1) 199(4): The Elliptical Tetrad

The Cartan tetrad is generalized in ECE theory to:

$$V^a = q^a_\mu V^\mu \quad -(1)$$

where

$$a = (1), (2), (3) \quad -(2)$$

$$\mu = 1, 2, 3, \quad -(3)$$

where attention is restricted to three dimensions. Eqs. (2) and (3) are any two representations of three dimensional space.

The ellipse is defined by:

$$\frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1. \quad -(4)$$

Define the vector:

$$\underline{r} = \frac{X}{a}\,\underline{i} + \frac{Y}{b}\,\underline{j} \quad -(5)$$

in Cartesian representation in two dimensions, in the place of an elliptical orbit. Then:

$$r^2 = \frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1 \quad -(6)$$

and

$$r_X = \frac{X}{a}, \quad r_Y = \frac{Y}{b}, \quad -(7)$$

$$\underline{r} = r_X\,\underline{i} + r_Y\,\underline{j}. \quad -(8)$$

Now define the vector:

$$\underline{r} = \frac{1}{\sqrt{2}}\left(\frac{X}{a} - i\frac{Y}{b}\right)\underline{e}^{(1)} + \frac{1}{\sqrt{2}}\left(\frac{X}{a} + i\frac{Y}{b}\right)\underline{e}^{(2)} \quad -(9)$$

$$= r^{(1)}\,\underline{e}^{(1)} + r^{(2)}\,\underline{e}^{(2)}$$

1) where:

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right), \quad \underline{e}^{(2)} = \frac{1}{\sqrt{2}}\left(\underline{i} + i\underline{j}\right), \quad r^{(1)} = \frac{1}{\sqrt{2}}\left(\frac{X}{a} - i\frac{Y}{b}\right); \quad r^{(2)} = \frac{1}{\sqrt{2}}\left(\frac{X}{a} + i\frac{Y}{b}\right) \quad -(10)$$

Then:

$$r^2 = \underline{r}\cdot\underline{r}^* = r^{(1)}r^{(2)} + r^{(2)}r^{(1)}$$
$$= \frac{X^2}{a^2} + \frac{Y^2}{b^2} = 1. \quad -(11)$$

<u>The elliptical tetrad</u> is then defined by:

$$r^a = r^a_\mu e^\mu \quad -(12) \qquad e^\mu = \begin{bmatrix} 1 \\ 1 \end{bmatrix} -(12a)$$

from which the <u>torsion</u> is defined by:

$$T^a_{\mu\nu} = \partial_\mu r^a_\nu - \partial_\nu r^a_\mu + \omega^a_{\mu b} r^b_\nu - \omega^a_{\nu b} r^b_\mu . \quad -(13)$$

Eq. (12) is:

$$\frac{1}{\sqrt{2}}\begin{bmatrix} \frac{X}{a} - i\frac{Y}{b} \\ \frac{X}{a} + i\frac{Y}{b} \end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{X}{a} & -i\frac{Y}{b} \\ \frac{X}{a} & i\frac{Y}{b} \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad -(14)$$

so

$$\boxed{r^a_\mu = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{X}{a} & -i\frac{Y}{b} \\ \frac{X}{a} & i\frac{Y}{b} \end{bmatrix}} \quad -(15)$$

The elliptical tetrad r^a_μ can be expressed a two conjugate vectors:

$$\underline{r}^{(1)} = \frac{1}{\sqrt{2}}\left(\frac{X}{a}\,\underline{i} - i\frac{Y}{b}\,\underline{j}\right) \quad -(16)$$

and

$$\underline{r}^{(2)} = \frac{1}{\sqrt{2}}\left(\frac{X}{a}\,\underline{i} + i\frac{Y}{b}\,\underline{j}\right) \quad -(17)$$

The ellipse (4) is accompanied by the torsion (13), conversely, an elliptical orbit is due to torsion.

