APPENDIX TO UFT 188: THEOREM OF THE ANTISYMMETRIC CONNECTION.

Consider three metric compatibility conditions in cyclic permutation:

$$\partial_\rho g_{\mu\nu} - \Gamma^\lambda_{\rho\mu} g_{\lambda\nu} - \Gamma^\lambda_{\rho\nu} g_{\mu\lambda} = 0 \quad -(A1)$$

$$\partial_\mu g_{\nu\rho} - \Gamma^\lambda_{\mu\nu} g_{\lambda\rho} - \Gamma^\lambda_{\mu\rho} g_{\nu\lambda} = 0 \quad -(A2)$$

$$\partial_\nu g_{\rho\mu} - \Gamma^\lambda_{\nu\rho} g_{\lambda\mu} - \Gamma^\lambda_{\nu\mu} g_{\rho\lambda} = 0. \quad -(A3)$$

Subtract Eqs. (2) and (3) from Eq. (1):

$$\partial_\rho g_{\mu\nu} - \partial_\mu g_{\nu\rho} - \partial_\nu g_{\rho\mu} - \Gamma^\lambda_{\rho\mu} g_{\lambda\nu} - \Gamma^\lambda_{\rho\nu} g_{\mu\lambda}$$
$$+ \Gamma^\lambda_{\mu\nu} g_{\lambda\rho} + \Gamma^\lambda_{\mu\rho} g_{\nu\lambda} + \Gamma^\lambda_{\nu\rho} g_{\lambda\mu} + \Gamma^\lambda_{\nu\mu} g_{\rho\lambda} = 0 \quad -(A4)$$

Subtract Eqs. (1) and (2) from Eq. (3):

$$\partial_\nu g_{\rho\mu} - \partial_\mu g_{\nu\rho} - \partial_\rho g_{\mu\nu} - \Gamma^\lambda_{\nu\rho} g_{\lambda\mu} - \Gamma^\lambda_{\nu\mu} g_{\rho\lambda}$$
$$+ \Gamma^\lambda_{\mu\nu} g_{\lambda\rho} + \Gamma^\lambda_{\mu\rho} g_{\nu\lambda} + \Gamma^\lambda_{\rho\mu} g_{\lambda\nu} + \Gamma^\lambda_{\rho\nu} g_{\mu\lambda} = 0 \quad -(A5)$$

Subtract Eqs. (1) and (3) from Eq. (2):

$$\partial_\mu g_{\nu\rho} - \partial_\rho g_{\mu\nu} - \partial_\nu g_{\rho\mu} - \Gamma^\lambda_{\mu\nu} g_{\lambda\rho} - \Gamma^\lambda_{\mu\rho} g_{\nu\lambda}$$
$$+ \Gamma^\lambda_{\rho\mu} g_{\lambda\nu} + \Gamma^\lambda_{\rho\nu} g_{\mu\lambda} + \Gamma^\lambda_{\nu\rho} g_{\lambda\mu} + \Gamma^\lambda_{\nu\mu} g_{\rho\lambda} = 0 \quad -(A6)$$

Now apply antisymetry:

$$\Gamma^\lambda_{\mu\nu} = -\Gamma^\lambda_{\nu\mu} \quad -(A7)$$

to obtain:

$$\partial_\rho g_{\mu\nu} - \partial_\mu g_{\nu\rho} - \partial_\nu g_{\rho\mu} = 2\left(\Gamma^{\lambda}_{\rho\mu} g_{\lambda\nu} + \Gamma^{\lambda}_{\rho\nu} g_{\lambda\mu}\right) \quad -(A8)$$

$$\partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} - \partial_\mu g_{\nu\rho} = 2\left(\Gamma^{\lambda}_{\nu\rho} g_{\lambda\mu} + \Gamma^{\lambda}_{\nu\mu} g_{\lambda\rho}\right) \quad -(A9)$$

$$\partial_\mu g_{\nu\rho} - \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu} = 2\left(\Gamma^{\lambda}_{\mu\nu} g_{\lambda\rho} + \Gamma^{\lambda}_{\mu\rho} g_{\lambda\nu}\right). \quad -(A10)$$

Add Eqs. (8) and (10):

$$\partial_\nu g_{\rho\mu} = -\left(\Gamma^{\lambda}_{\rho\nu} g_{\lambda\mu} + \Gamma^{\lambda}_{\mu\nu} g_{\lambda\rho}\right) \quad -(A11)$$

This equation relates the general metric to the antisymmetric connection.

For a diagonal metric:

$$\rho = \mu \quad -(A12)$$

so

$$\Gamma^{\mu}_{\nu\mu} = \frac{1}{2 g_{\mu\mu}} \partial_\nu g_{\mu\mu} \quad -(A13)$$

$$\nu \neq \mu \quad -(A14)$$

This result is given the appellation "Theorem of the Antisymmetric Connection".