## 199(3): Position Vector as a Tetrad and Geometrical Development for the Ellipse

Define the position tetrad as:

$$\underline{r}^{(1)} = \left(AX\,\underline{i} - iBY\,\underline{j}\right) \quad -(1)$$

then:

$$r^2 = \underline{r}^{(1)} \cdot \underline{r}^{(2)} = A^2X^2 + B^2Y^2 \quad -(2)$$

i.e.

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = r^2 \quad -(3)$$

where

$$A^2 = \frac{b}{a^2+b^2}, \quad B^2 = \frac{a}{a^2+b^2}. \quad -(4)$$

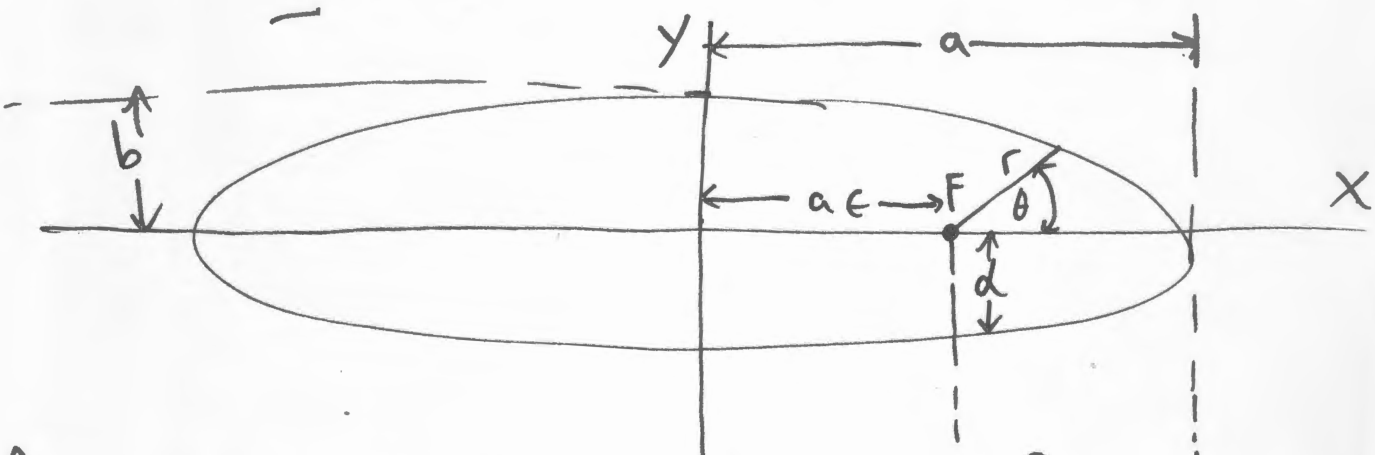
Here:

$$X = \epsilon a + r\cos\theta, \quad -(5)$$
$$Y = r\sin\theta, \quad -(6)$$

$$\underline{i} = \underline{e}_r\cos\theta - \underline{e}_\theta\sin\theta \quad -(7)$$
$$\underline{j} = \underline{e}_r\sin\theta + \underline{e}_\theta\cos\theta \quad -(8)$$

so:

$$\underline{r} = \epsilon a\,\underline{i} + r\,\underline{e}_r \quad -(9)$$




As in UFT 143 the velocity is a kind of torsion:

2)
$$V^a_{\mu\nu} = c\left(\partial_\mu r^a_\nu - \partial_\nu r^a_\mu + \omega^a_{\mu b} r^b_\nu - \omega^a_{\nu b} r^b_\mu\right) \quad -(10)$$

In vector notation:

$$\underline{v}^a = \frac{\partial \underline{r}^a}{\partial t} + c\underline{\nabla} r^a_0 + c\,\omega^a_{0b}\underline{r}^b - c r^b_0 \underline{\omega}^a_b \quad -(11)$$

$$\underline{w}^a = c\left(\underline{\nabla} \times \underline{r}^a - \underline{\omega}^a_b \times \underline{r}^b\right) \quad -(12)$$

## Flat Space Classical Development

In plane cylindrical coordinates:

$$\underline{v} = \dot{\underline{r}} = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta \quad -(13)$$

$$v^2 = \dot{r}^2 + r^2\dot{\theta}^2 \quad -(14)$$

By definition:

$$\underline{F} = m\dot{\underline{v}}, \quad T = \frac{1}{2}mv^2 \quad -(15)$$

where $T$ is the kinetic energy and $m$ the mass of a particle. Here $\underline{F}$ is the definition of force. This is a definition, not a law of physics,

The ellipse is:

$$r = \frac{\alpha}{1+\epsilon\cos\theta}. \quad -(16)$$

Now proceed by assuming that the lagrangian is:

$$\mathcal{L} = T = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) \quad -(17)$$

3) There is no potential energy. This is the usual idea of line element general relativity, now an obsolete theory. It is applied here in the limit of vanishing connection. The Euler Lagrange eqn. is:

