187(9): Checks on The Schwarzschild Metric

We have the results:

$$R^0{}_{101} = -\frac{r_0(4r-3r_0)}{4r^2(r-r_0)^2} \quad -(1)$$

$$R^3{}_{123} = -\frac{2\cos\phi}{r\sin\phi} \quad -(2)$$

$$R^3{}_{223} = 1 \quad -(3)$$

Compatibility with the identity means:

$$D_\mu T^{\kappa\mu\nu} := R^\kappa{}_\mu{}^{\mu\nu} \quad -(4)$$

i.e.

$$D_0 T^{\kappa 0\nu} + D_1 T^{\kappa 1\nu} + D_2 T^{\kappa 2\nu} + D_3 T^{\kappa 3\nu} \quad -(5)$$

$$:= R^\kappa{}_0{}^{0\nu} + R^\kappa{}_1{}^{1\nu} + R^\kappa{}_2{}^{2\nu} + R^\kappa{}_3{}^{3\nu} \quad -(6)$$

We have:

$$\Gamma^0{}_{10} = \frac{r_0}{2r^2\left(1-\frac{r_0}{r}\right)}, \quad \Gamma^2{}_{12} = \Gamma^3{}_{13} = \frac{1}{r},$$

$$\Gamma^3{}_{23} = \frac{\cos\phi}{\sin\phi} \quad -(7)$$

and

$$T^\kappa{}_{\mu\nu} = 2\Gamma^\kappa{}_{\mu\nu} \quad -(8)$$

For planar motion as in orbits all the terms in eq. (5) are relevant in general. From eq. (1):

$$\kappa = 0 \quad -(9)$$

2) and:

$$R^{0}{}_{1}{}^{10} = g^{11} g^{00} R^{0}{}_{110} = - R^{0}{}_{110} \quad -(10)$$

$$= - \frac{r_0 (4r - 3r_0)}{4r^2 (r - r_0)^2}.$$

There is no time dependence in any of the connections, so

$$D_1 T^{010} + D_2 T^{020} + D_3 T^{030} = R^{0}{}_{1}{}^{10} \quad -(11)$$

i.e. $$2\left(D_1 \Gamma^{010} + D_2 \Gamma^{020} + D_3 \Gamma^{030}\right) = R^{0}{}_{1}{}^{10}. \quad -(12)$$

By definition:

$$\Gamma^{010} = g^{11} g^{00} \Gamma^{0}{}_{10} = - \Gamma^{0}{}_{10} \quad -(13)$$

$$\Gamma^{020} = g^{22} g^{00} \Gamma^{0}{}_{20} = 0 \quad -(14)$$

$$\Gamma^{030} = g^{33} g^{00} \Gamma^{0}{}_{30} = 0 \quad -(15)$$

so $$-2 D_1 \Gamma^{0}{}_{10} = R^{0}{}_{1}{}^{10} \quad -(16)$$

i.e. $$-2 \partial_1 \Gamma^{0}{}_{10} = R^{0}{}_{1}{}^{10} \quad -(17)$$

The left hand side is:

$$-\frac{\partial}{\partial r}\left(\frac{r_0}{r^2\left(1 - \frac{r_0}{r}\right)}\right) = -\frac{\partial}{\partial r}\left(\frac{r_0}{r(r - r_0)}\right)$$

$$= \frac{(2r - r_0) r_0}{r^2 (r - r_0)^2} \neq R^{0}{}_{1}{}^{10} \quad -(18)$$

<u>So the Schwarzschild</u> metric is <u>incorrect</u>.