187(8): Self Consistent Solution, Recalculation.

For the "Schwarzschild" metric, it was found in UFT 186 that:

$$\Gamma^0_{10} = -\Gamma^0_{01} = \frac{r_0}{2r^2\left(1-\frac{r_0}{r}\right)} \quad -(1)$$

$$\Gamma^2_{12} = \Gamma^3_{13} = \frac{1}{r} \quad -(2)$$

$$\Gamma^3_{23} = \frac{\cos\phi}{\sin\phi} \quad -(3)$$

Now compute elements of the Riemann tensor:

$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho_{\nu\sigma} - \partial_\nu \Gamma^\rho_{\mu\sigma} + \Gamma^\rho_{\mu\lambda}\Gamma^\lambda_{\nu\sigma} - \Gamma^\rho_{\nu\lambda}\Gamma^\lambda_{\mu\sigma} \quad -(4)$$

with the above connections.

1) when $\rho = 0$, $\nu = 1$, $\sigma = 0$, $\mu = 0, 1, 2, 3$:

$$R^0{}_{001} = R^0{}_{011} = R^0{}_{021} = R^0{}_{031} = 0 \quad -(5)$$

2) when $\rho = 2$, $\nu = 1$, $\sigma = 2$, $\mu = 0, 1, 2, 3$:

$$R^2{}_{201} = R^2{}_{211} = R^2{}_{221} = R^2{}_{231} = 0 \quad -(6)$$

Similarly:

$$R^3{}_{302} = R^3{}_{312} = R^3{}_{322} = R^3{}_{332} = 0 \quad -(7)$$

and all elements vanish.

Therefore the identity:

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa}{}_{\mu}{}^{\mu\nu} \quad -(8)$$

reduces to

$$D_\mu T^{\kappa\mu\nu} = 0 \quad -(9)$$

The covariant derivative of the torsion tensor is:

$$\partial_\mu T^\kappa{}_{\nu\sigma} + \Gamma^\kappa_{\mu\lambda} T^\lambda{}_{\nu\sigma} - \Gamma^\lambda_{\mu\nu} T^\kappa{}_{\lambda\sigma} - \Gamma^\lambda_{\mu\sigma} T^\kappa{}_{\nu\lambda} = 0 \quad -(10)$$

2)

with:
$$T^{\lambda}_{\nu\sigma} = 2\Gamma^{\lambda}_{\nu\sigma} \quad -(11)$$

1) For:
$$\kappa = 0,\ \mu = 1,\ \lambda = 0,\ \nu = 1,\ \sigma = 0 \quad -(12)$$

$$\partial_1 \Gamma^0_{10} + \Gamma^0_{10}\Gamma^0_{10} - \Gamma^0_{11}\Gamma^0_{00} - \Gamma^0_{10}\Gamma^0_{10} = 0 \quad -(13)$$

i.e.
$$\partial_1 \Gamma^0_{10} = 0. \quad -(14)$$

In this case the identity (9) reduces to:
$$D_1 \Gamma^{010} = 0. \quad -(15)$$

Now use:
$$\Gamma^{010} = g^{11} g^{00} \Gamma^0_{10} \quad -(16)$$

and the result $D_1\left(g^{11} g^{00}\right) = 0 \quad -(17)$

from metric compatibility.

So:
$$D_1 \Gamma^0_{10} = 0 \quad -(18)$$

From eq. (14):
$$\boxed{\partial_1 \Gamma^0_{10} = 0} \quad -(19)$$

From eqs. (1) and (19):
$$\frac{\partial}{\partial r}\left(\frac{r_0}{2r^2\left(1 - \frac{r_0}{r}\right)}\right) = 0 \quad -(20)$$

i.e
$$\frac{1}{2}\frac{\partial}{\partial r}\left(\frac{r_0}{r(r - r_0)}\right) = 0 \quad -(21)$$

$$\frac{r_0}{r(r - r_0)^2} = 0 \quad -(22)$$

1) Since

$$r_0 = \frac{2mG}{c^2} \quad -(23)$$

the solution of eq. (22) is:

$$\boxed{r \to \infty.} \quad -(24)$$

The "Schwarzschild" metric is valid only if the distance r becomes infinite, and is a very restricted type of metric.

2) If in eq. (10):

$$\kappa = 2,\ \nu = 1,\ \sigma = 2,\ \mu = 1,\ \lambda = 2 \quad -(25)$$

then:

$$\partial_1 \Gamma^2_{12} + \Gamma^2_{12}\Gamma^2_{12} - \Gamma^2_{11}\Gamma^2_{22} - \Gamma^2_{12}\Gamma^2_{12} = 0. \quad -(26)$$

i.e

$$\partial_1 \Gamma^2_{12} = 0 \quad -(27)$$

or

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\right) = 0. \quad -(28)$$

The solution is again:

$$\boxed{r \to \infty} \quad -(29)$$

These results are consistent with the fact that the "Schwarzschild" metric is a vacuum metric.