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\theta}} = 0, \quad -(18)$$

giving the total angular momentum:

$$L = \frac{\partial \mathcal{L}}{\partial \dot{\theta}} = m r^2 \dot{\theta} = \text{constant} \quad -(19)$$

The acceleration is:

$$\underline{a} = \dot{\underline{v}} = \left(\ddot{r} - r\dot{\theta}^2\right)\underline{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\underline{e}_\theta \quad -(20)$$

As in UFT196:

$$r\ddot{\theta} + 2\dot{r}\dot{\theta} = 0 \quad -(21)$$

So

$$\underline{a} = \left(\ddot{r} - r\dot{\theta}^2\right)\underline{e}_r, \quad -(22)$$

which is the radially directed acceleration due to the ellipse, eq. (16).

Therefore:

$$\underline{a} = \left(\ddot{r} - \frac{L^2}{m^2 r^3}\right)\underline{e}_r \quad -(23)$$

where, as in UFT196:

4)

$$\ddot{r} = \frac{\epsilon L^2}{m^2 \alpha} \frac{1}{r^2} \cos\theta \quad -(24)$$

Using eq. (19):

$$\underline{a} = r \dot{\theta}^2 \left( \frac{\epsilon r}{\alpha} \cos\theta - 1 \right) \underline{e}_r \quad -(25)$$

and using eq. (16):

$$\boxed{\underline{a} = - r \dot{\theta}^2 \underline{e}_r} \quad -(26)$$

This is the inward directed acceleration needed for the function (16), given the definition (15).

Using eq. (19):

$$\boxed{\underline{a} = - \frac{L^2}{m^2 r^3} \underline{e}_r} \quad -(27)$$

This is a purely geometrical result, it uses no potential energy. The force by definition is:

$$\underline{F} = m \underline{a} = \frac{-L^2}{m r^3} \underline{e}_r \quad -(28)$$

In the usual Newtonian development, the "centrifugal force" is $-\underline{F}$, and is directed outward.

As in UFT196 it was found that:

$$\underline{F} = -\frac{mMG}{r^2}\underline{e}_r \quad -(29)$$

using:

$$\alpha = \frac{L^2}{m^2 MG}. \quad -(30)$$

So:

$$\boxed{\frac{mMG}{r^2} = \frac{L^2}{mr^3}}. \quad -(31)$$

The eq. (30) comes from assuming:

$$\mathcal{L}_1 = T - V \quad -(32)$$

where

$$T = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) \quad -(33)$$

and

$$V = -\frac{mMG}{r} \quad -(34)$$

<u>Conclusion</u>

The Newtonian theory of the ellipse is incorrect because its "centrifugal force" is incorrectly defined, and has the wrong sign. The theory produces an ellipse if eq. (30) is assumed, but differentiation of the same ellipse produces eq. (31).

The Newtonian derivation of the ellipse is as follows.

6) The total energy is:

$$E_1 = \frac{1}{2} m \dot{r}^2 + \frac{1}{2}\frac{L^2}{m r^2} + V, \quad -(35)$$

so:

$$\dot{r} = \frac{dr}{dt} = \left( \frac{2}{m}\left(E_1 - V\right) - \frac{L^2}{m^2 r^2} \right)^{1/2} \quad -(36)$$

use

$$\frac{d\theta}{dr} = \frac{d\theta}{dt}\frac{dt}{dr} \quad -(37)$$

so:

$$\theta(r) = \int \frac{1}{r^2}\left( 2m\left( E_1 - V - \frac{L^2}{2mr^2} \right)\right)^{1/2} dr \quad -(38)$$

which gives the ellipse:

$$\frac{\alpha}{r} = 1 + \epsilon \cos\theta \quad -(39)$$

if

$$\alpha = \frac{L^2}{m^2 M G} \quad -(40)$$

and

$$\epsilon = \left( 1 + \frac{2 E_1 L^2}{m k^2} \right) \quad -(41)$$

with

$$k = m M G. \quad -(42)$$

However, in this note it has been seen that the ellipse corresponds to the result (31), using only:

$$E = \frac{1}{2} m v^2 = \frac{1}{2}\left( m \dot{r}^2 + \frac{L^2}{m r^2} \right) \quad -(43)$$

and

$$V = 0. \quad -(44)$$

So Newton introduced $V$ and

incorrectly described it as a "force of attraction." This had to be counterbalanced by a "force of repulsion" in order to obtain a stable ellipse.

This force of repulsion was incorrectly attributed to:

$$F_c = -\frac{\partial U_c}{\partial r} = \frac{L^2}{mr^3}$$

$$= m r \dot{\theta}^2 \qquad -(35)$$

where

$$U_c \equiv \frac{L^2}{2mr^2} \qquad -(36)$$

From a comparison of eqns. (35) and (46) it is seen that in Newtonian dynamics, the rotational kinetic energy is incorrectly redefined as the centrifugal potential energy.

The true origin of the elliptical orbit is torsion, not an inverse square force of attraction.

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