right)^2 \cdot \frac{1}{r^2 \sin^2(x\theta)} \quad -(24)$$

$$= \left(\frac{\alpha}{x\epsilon}\right)^2 \left(\frac{1 + \epsilon\cos(x\theta)}{\sin(x\theta)}\right)^2 \cdot \frac{1}{\alpha^2}$$

$$\boxed{f = \frac{1}{x^2 \epsilon^2} \left(\frac{1 + \epsilon\cos(x\theta)}{\sin(x\theta)}\right)^2} \quad -(25)$$