197(2): Relativistic Force Law for Various Orbits

If it is assumed that the connection is:

$$\omega = -\frac{1}{r}, \quad \frac{d\omega}{dr} = \frac{1}{r^2} \quad -(1)$$

then:

$$F(r) = -\frac{L^2}{mr^2}\left[\left(1-\frac{ct_g}{r}\right)^{-1}\left(\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right)+\frac{1}{r}\right) + \frac{1}{2}ct_g\left(1-\frac{ct_g}{r}\right)^{-2}\left(\frac{1}{r^2}+\left(\frac{d}{d\theta}\left(\frac{1}{r}\right)\right)^2\right)\right] \quad -(2)$$

This can be worked out for various orbits as follows by computer.

1) Precessing Ellipse

$$\frac{1}{r} = \frac{1}{\alpha}\left(1+\epsilon\cos(x\theta)\right) \quad -(3)$$

2) Binary Pulsar

$$\frac{1}{r} = \frac{1}{\alpha}e^{\beta\theta}\left(1+\epsilon\cos(x\theta)\right) \quad -(4)$$

3) Hyperbolic Spiral

$$\frac{1}{r} = \frac{1}{a}\theta \quad -(5)$$

4) Logarithmic Spiral

$$\frac{1}{r} = \frac{1}{r_0}\exp(-\beta\theta) \quad -(6)$$

2)

5) <u>Archimedes Spiral</u>

$$r = a + b\theta \quad - (7)$$

6) <u>Fermat's Spiral</u>

$$\frac{1}{r} = \frac{1}{\theta^{1/2}} \quad - (8)$$

7) <u>The Lituus</u> $\frac{1}{r} = \theta^{1/2} \quad - (9)$

8) <u>Euler's Spiral</u>

$$B(r) = S(r) + i\,C(r) \quad - (10)$$

where

$$C(r) = \int_0^r \cos\left(\frac{1}{2}\pi x^2\right) dx \quad - (11)$$

$$S(r) = \int_0^r \sin\left(\frac{1}{2}\pi x^2\right) dx \quad - (12)$$

are ~~Fermat~~ Fresnel's integrals.

9) <u>The torsional Spiral</u>

$$T = -k\theta \quad - (13)$$

where k is a constant. This is <u>the angular Hooke</u> Law. The energy of (13) is:

$$\bar{U} = \frac{1}{2} k \theta^2 \quad -(14)$$

It is seen immediately from eq. (13) that the reason for a spiral is torsion. The angular Hooke law is a kind of Archimedes spiral, with

$$a = 0, \quad b = -k. \quad -(15)$$

In the spiral itself, r is proportional to θ, in the angular Hooke law, torsion is proportional to θ.

Therefore the precessing ellipse is due to the torsion:

$$T = \frac{-k\alpha}{1 + \epsilon \cos(x\theta)} \quad -(16)$$
