207(9): Development of the Identity Equations

The two identity equations are:

$$D_0 T^1{}_{01} = R^1{}_{001} \quad -(1)$$

and

$$D_1 T^1{}_{10} = R^1{}_{110}. \quad -(2)$$

These become:

$$6(1+f)\frac{\partial}{\partial t}\left(\frac{\partial f}{\partial t}\right) = 5\left(\frac{\partial f}{\partial t}\right)^2 \quad -(3)$$

and

$$(1+f)\frac{\partial}{\partial r}\left(\frac{\partial f}{\partial t}\right) = \frac{\partial f}{\partial r}\frac{\partial f}{\partial t} \quad -(4)$$

These must be solved simultaneously, so if:

$$x = \frac{\partial f}{\partial t}, \quad y = \frac{\partial f}{\partial r} \quad -(5)$$

$$(1+f)\frac{\partial x}{\partial r} = xy \quad -(6)$$

$$6(1+f)\frac{\partial x}{\partial t} = 5x^2. \quad -(7)$$

Divide (7) by (6):

$$6\frac{\partial x}{\partial t}\frac{\partial r}{\partial x} = \frac{5x}{y} \quad -(8)$$

i.e. $$\boxed{6y\frac{\partial r}{\partial t} = 5x} \quad -(9)$$

The equation of motion or field equation is:

$$6\frac{\partial x}{\partial t} = 5\frac{\partial x}{\partial t} \quad -(10)$$

so:

$$\boxed{\frac{\partial x}{\partial t} = 0} \quad -(11)$$

This means that:

$$\frac{\partial}{\partial t}\left(\frac{\partial f}{\partial t}\right) = 0 \quad -(12)$$

where

$$\frac{\partial f}{\partial t} = \frac{1}{2}\,\omega\,\frac{df}{d\theta} \quad -(13)$$

so

$$\frac{\partial}{\partial t}\left(\omega\frac{df}{d\theta}\right) = 0, \quad -(14)$$

or

$$\boxed{\frac{\partial \omega}{\partial t}\frac{df}{d\theta} + \omega\frac{\partial}{\partial t}\left(\frac{df}{d\theta}\right) = 0} \quad -(15)$$

using the Leibniz rule.

It is seen that eq. (15) is an equation of motion, giving:

$$\boxed{\frac{\partial \omega}{\partial t} = -\frac{\omega}{x_1}\frac{\partial x_1}{\partial t} \quad -(16) \qquad \text{where} \quad x_1 = \frac{df}{d\theta}.}$$

3)

The field equation or equation of motion is therefore:

$$\boxed{\frac{\partial \omega}{\partial t} = -\,\omega \frac{\frac{\partial}{\partial t}\left(\frac{df}{d\theta}\right)}{\left(\frac{df}{d\theta}\right)}} \quad -(17)$$

where

$$f = r^2 \left(\frac{d\theta}{dr}\right)^2 \quad -(18)$$

<u>This is true for all orbits</u>.