198(2) : The Spin Connection of a Whirlpool Galaxy / Solar System

From eq. (6) of note 198(1) the orbital linear velocity of a star in a whirlpool galaxy is given by:

$$v^2 = \left(\frac{L}{mr^2 (1 + \omega c t_g)} \right)^2 \left(\left(\frac{dr}{d\theta} \right)^2 + r^2 \right) \quad -(1)$$

where ω is the magnitude of the spin connection, and where t_g is a constant time interval. From astronomy:

$$v \xrightarrow[r \to \infty]{} \text{constant} \quad -(2)$$

which is known as the velocity curve of the spiral galaxy, discovered about 1958. Eq. (2) cannot be explained by Einstein's general relativity, nor can it be explained by Newtonian dynamics.

From eq. (1):

$$\left(\frac{dr}{d\theta} \right)^2 + r^2 = A r^4 (1 + \omega c t_g)^2 \quad -(3)$$

where

$$A = \left(\frac{v m}{L} \right)^2 \quad -(4)$$

So

$$(1 + \omega c t_g)^2 = \frac{1}{A r^4} \left(\left(\frac{dr}{d\theta} \right)^2 + r^2 \right) \quad -(4)$$

$$1 + \omega c t_g = \frac{1}{A^{1/2} r^2} \left(\left(\frac{dr}{d\theta} \right)^2 + r^2 \right)^{1/2} \quad -(5)$$

$$\boxed{\omega = \frac{1}{c t_g} \left[\frac{1}{A^{1/2} r^2} \left(\left(\frac{dr}{d\theta} \right)^2 + r^2 \right)^{1/2} - 1 \right]} \quad -(6)$$

In eq. (6):

$$A \xrightarrow[r \to \infty]{} \text{constant.} \quad -(7)$$

For each type of spiral in note 197(2) there is a specific spin connection magnitude.

1) Hyperbolic Spiral

$$\left(\frac{dr}{d\theta}\right)^2 = \frac{r^4}{r_0^2} \quad -(8)$$

so from eqs. (6) and (8):

$$\omega = \frac{1}{ct_f}\left[\frac{1}{A^{1/2} r^2}\left(\frac{r^4}{r_0^2} + r^2\right)^{1/2} - 1\right]$$

$$\boxed{\omega = \frac{1}{ct_f}\left[\frac{1}{A^{1/2} r}\left(1 + \left(\frac{r}{r_0}\right)^2\right)^{1/2} - 1\right]} \quad -(9).$$

As $r \to \infty$:

$$\omega \to \frac{1}{ct_f}\left[\frac{1}{A^{1/2} r_0} - 1\right] \quad -(10)$$

which is a constant.

2) Logarithmic Spiral

$$r = r_0 e^{\beta\theta} \quad -(11)$$

$$\frac{dr}{d\theta} = \beta r. \quad -(12)$$

From eqs. (6) and (12):

$$\omega = \frac{1}{c t_g}\left[\frac{1}{A^{1/2}}\left(1+\beta^2\right)^{1/2} - 1\right] \quad (13)$$

where

$$A^{1/2} = \frac{v m}{L} \quad (14)$$

In the case of the logarithmic spiral:

$$\boxed{\omega = \frac{1}{c t_g}\left[\frac{L}{v m}\left(1+\beta^2\right)^{1/2} - 1\right]} \quad (15)$$

As $r \to \infty$, ω and v become constant.

