1) Note 188(3): Two Examples

Consider
$$\partial_\nu g_{\rho\mu} = -\left(\Gamma^\lambda_{\rho\nu} g_{\lambda\mu} + \Gamma^\lambda_{\mu\nu} g_{\lambda\rho}\right) \quad -(1)$$

1) For diagonal metrics
$$\rho = \mu = \lambda \quad -(2)$$

For example:
$$\rho = \mu = \lambda = 0 \quad -(3)$$

then
$$\partial_\nu g_{00} = -\left(\Gamma^0_{0\nu} g_{00} + \Gamma^0_{0\nu} g_{00}\right) - (4)$$

i.e.
$$\boxed{\Gamma^0_{\nu 0} = \frac{1}{2 g_{00}} \partial_\nu g_{00} \quad \text{with} \quad \nu \neq 0} \quad -(5)$$

because if $\nu = 0$ the connection is symmetric, and eq. (1) has already eliminated symmetric connections, i.e. is valid only for antisymmetric connections.

2) General Metrics

For example:
$$\nu = 0, \rho = 1, \mu = 2 \quad -(6)$$

then:

2)
$$\partial_0 g_{12} = -\left(\Gamma^{\lambda}_{10} g_{\lambda 2} + \Gamma^{\lambda}_{20} g_{\lambda 1}\right) \quad -(7)$$
$$= -\left(\Gamma^{0}_{10} g_{02} + \Gamma^{1}_{10} g_{12} + \Gamma^{3}_{10} g_{32} + \Gamma^{0}_{20} g_{01} + \Gamma^{2}_{20} g_{21} + \Gamma^{3}_{20} g_{31}\right)$$

and so on.

The diagonal metric is :

$$g_{\mu\nu} = \begin{bmatrix} g_{00} & 0 & 0 & 0 \\ 0 & g_{11} & 0 & 0 \\ 0 & 0 & g_{22} & 0 \\ 0 & 0 & 0 & g_{33} \end{bmatrix} \quad -(8)$$

and has only four elements.

The general metric has <u>ten</u> <u>independent</u> <u>elements</u>

because
$$g_{12} = g_{21} \text{ etc.} \quad -(9)$$

so needs ten independent equations.
