The force law is calculated from the lagrangian:

$$\mathcal{L} = \frac{1}{2} m \left(\dot{r}^2 + r^2 \dot{\theta}^2 \right) \left(1 + \omega c t_g \right) - U(r) \quad -(1)$$

and the Euler Lagrange equations:

$$\frac{\partial \mathcal{L}}{\partial r} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{r}} \quad -(2)$$

$$\frac{\partial \mathcal{L}}{\partial \theta} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{\theta}} \quad -(3)$$

Assume that the spin connection is a function of r, but not of θ. Then:

$$\frac{\partial \mathcal{L}}{\partial r} = m r \dot{\theta}^2 \left(1 + \omega c t_g \right) + \frac{1}{2} m c t_g r \dot{\theta}^2 \left(2\omega + r \frac{\partial \omega}{\partial r} \right) - \frac{\partial U(r)}{\partial r} \quad -(4)$$

and

$$\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{r}} = m \ddot{r} \left(1 + \omega c t_g \right) \quad -(5)$$

So:

$$F(r) = -\frac{\partial U(r)}{\partial r} = \left(m \ddot{r} - m r \dot{\theta}^2 \right) \left(1 + \omega c t_g \right) - \frac{1}{2} m c t_g r \dot{\theta}^2 \left(2\omega + r \frac{\partial \omega}{\partial r} \right) \quad -(6)$$

From eq. (3):

$$L = m r^2 \dot{\theta} = \text{constant} \quad -(7)$$

where L is the total angular momentum.

Therefore:

$$\ddot{r} = -\frac{L^2}{m^2} u^2 \frac{d^2 u}{d\theta^2}, \quad r \dot{\theta}^2 = \frac{L^2}{m^2} u^3 \quad -(8)$$