180(3): Plane Wave Solution of the ECE Wave Equation

In general, it was shown in previous notes for UFT 180 that the ECE wave equation is:

$$\left(\square + \frac{\omega^2}{c^2} - \kappa^2 \right) q^a_\mu = 0 \quad -(1)$$

A solution of eq. (1) is the plane wave:

$$\boxed{q^a_\mu = q^a_\mu(0) \exp\left(-i(\omega t - \kappa Z)\right)} \quad -(2)$$

where:

$$\frac{\omega^2}{c^2} - \kappa^2 = \left(\frac{mc}{\hbar}\right)^2 \quad -(3)$$

in which m is the covariant mass.

In terms of Cartan geometry:

$$R = q^\nu_a \partial^\mu \Omega^a_{\mu\nu} = \frac{\omega^2}{c^2} - \kappa^2 \quad -(4)$$

i.e.

$$\partial^\mu \Omega^a_{\mu\nu} = \left(\frac{\omega^2}{c^2} - \kappa^2 \right) q^a_\nu = - \square q^a_\mu \quad -(5)$$

Eq. (5) also follows from the tetrad postulate:

$$D_\mu q^a_\nu = \partial_\mu q^a_\nu + \omega^a_{\mu b} q^b_\nu - \Gamma^\kappa_{\mu\nu} q^a_\kappa = 0. \quad -(6)$$

Differentiating both sides of eq. (6) gives:

$$\square q^a_\nu + \partial^\mu \Omega^a_{\mu\nu} = 0 \quad -(7)$$

2) where:

$$\left.\begin{aligned}\Omega^a_{\mu\nu} &= \omega^a_{\mu b} q^b_\nu - \Gamma^\kappa_{\mu\nu} q^a_\kappa \\ &= \omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}\end{aligned}\right\} \quad -(8)$$

so

$$\partial^\mu \left(\omega^a_{\mu\nu} - \Gamma^a_{\mu\nu}\right) = \left(\frac{\omega^2}{c^2} - \kappa^2\right) q^a_\nu \quad -(9)$$

From eqs. (2) and (9):

$$\boxed{\partial^\mu \Omega^a_{\mu\nu} = \left(\frac{\omega^2}{c^2} - \kappa^2\right) q^a_\nu(0)\, e^{-i(\omega t - \kappa Z)}} \quad -(10)$$

Restricting consideration to the Z axis:

$$\partial^0 \Omega^a_{0\nu} + \partial^3 \Omega^a_{3\nu} = \left(\frac{\omega^2}{c^2} - \kappa^2\right) q^a_\nu(0)\, e^{-i(\omega t - \kappa Z)} \quad -(11)$$

i.e.

$$\frac{1}{c}\frac{\partial \Omega^a_{0\nu}}{\partial t} + \frac{\partial}{\partial Z}\Omega^a_{3\nu} = \left(\frac{\omega^2}{c^2} - \kappa^2\right) q^a_\nu(0)\, e^{-i(\omega t - \kappa Z)} \quad -(12)$$

i.e.

$$\boxed{\begin{aligned}\Omega^a_{0\nu} &= i\frac{\omega}{c} q^a_\nu(0)\, e^{-i(\omega t - \kappa Z)} && -(13)\\ \Omega^a_{3\nu} &= i\kappa\, q^a_\nu(0)\, e^{-i(\omega t - \kappa Z)} && -(14)\\ q^a_\nu &= q^a_\nu(0)\, e^{-i(\omega t - \kappa Z)} && -(15)\end{aligned}}$$