179(3): Classical and Quantum Covariant Mass.

The idea of covariant mass originates in the ECE wave equation:

$$\left( \Box + R \right) q^a_\mu = 0 \quad -(1)$$

where $R$ is well defined in terms of geometry. Eq. (1) is an identity of geometry. It may be transformed into a classical equation using Schroedinger's axiom:

$$\hat{p}^\mu \phi = -i\hbar \partial^\mu \phi \quad -(2)$$

so

$$\hat{p}^\mu \hat{p}_\mu = -\hbar^2 \partial^\mu \partial_\mu = -\hbar^2 \Box .$$

Therefore eq. (1) becomes:

$$\left( \hat{p}^\mu \hat{p}_\mu - \hbar^2 R \right) q^a_\mu = 0 . \quad -(3)$$

Replacing the operator $\hat{p}^\mu$ by the momentum $p^\mu$ gives:

$$\boxed{p^\mu p_\mu = \hbar^2 R} \quad -(4)$$

This is an equation of general relativity in which:

$$p^\mu = \left( \frac{E}{c}, \underline{p} \right) ; \; p_\mu = \left( \frac{E}{c}, -\underline{p} \right) -(5)$$

so

$$\frac{E^2}{c^2} - p^2 = \hbar^2 R \quad -(6)$$

i.e.

$$\boxed{E^2 = c^2 p^2 + c^2 \hbar^2 R} \quad -(7)$$

If

$$R = \left(\frac{mc}{\hbar}\right)^2 \quad -(8)$$

then eq. (7) has the same format as the Einstein energy equation of special relativity:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(9)$$

but eq. (9) is an equation of general relativity in which the covariant mass $m$ is defined by geometry:

$$\boxed{m^2 = \frac{\hbar^2}{c^2} q^{\nu}_{a} \partial^{\mu} \Omega^{a}_{\mu\nu}} \quad -(10)$$

where

$$\left.\begin{aligned} \Omega^{a}_{\mu\nu} &= \omega^{a}_{\mu\nu} - \Gamma^{a}_{\mu\nu} \\ &= \omega^{a}_{\mu b} q^{b}_{\nu} - \Gamma^{\lambda}_{\mu\nu} q^{a}_{\lambda} \end{aligned}\right\} \quad -(11)$$

In order to distinguish the covariant mass $m$ from the mass $m_0$ of special relativity, the Einstein equation of special relativity is written as:

$$E_0^2 = c^2 p_0^2 + m_0^2 c^4 \quad -(12)$$

In special relativity:

$$\underline{p_0} = \gamma m_0 \underline{v} \quad -(13)$$

and in general relativity:

$$\underline{p} = \gamma m \underline{v} \quad -(14)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(15)$$

3) In special relativity:

$$R_0 = \left(\frac{m_0 c}{\hbar}\right)^2 \quad -(16)$$

is a constant defined by the measured mass $m_0$ of a particle such as an electron.

In general relativity:

$$R = \left(\frac{mc}{\hbar}\right)^2 \quad -(17)$$

is not a constant. In general relativity mass $m$ is quantized according to:

$$\boxed{m = \hbar\left(\frac{R^{1/2}}{c}\right)} \quad -(18)$$

or

$$m = \alpha R^{1/2} \quad -(19)$$

where

$$\alpha = \frac{\hbar}{c} = 3.51773 \times 10^{-43} \text{ kgm m} \quad -(20)$$

Therefore ECE theory automatically achieves mass quantization in physics.

The Einsteinian rest energy is also quantized:

$$\boxed{E_0 = \hbar c R^{1/2}} \quad -(21)$$

In this notation, the Einsteinian rest energy of special relativity is denoted:

$$E_{00} = m_0 c^2. \quad -(22)$$

4.) In previous work in GCUFT1 the quantity $R$ was assumed to be proportional to mass density $m/V$ via the Einstein constant:

$$k = \frac{8\pi G}{c^2} = 1.86595 \times 10^{-26} \, N s^2 kg^{-2} \quad -(23)$$

i.e.

$$R = k \frac{m}{V} \quad -(24)$$

However, in the work of eight years ago $m$ was assumed to be constant. The work of UFT 158 ff shows that $m$ cannot be constant. If this is accepted, special relativity is generalized with great elegance.

From eqs. (8) and (24):

$$\boxed{m V = \left(\frac{\hbar}{c}\right)^2 k} \quad -(25)$$

in which both $m$ and $V$ may vary. We denote $m$ as the covariant mass, and $V$ the covariant volume.

The Einstein de Broglie equations in general relativity become:

$$E = \hbar \omega = \gamma m c^2 \quad -(26)$$

$$\underline{p} = \hbar \underline{\kappa} = \gamma m \underline{v} \quad -(27)$$

$$m = \hbar \frac{R^{1/2}}{c} \quad -(28)$$

and these are the October Postulates of ECE theory. Mass quantization is incorporated into Einstein / de Broglie.