1) 188(5): Some Calculations of Antisymmetric Connection for the General Metric

The starting equation is:

$$\partial_\nu g_{\mu\rho} = -\left(\Gamma^\lambda_{\rho\nu} g_{\lambda\mu} + \Gamma^\lambda_{\mu\nu} g_{\lambda\rho}\right) \quad -(1)$$

$$\nu \neq \rho, \quad \nu \neq \mu. \quad -(2)$$

Let

$$\nu = 0 \quad -(2)$$

then

$$\partial_0 g_{\mu\rho} = -\left(\Gamma^\lambda_{\rho 0} g_{\lambda\mu} + \Gamma^\lambda_{\mu 0} g_{\lambda\rho}\right). \quad -(3)$$

Let

$$\mu = 1, \rho = 1 \quad -(4)$$

then

$$\partial_0 g_{11} = -\left(\Gamma^\lambda_{10} g_{\lambda 1} + \Gamma^\lambda_{10} g_{\lambda 1}\right). \quad -(5)$$

Let

$$\mu = 1, \rho = 2 \quad -(6)$$

then

$$\partial_0 g_{12} = -\left(\Gamma^\lambda_{20} g_{\lambda 1} + \Gamma^\lambda_{10} g_{\lambda 2}\right). \quad -(7)$$

Let

$$\mu = 1, \rho = 3 \quad -(8)$$

then

$$\partial_0 g_{13} = -\left(\Gamma^\lambda_{30} g_{\lambda 1} + \Gamma^\lambda_{10} g_{\lambda 3}\right) \quad -(9)$$

Let

$$\mu = 2, \rho = 1$$

then

$$\partial_0 g_{21} = -\left(\Gamma^\lambda_{10} g_{\lambda 2} + \Gamma^\lambda_{20} g_{\lambda 1}\right) \quad -(10)$$

From eqs. (7) and (10):

$$\Gamma^\lambda_{20} g_{\lambda 1} + \Gamma^\lambda_{10} g_{\lambda 2} = \Gamma^\lambda_{10} g_{\lambda 2} + \Gamma^\lambda_{20} g_{\lambda 1} \quad \checkmark\checkmark \quad -(11)$$

This is self consistent.

2) ~~From~~ Let $\mu = 2, \rho = 2$ — (12)

then $\partial_0 g_{22} = -\left(\Gamma^{\lambda}_{20} g_{\lambda 2} + \Gamma^{\lambda}_{20} g_{22}\right)$ — (13)

Let $\mu = 2, \rho = 3$ — (14)

then $\partial_0 g_{23} = -\left(\Gamma^{\lambda}_{30} g_{\lambda 3} + \Gamma^{\lambda}_{20} g_{\lambda 3}\right)$ — (15)

Let $\mu = 3, \rho = 1$ — (16)

then $\partial_0 g_{31} = -\left(\Gamma^{\lambda}_{10} g_{\lambda 3} + \Gamma^{\lambda}_{30} g_{\lambda 1}\right)$ — (17)

From eqs. (9) and (17)

$$\Gamma^{\lambda}_{30} g_{\lambda 1} + \Gamma^{\lambda}_{10} g_{\lambda 3} = \Gamma^{\lambda}_{10} g_{\lambda 3} + \Gamma^{\lambda}_{30} g_{\lambda 1} \quad (18)$$

This is self-consistent

Let $\mu = 3, \rho = 2$ — (19)

then $\partial_0 g_{32} = -\left(\Gamma^{\lambda}_{20} g_{\lambda 3} + \Gamma^{\lambda}_{30} g_{\lambda 2}\right)$ — (20)

From eqs. (15) and (20):

$$\Gamma^{\lambda}_{30} g_{\lambda 3} + \Gamma^{\lambda}_{20} g_{\lambda 3} = \Gamma^{\lambda}_{20} g_{\lambda 3} + \Gamma^{\lambda}_{30} g_{\lambda 2} \quad (21)$$

This is self-consistent.

Let $\mu = 3, \rho = 3$ — (22)

then $\partial_0 g_{33} = -\left(\Gamma^{\lambda}_{30} g_{\lambda 3} + \Gamma^{\lambda}_{30} g_{\lambda 3}\right)$ — (23)

Similarly all equations are self consistent for

$\nu = 1, 2, 3$ — (24)