Rule (3): General Expression for Spiral Curves

The two expressions for force are:

$$F = -\frac{L^2}{mr^3}\left(\frac{x^2}{\alpha} + \frac{1-x^2}{r}\right) \quad -(1)$$

from UFT 194, and:

$$F = -\frac{mMG}{r}\left(\frac{1}{r} + \omega\right) \quad -(2)$$

from note 196(2). So in general:

$$\frac{mMG}{r}\left(\frac{1}{r} + \omega\right) = \frac{L^2}{mr^3}\left(\frac{x^2}{\alpha} + \frac{1-x^2}{r}\right).$$

i.e $$\frac{1}{r} + \omega = \frac{\alpha}{r}\left(\frac{x^2}{\alpha} + \frac{1-x^2}{r}\right) \quad -(3)$$

where $$\alpha = \frac{L^2}{m^2 MG} \quad -(4)$$

So:

$$\boxed{\omega = -\frac{1}{r} + \frac{\alpha}{r}\left(\frac{x^2}{\alpha} + \frac{1-x^2}{r}\right)} \quad -(5)$$

for a precessing ellipse.

In the solar system:

2)

$$x = 1 \quad -(6)$$

to an excellent approximation. So for a static ellipse:

$$\omega \to 0 \quad -(7)$$

Astronomical measurements show that the orbit of objects of mass m in the solar system is a precessing ellipse.

Conclusion

The precession of the perihelion is caused by the spin connection.

This is a very simple first theory which can be developed in many ways. The gravitational potential has been chosen to be:

$$\phi = -\frac{MG}{r} \quad -(8)$$

to give the Newton / Hooke law in the limit of $x = 1$:

$$F = -\frac{mMG}{r^2} \quad -(9)$$