198(1): Orbital Velocity for Various Orbits.

In cylindrical polar coordinates the orbital velocity is defined by:

$$\underline{v} = \frac{dr}{dt}\underline{e}_r + r\frac{d\theta}{dt}\underline{e}_\theta \quad -(1)$$

$$= \frac{d\theta}{dt}\left[\frac{dr}{d\theta}\underline{e}_r + r\,\underline{e}_\theta\right] \quad -(2)$$

In note 196(8) the constant angular momentum was calculated to be:

$$L = mr^2(1+\omega_0 c t_f)\frac{d\theta}{dt} \quad -(3)$$

where ω_0 is the spin connection and t_f a characteristic time. So:

$$\frac{d\theta}{dt} = \frac{L}{mr^2(1+\omega_0 c t_f)} \quad -(4)$$

$$-(5)$$

and

$$\boxed{\underline{v} = \frac{L}{mr^2(1+\omega_0 c t_f)}\left[\frac{dr}{d\theta}\underline{e}_r + r\,\underline{e}_\theta\right]}$$

The square of $\underline{v}$ is:

$$v^2 = \left(\frac{L}{mr^2(1+\omega_0 c t_f)}\right)^2\left(\left(\frac{dr}{d\theta}\right)^2 + r^2\right) \quad -(6)$$

In a whirlpool galaxy, v is constant as

2)

As r becomes infinite in eq. (6):

$$v \xrightarrow[r \to \infty]{} \frac{L}{mr^2(1+\omega c t_f)} \frac{dr}{d\theta} \quad -(7)$$

$$= \text{constant}$$

It is seen that this result is given by the hyperbolic spiral:

$$\boxed{\theta = \frac{r_0}{r}} \quad -(8)$$

so

$$\frac{d\theta}{dr} = -\frac{r_0}{r^2}, \quad \frac{dr}{d\theta} = -\frac{r^2}{r_0} \quad -(9)$$

and

$$\boxed{v \to \frac{L}{m r_0 (1+\omega c t_f)}} \quad -(10).$$

It is not given by any other type of spiral.

The hyperbolic spiral (8) is given by the non-linear rotational Hooke law:

$$Tq = k\theta^2 \quad -(11)$$

coming from a basic spacetime torsion. The torque is:

$$Tq = \omega L = m\omega r^2 \left(1+\omega_0 c t_f\right) \frac{d\theta}{dt}$$

$$= L \frac{d\theta}{dt} = \frac{L^2}{mr^2(1+\omega_0 c t_f)} \quad -(12)$$

$$= k\theta^2 \quad -(13)$$

$$\theta^2 = \left(\frac{L^2}{k(1+\omega_0 c t_f)} \right) \frac{1}{r^2}$$

3) Comparing eqs. (8) and (13):

$$r_0 = \left(\frac{L^2}{mk(1+\omega_0 c t_g)} \right)^{1/2} \quad -(14)$$

The rotational Hooke law (11) gives the hyperbolic spiral (8), which gives the velocity curve of the spiral galaxy. The hyperbolic spiral of stars observed in a whirlpool galaxy is consistent with the observed velocity curve.

The force law is given by:

$$F(r) = -\frac{L^2}{mr^2}\left[\left(1-\frac{ct_g}{r}\right)^{-1}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)+\frac{1}{r}\right) + \frac{1}{2}ct_g\left(1-\frac{ct_g}{r}\right)^{-2}\left(\frac{1}{r^2}+\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2\right)\right] \quad -(15)$$

with:

$$\frac{1}{r} = \frac{1}{r_0}\theta \quad -(16)$$

$$\frac{d}{d\theta}\left(\frac{1}{r}\right) = \frac{1}{r_0} \quad -(17)$$

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) = 0 \quad -(18)$$