Note 201(2): Spin Connections for the Precessing and Static Ellipses.

As shown in UFT 199 for any planar orbit:

$$\underline{v} = rc\left(\omega^1{}_{01}\,\underline{e}_r - \omega^1{}_{02}\,\underline{e}_\theta\right) \quad -(1)$$

where the spin connections are defined by:

$$\dot{r} = rc\omega^1{}_{01}, \quad r\dot{\theta} = -rc\omega^1{}_{02}. \quad -(2)$$

For the precessing ellipse:

$$\dot{r} = A\sin(x\theta) \quad -(3)$$

$$r\dot{\theta} = \frac{L}{mr} \quad -(4)$$

where

$$A = \frac{xL\epsilon}{m\alpha} \quad -(5)$$

and

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)}. \quad -(6)$$

Here L is the constant angular momentum, m the mass of the orbiting object, 2α the right latitude, ϵ the eccentricity, and x the precession constant.

So:

$$\omega^1{}_{01} = \frac{A}{rc}\sin(x\theta), \quad \omega^1{}_{02} = -\frac{L}{cmr^2}. \quad -(7)$$

It is convenient to define:

$$\Omega^1{}_{01} := c\omega^1{}_{01}, \quad \Omega^1{}_{02} := c\omega^1{}_{02} \quad -(8)$$

So:

$$\boxed{\Omega^1{}_{01} = \frac{A}{r}\sin(x\theta) \, , \quad \Omega^1{}_{02} = -\frac{L}{mr^2}} \quad - (9)$$

This is a simple, fully relativistic description of the precessing elliptical orbit.

For a static ellipse:

$$x = 1 \quad - (10)$$

and as shown in notes for UFT 199:

$$\underline{a} = \underline{\dot{v}} = -\frac{L^2}{m^2 r^3} \underline{e}_r \quad - (11)$$

So:

$$\boxed{\underline{a} = \underline{\dot{v}} = \frac{L}{mr} \Omega^1{}_{02} \underline{e}_r} \quad - (12)$$

If the definition is accepted of force:

$$\underline{F} = m \underline{a} \quad - (13)$$

then

$$\boxed{\underline{F} = \frac{L}{r} \Omega^1{}_{02} \underline{e}_r} \quad - (14)$$

The concept of Newtonian force is shown to be due to the spin connection $\Omega^1{}_{02}$ of the moving frame, a fully relativistic result.