3) It can be seen that the two expressions (8) and (10) are similar, but eq. (8) uses only the experimental observation (1).

The most elegant approach is the lagrangian. For

$$x = 1 \quad - (11)$$

then

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \alpha, \quad - (12)$$

i.e

$$\frac{d^2 u}{d\theta^2} + u = \alpha \quad - (13)$$

with

$$u = 1/r. \quad - (14)$$

So

$$\frac{du}{dr} = -\frac{1}{r^2}, \quad \frac{d^2 u}{dr^2} = \frac{2}{r^3} \quad - (15)$$

and

$$u = \frac{1}{\alpha}\left(1 + \epsilon\cos(x\theta)\right) \quad - (16)$$

so

$$\frac{du}{d\theta} = -\frac{x\epsilon}{\alpha}\sin(x\theta)$$

$$= -\frac{x\epsilon}{\alpha}\left(1 - \frac{1}{\epsilon^2}\left(\alpha u - \frac{1}{\cancel{\alpha}}\right)^2\right)^{1/2}$$

$$\cancel{= -\frac{x\epsilon}{\alpha}\left(\frac{\epsilon^2\alpha^2 - (\alpha u - 1)^2}{\epsilon^2\alpha^2}\right)^{1/2}}$$

$$= -\frac{x}{\alpha^2}\left(\epsilon^2 \cancel{\alpha^2} - (\alpha u - 1)^2\right)^{1/2} \quad - (16)$$

So

$$\frac{d\theta}{du} = -\frac{\alpha^2}{x}\left(\epsilon^2\alpha^2 - (\alpha u - 1)^2\right)^{-1/2} \quad -(17)$$

and:

$$\boxed{\Delta\theta = -\frac{2\alpha^2}{x}\int_{1/R_0}^{0}\left(\epsilon^2\alpha^2 - (\alpha u - 1)^2\right)^{-1/2} du \; - \pi} \quad -(18)$$

If the path of the photon of mass m is assumed to be a parabola then

$$\epsilon = 1. \quad -(19)$$

In the solar system:

$$x = 1 \quad -(20)$$

to an excellent approximation, so:

$$\boxed{\Delta\theta = -2\alpha^2\int_{1/R_0}^{0}\left(1\alpha^2 - (\alpha u - 1)^2\right)^{-1/2} du \; - \pi} \quad -(21)$$

Therefore α may be found from the experimental $\Delta\theta$.