1)

187(7): Vanishing Curvature for the Metric of Note 187(6)

The self consistent metric derived in note 187(6) is:

$$g_{00} = 1 - \frac{r_0}{r} + \frac{1}{f(r)} \quad -(1)$$

$$g_{11} = -\left(1 - \frac{r_0}{r} + \frac{1}{f(r)}\right)^{-1} \quad -(2)$$

$$g_{22} = -r^2 \quad -(3)$$

$$g_{33} = -r^2 \sin^2\phi . \quad -(4)$$

From eq. (1) for example:

$$\Gamma^0_{10} = \frac{1}{2\left(1 - \frac{r_0}{r} + f(r)\right)} \frac{\partial}{\partial r}\left(1 - \frac{r_0}{r} + \frac{1}{f(r)}\right), \quad -(5)$$

Then as in note 186(5):

$$R^0{}_{0\mu 1} = \partial_\mu \Gamma^0_{10} - \partial_1 \Gamma^0_{\mu 0} + \Gamma^0_{\mu\lambda}\Gamma^\lambda_{10} - \Gamma^0_{1\lambda}\Gamma^\lambda_{\mu 0} . \quad -(6)$$

The only possibility is:

$$\mu = 1 \quad -(7)$$

in which case:

$$\boxed{R^0{}_{011} = 0 .} \quad -(8)$$

For the metric of a spherically symmetric spacetime:

$$g_{00} = e^{2\alpha}; \; g_{11} = -e^{2\beta}, \; g_{22} = -r^2,$$

$$g_{33} = -r^2 \sin^2\phi \quad -(9)$$

2) In general α and β are functions of r and t, but almost all known orbits are described by

$$\alpha = \alpha(r), \quad \beta = \beta(r) \quad - (10)$$

The metric compatibility equation:

$$\partial_\rho g_{\mu\nu} - \Gamma^\lambda_{\rho\mu} g_{\lambda\nu} - \Gamma^\lambda_{\rho\nu} g_{\mu\lambda} = 0 \quad - (11)$$

then produces, for g_{00}:

$$\Gamma^0_{10} = \frac{1}{2g_{00}} \partial_1 g_{00} \quad - (12)$$

in which case: $R^0{}_{011} = 0 \quad - (13)$

Similarly all curvature elements vanish for a metric of type (9).

If however:

$$\alpha = \alpha(r,t), \quad \beta = \beta(r,t) \quad - (14)$$

there are results such as:

$$\Gamma^1_{01} = \frac{1}{2g_{11}} \partial_0 g_{11} = f(r,t) \quad - (15)$$

and it is possible that elements of curvature exist such as:

$$R^0{}_{\mu 1} = \partial_\mu \Gamma^0_{10} - \partial_1 \Gamma^0_{\mu 0} + \Gamma^0_{\mu\lambda}\Gamma^\lambda_{10} - \Gamma^0_{1\lambda}\Gamma^\lambda_{\mu 0} \quad - (16)$$

when

$$\mu = 1 \quad - (17)$$

) then

$$R^{0}{}_{011} = 0, \quad -(18)$$

but when:

$$\mu = 0 \quad . \quad -(19)$$

then

$$R^{0}{}_{001} = \partial_0 \Gamma^{0}_{10} - \partial_1 \Gamma^{0}_{00} + \Gamma^{0}_{0\lambda}\Gamma^{\lambda}_{10} - \Gamma^{0}_{1\lambda}\Gamma^{\lambda}_{00} \quad -(20)$$

$$= \partial_0 \Gamma^{0}_{10} + \Gamma^{0}_{0\lambda}\Gamma^{\lambda}_{10}$$

$$= \partial_0 \Gamma^{0}_{10} + \Gamma^{0}_{01}\Gamma^{1}_{10} + \Gamma^{0}_{02}\Gamma^{2}_{10} + \Gamma^{0}_{03}\Gamma^{3}_{10}$$

$$= \partial_0 \Gamma^{0}_{10} - \left(\Gamma^{0}_{01}\right)^2$$

$$\neq 0$$

However, when α and β are functions only of r, the curvature elements all vanish, a result of major importance.

From eqs. (9), (10) and (12):

$$\Gamma^{0}_{10} = \frac{1}{2} e^{-2\alpha(r)} \frac{\partial}{\partial r} e^{2\alpha(r)} \quad -(21)$$

i.e.

$$\boxed{\Gamma^{0}_{10} = \frac{\partial \alpha(r)}{\partial r}} \quad -(22)$$

The only relevant connection for a spherically symmetric spacetime is eq. (22).

This is another very important result that very greatly simplifies the mathematics of cosmology.

The Evans identity adds the general constraint

$$\boxed{\frac{\partial^2 \alpha(r)}{\partial r^2} = 0} \quad -(23)$$

another result of great importance

Eq. (1) is an example with:

$$e^{2\alpha(r)} = 1 - \frac{r_0}{r} + \frac{1}{f(r)}, \quad -(24)$$

$$\boxed{\alpha(r) = \frac{1}{2}\log_e\left(1 - \frac{r_0}{r} + \frac{1}{f(r)}\right)} \quad -(25)$$

Computer Algebra

Following the method of UFT 108, it would be optimal for UFT 187 to compute orbits from precessing elliptical orbits using various function $f(r)$. This could produce both inward spiralling orbits of binary pulsars, or outwardly spiralling orbits such as that of the moon.