207(4) : Meaning of the First Equation of the Evans Identity.

This is :

$$6\left(1+\frac{dr}{d\theta}\right)\frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}\left(\frac{dr}{d\theta}\right)\right) = 5\left(\frac{\partial}{\partial t}\left(\frac{dr}{d\theta}\right)\right)^2 \quad -(1)$$

and is an exact identity of Cartan geometry.

Proof of Eq. (1)

Denote :

$$f = \frac{dr}{d\theta} \quad -(2)$$

then

$$\frac{\partial f}{\partial t} = \frac{df}{dr}\frac{\partial r}{\partial t} . \quad -(3)$$

Now consider :

$$g = g(x, y) \quad -(4)$$

then

$$\frac{dg}{dt} = \frac{\partial g}{\partial x}\frac{dx}{dt} + \frac{\partial g}{\partial y}\frac{dy}{dt} \quad -(5)$$

$$\frac{dg}{dx} = \frac{\partial g}{\partial x} + \frac{\partial g}{\partial y}\frac{dy}{dx} \quad -(6)$$

$$\frac{dg}{dy} = \frac{\partial g}{\partial y} + \frac{\partial g}{\partial x}\frac{dx}{dy} \quad -(7)$$

If :

$$\theta = \theta(r, t) \quad -(8)$$

2) then

$$\frac{d\theta}{dr} = \frac{\partial\theta}{\partial r} + \frac{\partial\theta}{\partial t}\frac{dt}{dr} \quad - (9)$$

If

$$r = r(\theta, t) \quad - (10)$$

then

$$\frac{dr}{d\theta} = \frac{\partial r}{\partial\theta} + \frac{\partial r}{\partial t}\frac{dt}{d\theta} \quad - (11)$$

Multiply eqs. (9) and (11):

$$\left(\frac{\partial\theta}{\partial r} + \frac{\partial\theta}{\partial t}\frac{dt}{dr}\right)\left(\frac{\partial r}{\partial\theta} + \frac{\partial r}{\partial t}\frac{dt}{d\theta}\right) = 1 \quad - (12)$$

i.e.

$$\frac{\partial r}{\partial t}\frac{dt}{dr} + \frac{\partial\theta}{\partial t}\frac{dt}{d\theta} + \frac{\partial\theta}{\partial t}\frac{\partial r}{\partial t}\frac{dt}{dr}\frac{dt}{d\theta} = 0. \quad - (13)$$

This is true if:

$$\frac{\partial\theta}{\partial t} = 0 \quad - (14)$$

$$\frac{\partial r}{\partial t} = 0 \quad - (15)$$

$$\frac{d\theta}{dr} = \frac{\partial\theta}{\partial r} \quad - (16)$$

$$\frac{dr}{dt} = \frac{\partial r}{\partial t} + \frac{\partial r}{\partial\theta}\frac{d\theta}{dt} = \frac{dr}{d\theta}\frac{d\theta}{dt} \quad - (17)$$

$$\frac{d\theta}{dt} = \frac{\partial\theta}{\partial t} + \frac{\partial\theta}{\partial r}\frac{dr}{dt} = \frac{d\theta}{dr}\frac{dr}{dt} \quad - (18).$$

3) From eq. (3) and (15) it is seen that eq (1) is the exact identity:

$$\boxed{0 = 0} \quad -(19)$$

QED.

Therefore the code output 3b is fully self consistent and correct. The metric is:

$$g_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -(1+r^2/f^2) & 0 \\ 0 & 0 & -1 \end{bmatrix} \quad -(20)$$

i.e.

$$ds^2 = g_{\mu\nu}\, dx^\mu dx^\nu \quad -(21)$$

$$= c^2 dt^2 - dr^2 - r^2 d\theta^2 \quad -(22)$$

with

$$f = \frac{dr}{d\theta} \quad -(23)$$