180(1): Comparison of the Metrical and Covariant Mass Methods of Gravitational Theory.

In the received opinion, the effect of gravitation was thought to be a change in the metric. The metric in the absence of gravitation was the Minkowski metric. Its line element is:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 \quad -(1)$$

using cylindrical polar coordinates for planar motion. The speed of light c is constant:

$$c^2 = \left(\frac{ds}{d\tau}\right)^2 = c^2\left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\phi}{d\tau}\right)^2 \quad -(2)$$

The hamiltonian was defined by:

$$H_0 = \frac{1}{2} m_0 c^2 \quad -(3)$$

where m_0 is the measured mass of a particle. The equation of motion of the free particle was:

$$H_0 = \frac{1}{2} m_0 c^2 = \frac{1}{2} m_0 \left(c^2\left(\frac{dt}{d\tau}\right)^2 - \left(\frac{dr}{d\tau}\right)^2 - r^2\left(\frac{d\phi}{d\tau}\right)^2\right) \quad -(4)$$

As shown in previous work, eq. (4) is the same as:

$$E^2 = p^2 c^2 + m_0^2 c^4, \quad -(5)$$

which is the Einstein energy equation. In eq. (5), E is the total relativistic energy, and $\underline{p}$ is the total relativistic momentum. The mass m_0 appearing in eq.

) (5) is the measured particle mass of the standard laboratories. In eq. (5):

$$E = \gamma m c^2 = m c^2 \frac{dt}{d\tau}, \quad -(6)$$

$$\underline{p} = \gamma m \underline{v}, \quad -(7)$$

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(8)$$

where $\underline{v}$ is the velocity of the particle in the laboratory frame, the frame of the observer. The total momentum is:

$$p^2 = p_\ell^2 + \frac{L^2}{r^2} \quad -(9)$$

where L is the angular momentum:

$$L = m_0 r^2 \frac{d\phi}{d\tau} \quad -(10)$$

and p_ℓ the linear momentum:

$$p_\ell = m_0 \frac{dr}{d\tau} \quad -(11)$$

From eq. (6):

$$\frac{dt}{d\tau} = \gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(12)$$

and

$$d\tau = \left(1 - \frac{v^2}{c^2}\right)^{1/2} dt. \quad -(13)$$

The infinitesimal of proper time $d\tau$ is shorter than the infinitesimal of time as seen by the observer, dt.

3) The proper time infinitesimal $d\tau$ is that in the moving frame, a frame that moves with the particle.

In the received opinion the effect of gravitation was to change ds^2. Usually the following ~~~~ infinitesimal line element was used:

$$ds^2 = c^2 d\tau^2 = \left(1 - \frac{r_0}{r}\right)c^2 dt^2 - \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 - r^2 d\phi^2 \quad -(14)$$

where

$$r_0 = \frac{2GM}{c^2} \quad -(15)$$

Here G is Newton's constant and M the mass of the object that attracts m_0 by gravitation. The hamiltonian in the received opinion remained the same in the presence of gravitation:

$$H_0 = \frac{1}{2} m_0 c^2 = \frac{1}{2} m_0 \left(\left(1 - \frac{r_0}{r}\right) c^2 \left(\frac{dt}{d\tau}\right)^2 - \left(1 - \frac{r_0}{r}\right)^{-1} \left(\frac{dr}{d\tau}\right)^2 - r^2 \left(\frac{d\phi}{d\tau}\right)^2 \right) \quad -(16)$$

This means that the classical idea of gravitational potential energy was replaced by geometry.

Eq. (16) can be written as:

$$E_1^2 = p_1^2 c^2 + m_0^2 c^4 \quad -(17)$$

where

$$E_1 = \left(1 - \frac{r_0}{r}\right) m_0 c^2 \frac{dt}{d\tau} \quad -(18)$$

and:
$$p_1^2 = p_{1r}^2 + \frac{L^2}{m_0 r^2} \quad -(19)$$
in which:
$$p_{1r} = \left(1 - \frac{r_0}{r}\right)^{-1} m_0 \frac{dr}{d\tau} \quad -(20)$$
$$L = m_0 r^2 \frac{d\phi}{d\tau} \quad -(21)$$
So by comparing eqs. (17) and (5), the received opinion asserts that:
$$\boxed{E^2 - c^2 p^2 = E_1^2 - c^2 p_1^2} \quad -(22)$$
and asserts that the mass m_0 does not change. For this reason the hamiltonian does not change. In the received opinion of metrical general relativity there is no potential energy and no force. Orbits are described by changes in geometry.

In the new view of gravitation, based on the ECE theory, spacetime is described by the ECE wave equation:
$$(\Box + R)\, q^a_\mu = 0 \quad -(23)$$
where q^a_μ is the Cartan tetrad. Eq. (23) is the most basic theorem of differential geometry, and is a re-arrangement of the tetrad postulate:
$$D_\mu q^a_\nu = 0 \quad -(24)$$
In the absence of gravitation, eq. (23) is the

5) free particle wave equation:

$$\left(\Box + \frac{m_0^2 c^2}{\hbar^2}\right) q^a_\mu = 0 \quad -(25)$$

of quantum mechanics. The latter is inbuilt into gravitational theory automatically. The received opinion failed to achieve this in a century of trying. Using the Schroedinger axiom:

$$\hat{p}^\mu = i\hbar\,\partial^\mu \quad -(26)$$

eq. (25) becomes eq. (5). Written out fully, eq. (26) is:

$$\hat{p}^\mu q^a_\mu = i\hbar\,\partial^\mu q^a_\mu \quad -(27)$$

where:

$$p^\mu = \left(\frac{E}{c}, \underline{p}\right), \quad \partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\underline{\nabla}\right), \quad -(28)$$

with

$$\Box = \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \nabla^2. \quad -(29)$$

The effect of gravitation is:

$$\boxed{\left(\frac{m_0 c}{\hbar}\right)^2 \longrightarrow R} \quad -(30)$$

where R can be expressed in terms of the covariant mass m:

$$\boxed{R := \left(\frac{mc}{\hbar}\right)^2} \quad -(31)$$

From eqs. (23) and (27):

6)

$$R = \frac{1}{c^2 \hbar^2} \left(E^2 - c^2 p^2 \right) \quad -(32)$$

in which

$$E = m_0 c^2 \frac{dt}{d\tau} = \gamma m_0 c^2, \quad -(33)$$

$$p^2 = \gamma^2 m_0^2 v^2 + \frac{L^2}{r^2} \quad -(34)$$

Using the Planck / de Broglie equation:

$$E = \hbar \omega, \quad \underline{p} = \hbar \underline{\kappa} \quad -(35)$$

it is found that:

$$\boxed{R = \frac{\omega^2}{c^2} - \kappa^2 = \kappa^\mu \kappa_\mu} \quad -(36)$$

where

$$\kappa^\mu = \left(\frac{\omega}{c}, \underline{\kappa} \right) \quad -(37)$$

$$\kappa_\mu = \left(\frac{\omega}{c}, -\underline{\kappa} \right) \quad -(38)$$

The covariant mass is

$$\boxed{m = \frac{\hbar}{c} \left(\kappa^\mu \kappa_\mu \right)^{1/2}} \quad -(39)$$

and the effect of gravitation on the Einstein energy equation is to change it to:

$$\boxed{E^2 = c^2 \left(p^2 + \hbar^2 R \right)} \quad -(40)$$

i.e.

$$\boxed{p^2 \longrightarrow p^2 + \hbar^2 R} \quad -(41)$$