197(3): The Role of the Rotational Hooke Law in the Theory of Orbits.

In the linear approximation the rotational Hooke law is:

$$T_q = -k\theta \qquad (1)$$

where T_q is torque magnitude and θ is angle. The first question is what kind of orbit does eqn (1) give. If the lagrangian is assumed to be:

$$\mathcal{L} = \frac{1}{2}m\left(\dot{r}^2 + r^2\dot{\theta}^2\right) - U(r) \qquad (2)$$

the angular momentum is:

$$L = mr^2 \frac{d\theta}{dt} \qquad (3)$$

and is a constant.

As in UFT 119 the torque may be defined as:

$$T_q = \omega L \qquad (4)$$

where

$$\omega = \frac{d\theta}{dt} \qquad (5)$$

Therefore:

$$\boxed{T_q = L\frac{d\theta}{dt} = \frac{L^2}{mr^2}} \qquad (6)$$

The orbit given by Hooke's rotational law is therefore:

$$\frac{L^2}{mr^2} = -k\theta \qquad (7)$$

2) which is the spiral:

$$\theta = -\left(\frac{L^2}{mk}\right)\frac{1}{r^2} \quad -(8)$$

The angular momentum is related to torsion as in UFT 124, so the torque is defined by the torsion. So torsion gives a spiral orbit if the Hooke law, eq. (1), is accepted.

For a Newtonian orbit:

$$\frac{1}{r^2} = \frac{1}{d^2}\left(1 + \epsilon\cos\theta\right)^2 \quad -(9)$$

so the torque from eq. (6) is:

$$Tq = \frac{L^2}{md^2}\left(1 + \epsilon\cos\theta\right)^2 \quad -(10)$$

and is not given by the simple Hooke law, eq. (1), but the orbit can be interpreted as a non linear rotational Hooke law. For small θ:

$$Tq = \frac{L^2}{md^2}\left(1 + \epsilon\left(1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \dots\right)\right)^2 \quad -(11)$$

For a nearly circular orbit:

$$\epsilon \sim 1 \quad -(12)$$

3)

$$T_q \sim \frac{L^2}{m d^2}\left(2 - \frac{\theta^2}{2}\right)^2 \quad -(13)$$

$$\sim \frac{L^2}{m d^2}\left(2 - \theta^2 + \frac{\theta^4}{4}\right) \quad -(14)$$

which is :

$$T_q \sim \frac{2L^2}{m d^2} - \frac{L^2\theta^2}{m d^2} \quad -(15)$$

i.e. a non linear Hooke law.

So orbits can be understood in terms of a rotational Hooke law.