3) <u>Archimedes Spiral</u>

This is:

$$r = a + b\theta \quad (16)$$

So

$$\left(\frac{dr}{d\theta}\right)^2 = b^2. \quad (17)$$

From eqs. (6) and (17):

$$\boxed{\omega = \frac{1}{c t_g}\left[\frac{1}{A^{1/2} r^2}\left(b^2 + r^2\right)^{1/2} - 1\right]} \quad (18)$$

As $r \to \infty$:

$$\omega \to \frac{1}{c t_g}\left[\frac{1}{A^{1/2} r} - 1\right] \quad (19)$$

So for the Archimedes spiral:

$$\omega \xrightarrow[r \to \infty]{} -1 \quad (20)$$

4) Fermat's Spiral

$$r = r_0 \theta^{1/2} \quad -(21)$$

So

$$\theta = \frac{r^2}{r_0^2} \quad -(22)$$

and

$$\frac{d\theta}{dr} = \frac{2r}{r_0^2} \quad -(23)$$

Therefore

$$\left(\frac{dr}{d\theta}\right)^2 = \frac{r_0^4}{4r^2} \quad -(24)$$

From eqs. (6) and (24):

$$\boxed{\omega = \frac{1}{ct_g}\left[\frac{1}{A^{1/2} r^2}\left(\frac{r_0^4}{4r^2} + r^2\right)^{1/2} - 1\right]} \quad -(25)$$

As $r \to \infty$:

$$\omega \to \frac{1}{ct_g}\left[\frac{1}{A^{1/2} r} - 1\right] \quad (26)$$

which is the same result as the Archimedes spiral.

5) The Lituus

$$\theta = \left(\frac{r_0}{r}\right)^2 \quad -(27)$$

so

$$\frac{d\theta}{dr} = -\frac{2r_0^2}{r^3} \quad -(28)$$

$$\left(\frac{dr}{d\theta}\right)^2 = \frac{r^6}{4r_0^4} \quad -(29)$$

5)

From eqs. (6) and (29):

$$\omega = \frac{1}{ct_g}\left[\frac{1}{A^{1/2} r^2}\left(\frac{r^6}{4r_0^4} + r^2\right)^{1/2} - 1\right]$$

$$\boxed{\omega = \frac{1}{ct_g}\left[\frac{1}{A^{1/2} r}\left(1 + \left(\frac{r}{r_0}\right)^4\right)^{1/2} - 1\right]} \quad -(30)$$

For the lituus, as $r \to \infty$, $\underline{\omega \to \infty}$.

<u>The Solar System</u>

$$r = \frac{\alpha}{1 + \epsilon\cos(x\theta)} \quad -(31)$$

so

$$\frac{dr}{d\theta} = \frac{\epsilon\alpha x \sin(x\theta)}{(1+\epsilon\cos(x\theta))^2} \quad -(32)$$

$$= \frac{\epsilon x}{\alpha} r^2 \sin(x\theta)$$

So

$$\left(\frac{dr}{d\theta}\right)^2 = \left(\frac{\epsilon x}{\alpha}\right)^2 r^4 \sin^2(x\theta) \quad -(33)$$

where

$$\sin^2(x\theta) + \cos^2(x\theta) = 1 \quad -(34)$$

and

$$\cos^2(x\theta) = \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2 \quad -(35)$$

6) Therefore:

$$\sin^2(x\theta) = 1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2 \quad -(36)$$

and

$$\left(\frac{dr}{d\theta}\right)^2 = \left(\frac{\epsilon x r^2}{\alpha}\right)^2\left[1 - \frac{1}{\epsilon^2}\left(\frac{\alpha}{r} - 1\right)^2\right] \quad -(37)$$

From eqs. (6) and (37):

$$\boxed{\omega = \frac{1}{ct_g}\left[\frac{1}{A^{1/2} r^2}\left[\left(\frac{\epsilon x r^2}{\alpha}\right)^2\left(1 - \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right)^2\right) + r^2\right]^{1/2} - 1\right]} \quad -(38)$$

where

$$A = \left(\frac{vm}{L}\right)^2 \quad -(39)$$

The spin connection can be expressed as:

$$\boxed{\omega = \frac{1}{ct_g}\left[\frac{1}{vmr}\left[1 + \left(\frac{\epsilon x r}{\alpha}\right)^2\left(1 - \frac{1}{\epsilon}\left(\frac{\alpha}{r} - 1\right)^2\right)\right]^{1/2} - 1\right]} \quad -(40)$$

So this theory describes all orbits in terms of ω and t_g.