The torsion equation (13) can be expressed as two vector equations:

$$\underline{v}^{a} = \frac{\partial \underline{r}^{a}}{\partial t} + c\underline{\nabla}\, r_{0}^{a} + c\,\omega^{a}_{0b}\,\underline{r}^{b} - c\, r_{0}^{b}\,\underline{\omega}^{a}_{b} \quad -(18)$$

and

$$\underline{w}^{a} = c\left(\underline{\nabla} \times \underline{r}^{a} - \underline{\omega}^{a}_{b} \times \underline{r}^{b}\right) \quad -(19)$$

In eq. (19), as in UFT143:

$$v^{a}_{\mu\nu} = \begin{bmatrix} 0 & -v^{a}_{X} & -v^{a}_{Y} & -v^{a}_{Z} \\ v^{a}_{X} & 0 & -w^{a}_{Z} & w^{a}_{Y} \\ v^{a}_{Y} & w^{a}_{Z} & 0 & -w^{a}_{X} \\ v^{a}_{Z} & -w^{a}_{Y} & w^{a}_{X} & 0 \end{bmatrix} \quad -(20)$$

$$\underbrace{\begin{matrix} v^{a}_{X} = -v^{a}_{01}, \\ v^{a}_{Y} = -v^{a}_{02}, \\ v^{a}_{Z} = -v^{a}_{03}, \end{matrix}}_{\text{orbital}} \qquad \underbrace{\begin{matrix} v^{a}_{12} = -w^{a}_{Z} \\ v^{a}_{13} = w^{a}_{Y} \\ v^{a}_{23} = -w^{a}_{X} \end{matrix}}_{\text{spin}} \quad -(21)$$

By antisymmetry:

4)

$$\frac{\partial \underline{r}^a}{\partial t} + c\,\omega^a_{\ 0b}\,\underline{r}^b = c\,\underline{\nabla}\,r_0^a - c\,r_0^b\,\underline{\omega}^a_{\ b} \quad -(22)$$

If it is assumed that:

$$\underline{\nabla}\,r_0^a = 0 \quad -(23)$$

then:

$$\boxed{\begin{aligned}\underline{v}^a &= -2c\,r_0^b\,\underline{\omega}^a_{\ b} \\ &= 2\left(\frac{\partial \underline{r}^a}{\partial t} + c\,\omega^a_{\ 0b}\,\underline{r}^b\right)\end{aligned}} \quad -(24)$$

Eq. (24) means that the orbital torsion can be expressed as the covariant derivative:

$$\underline{v}^a = 2\left(\frac{\partial \underline{r}^a}{\partial t} + c\,\omega^a_{\ 0b}\,\underline{r}^b\right) \quad -(25)$$

which is equal to:

$$\underline{v}^a = -2c\,r_0^b\,\underline{\omega}^a_{\ b}. \quad -(26)$$

The presence of the spin connection:

$$\omega^a_{\ \mu b} = \left(\omega^a_{\ 0b},\ -\underline{\omega}^a_{\ b}\right) \quad -(27)$$

means that $((1),(2))$ is moving with respect to the plane of the $(1, 2)$, or representation of the orbit is moving. To make this clearer the eq. (14) can be rewritten by defining:

$$r^a = r^a_{\ \mu}\,e^\mu \quad -(28)$$

$$e_\ \ = e^1 = 1,\quad e_y = e^2 = 1$$

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So:

$$\frac{1}{\sqrt{2}}\underbrace{\begin{bmatrix} \frac{X(t)}{a} - i\frac{Y(t)}{b} \\ \frac{X(t)}{a} + i\frac{Y(t)}{b} \end{bmatrix}}_{\text{moving}} = \frac{1}{\sqrt{2}}\begin{bmatrix} \frac{X(t)}{a} & -i\frac{Y(t)}{b} \\ \frac{X(t)}{a} & +i\frac{Y(t)}{b} \end{bmatrix}\underbrace{\begin{bmatrix} 1 \\ 1 \end{bmatrix}}_{\text{static}} \quad -(30)$$

This makes it clear that an elliptical orbit is:

$$\frac{X^2(t)}{a^2} + \frac{Y^2(t)}{b^2} = 1 \quad -(31)$$

i.e. X and Y of the mass m change with time as it orbits M is an ellipse.

With these definitions the field equations of the elliptical orbit may be deduced from:

$$D \wedge T := R \wedge q \quad -(32)$$

and

$$D \wedge \tilde{T} := \tilde{R} \wedge q \quad -(33)$$

and these are the equations of the ECE engineering model. These are four of them, and they are more general than both Einstein and Newton.
