1)

187(63): Self Consistent Metric

As in previous notes the connection for the solar system is:

$$\Gamma^0_{10} = \frac{1}{2\left(1-\frac{r_0}{r}\right)}\frac{\partial}{\partial r}\left(1-\frac{r_0}{r}\right) \quad -(1)$$

$$= \frac{r_0}{2r^2\left(1-\frac{r_0}{r}\right)}$$

This must also be a solution of the Evans identity, which reduces to:

$$\partial_\mu \Gamma^{\kappa\mu\nu} = 0 \quad -(2)$$

where

$$\Gamma^{\kappa\mu\nu} = g^{\mu\alpha} g^{\nu\beta} \Gamma^{\kappa}_{\alpha\beta}. \quad -(3)$$

Consider a metric of the type:

$$g_{00} = 1-\frac{r_0}{r}, \quad g_{11} = -\left(1-\frac{r_0}{r}\right)^{-1} \quad -(4)$$

Then

$$\Gamma^{010} = g^{00} g^{11} \Gamma^0_{10} \quad -(5)$$

$$= -\Gamma^0_{10}$$

and eq. (2) reduces to:

$$\frac{\partial \Gamma^{010}}{\partial r} = -\frac{\partial \Gamma^0_{10}}{\partial r} = \frac{r_0}{r(r-r_0)^2} \quad -(6)$$

i.e.

$$\frac{r_0}{r^2} \rightarrow 0 \text{ if } r \gg r_0 \quad -(7)$$

2) Consider a metric of the type used in UFT 108 for a binary pulsar:

$$g_{00} = 1 - \frac{r_0}{r} + \frac{b}{r^2} \quad -(8)$$

then:

$$\Gamma^0_{10} = \frac{1}{2g_{00}} \frac{\partial}{\partial r} g_{00} = \frac{1}{2}\left(\frac{r_0 - \frac{2b}{r}}{r^2 - r_0 r + b}\right) -(9)$$

The Evans identity means that:

$$\frac{\partial \Gamma^0_{10}}{\partial r} = \frac{1}{2}\left[\frac{\frac{2b}{r^2}\left(r^2 - r_0 r + b\right) - \left(2r - r_0\right)\left(r_0 - \frac{2b}{r}\right)}{\left(r^2 - r_0 r + b\right)^2}\right] = 0$$

$$\frac{2b^2}{r^2} + 2b\left(1 - \frac{r_0}{r}\right) + \frac{2b}{r}\left(2r - r_0\right) - \left(2r - r_0\right) r_0 = 0 \quad -(10)$$

$$\frac{2b^2}{r^2} + 2b\left(3 - \frac{2r_0}{r}\right) - \left(2r - r_0\right) r_0 = 0 \quad -(11)$$

$$2b^2 + 2br^2\left(3 - \frac{2r_0}{r}\right) - r^2 r_0\left(2r - r_0\right) = 0$$

This is a quadratic for b. In the limit:

$$r \gg r_0 \quad -(12)$$

$$2b^2 + 6br^2 - 2r^3 r_0 = 0 \quad -(13)$$

i.e.

$$b^2 + 3br^2 + r^3 r_0 = 0 \quad -(14)$$

$$b = \frac{1}{2}\left(-3r^2 \pm \left(9r^4 - 4r^3 r_0\right)^{1/2}\right) -(15)$$

i.e.

$$\boxed{b \rightarrow 0} \quad -(16)$$

because of eq. (12). This is self consistent because in UFT 108 b must be a very small constant

3)

but more accurately it is nearly constant.

From eq. (15)

$$b = \frac{1}{2}\left(-3r^2 + r\left(9r^2 - 4r\,r_0\right)^{1/2}\right)$$

$$b = \frac{1}{2}\left(-3r^2 + 3r^2\left(1 - \frac{4r_0}{9r}\right)^{1/2}\right) \quad -(17)$$

However b is defined as a constant, so eq. (16) is the only consistent solution.

Therefore:

$$\boxed{\left.\begin{array}{l} g_{00} = 1 - \frac{r_0}{r} + \frac{b}{r^2} \\ b \to 0 \end{array}\right\} \quad -(18)}$$

In general:

$$\Gamma^0_{10} = \frac{1}{2g_{00}} \frac{\partial g_{00}}{\partial r} \quad -(19)$$

and

$$\frac{\partial \Gamma^0_{10}}{\partial r} = \frac{1}{2}\left(\frac{1}{g_{00}} \frac{\partial^2 g_{00}}{\partial r^2} - \frac{1}{g_{00}^2}\left(\frac{\partial g_{00}}{\partial r}\right)^2\right) \quad -(20)$$

$$= 0$$

So

$$\boxed{\frac{\partial^2 g_{00}}{\partial r^2} = \frac{1}{g_{00}}\left(\frac{\partial g_{00}}{\partial r}\right)^2} \quad -(21)$$
