196(4) : Calculation of the Precession Constant x

The precession constant x is calculated from:

$$\frac{1}{r} + \omega = \frac{\alpha}{r}\left(\frac{x^2}{\alpha} + \frac{1-x^2}{r}\right) \quad -(1)$$

of note 196(3), so:

$$\frac{\alpha}{r}\left(\frac{rx^2 + \alpha(1-x^2)}{\alpha r}\right) = \frac{1+\omega r}{r} \quad -(2)$$

and

$$rx^2 + \alpha(1-x^2) = r(1+\omega r), \quad -(3)$$

$$(r-\alpha)x^2 = r(1+\omega r) - \alpha,$$

$$x^2 = \frac{r(1+\omega r) - \alpha}{r-\alpha}$$

i.e

$$\boxed{x^2 = 1 + \frac{\omega r^2}{r-\alpha}} \quad -(4)$$

and the orbit is:

$$r = \frac{\alpha}{1+\epsilon\cos(x\theta)} \quad -(5)$$

In eq. (4), the quantities x, r and α can be determined by astronomy, so ω can be found experimentally.

The force law is:

$$F = -\frac{mMG}{r}\left(\frac{1}{r} + \omega\right) \quad -(6)$$

2) and in the classical limit:

$$\omega \rightarrow 0 \quad -(7)$$

so the spacetime becomes flat and:

$$F \rightarrow -mMG / r^2 \quad -(8)$$

In the limit (7), eq. (4) gives:

$$x^2 \rightarrow 1 \quad -(9)$$

and the orbit is the static ellipse:

$$r = \frac{\alpha}{1 + \epsilon \cos\theta} \quad -(10)$$

The spin connection magnitude is given by:

$$\omega = -\frac{1}{r} + \frac{\alpha}{r}\left(\frac{x^2}{\alpha} + \frac{1 - x^2}{r}\right) \quad -(11)$$

From eq. (11):

$$\omega \rightarrow 0 \quad \text{as} \quad x \rightarrow 1 \quad -(12)$$

So tables of spin connections can be drawn up for any object of mass m orbiting an object of mass M. This method can be used for any kind of orbit in cosmology.