1) 201(3): Testing the Constraint (18) in UFT199

Assume that the fundamental equivalence theorem is:

$$\partial_\mu V^a = \omega^a_{\mu b} V^b \quad -(1)$$

where the covariant derivative is:

$$D_\mu V^a = \partial_\mu V^a + \omega^a_{\mu b} V^b \quad -(2)$$

If the equivalence theorem is defined as eq. (1), under what circumstances does it follow that:

$$\partial_\mu q^a_\nu = \omega^a_{\mu\nu} = \omega^a_{\mu b} q^b_\nu \quad -(3)$$

For rotation in a plane, UFT199 shows that:

$$q^a_\mu = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}, \quad \omega^a_{ob} = \frac{1}{c}\begin{bmatrix} \frac{1}{r}\frac{\partial r}{\partial t} & -\frac{\partial\theta}{\partial t} \\ \frac{\partial\theta}{\partial t} & \frac{1}{r}\frac{\partial r}{\partial t} \end{bmatrix} \quad -(4)$$

Let $a = 1$, then:

$$\partial_o q^1_\nu = \omega^1_{o1} q^1_\nu + \omega^1_{o2} q^2_\nu. \quad -(5)$$

Let $\nu = 1$, then:

$$\partial_o q^1_1 = \omega^1_{o1} q^1_1 + \omega^1_{o2} q^2_1. \quad -(6)$$

This equation implies that:

$$\frac{1}{c}\frac{\partial \cos\theta(t)}{\partial t} = -\frac{1}{c}\frac{\partial\theta}{\partial t}\sin\theta \quad -(7)$$

$$= \frac{1}{cr}\frac{\partial r}{\partial t} + \frac{1}{c}\frac{\partial\theta}{\partial t} \quad -(8)$$

i.e.

$$-\sin\theta\frac{\partial\theta}{\partial t} = \frac{1}{r}\frac{\partial r}{\partial t} + \frac{\partial\theta}{\partial t} \quad -(9)$$

or

$$\left(1-\sin\theta\right)\frac{\partial\theta}{\partial t} = \frac{1}{r}\frac{\partial r}{\partial t}. \quad -(10)$$

Now use:

$$\frac{\partial r}{\partial t} = \frac{\partial r}{\partial\theta}\frac{\partial\theta}{\partial t}$$

2) to find that:

$$\frac{\partial r}{\partial \theta} = r(1 - \sin\theta) \quad -(11)$$

It is concluded that eq. (3) does not follow from eq. (1) is general because eq. (11) is too restrictive.

Therefore the torsion is defined by:

$$T^a_{\mu\nu} = \partial_\mu q^a_\nu - \partial_\nu q^a_\mu + \omega^a_{\mu b} q^b_\nu - \omega^a_{\nu b} q^b_\mu \quad -(12)$$

in general. The tetrad postulate is:

$$D_\mu q^a_\nu = \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu - \Gamma^\lambda_{\mu\nu} q^a_\lambda \quad -(13)$$

$$= 0.$$

Therefore:

$$\Gamma^a_{\mu\nu} = \Gamma^\lambda_{\mu\nu} q^a_\lambda = \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu \quad -(14)$$

$$= -\Gamma^a_{\nu\mu} \quad -(15)$$

so <u>the antisymmetry</u> <u>constraint</u> is:

$$\partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu = -\left(\partial_\nu q^a_\mu + \omega^a_{\nu b} q^b_\mu\right)$$

and <u>the</u> <u>equivalence theorem</u> <u>is</u> eq. <u>(1).</u>