193(8): The Parabola in Polar and Cartesian Coordinates

In polar coordinates the parabola is:

$$r = \frac{\alpha}{1 + \cos\theta} \quad -(1)$$

where

$$r = (x^2 + y^2)^{1/2}, \quad \cos\theta = \frac{X}{r} \quad -(2)$$

So:

$$1 + \cos\theta = \frac{\alpha}{r} = 1 + \frac{X}{(x^2+y^2)^{1/2}} = \frac{\alpha}{(x^2+y^2)^{1/2}} \quad -(3)$$

So:

$$(x^2 + y^2)^{1/2} + X = \alpha \quad -(4)$$

$$x^2 + y^2 = (\alpha - X)^2$$

$$= \alpha^2 - 2\alpha X + X^2$$

and

$$\boxed{\alpha^2 - 2\alpha X = y^2} \quad -(5)$$

A plot of eq. (5) will give the meaning of α in Cartesian coordinates.

The ellipse is:

$$r = \frac{\alpha}{1 + \epsilon\cos\theta} \quad -(6)$$

so

$$1 + \epsilon\cos\theta = \frac{\alpha}{r} \quad -(7)$$

$$1 + \epsilon\frac{X}{(x^2+y^2)^{1/2}} = \frac{\alpha}{(x^2+y^2)^{1/2}} \quad -(8)$$

2)

i.e.

$$\left(X^2+Y^2\right)^{1/2} + \epsilon X = \alpha \quad -(9)$$

$$X^2+Y^2 = \left(\alpha - \epsilon X\right)^2$$

$$= \alpha^2 - 2\epsilon X\alpha + \epsilon^2 X^2 \quad -(10)$$

$$\boxed{\alpha^2 - 2\epsilon X\alpha = X^2\left(1-\epsilon^2\right) + Y^2} \quad -(11)$$

I.e. of Newtonian ellipse:

$$\alpha = \frac{L^2}{m^2 M G} \quad -(12)$$

so if light is trapped in an elliptical orbit around a mass M, its mass m can be found.