179(2): Development of the Hamiltonian.

The generally covariant fermion equation can be written as:

$$\hat{H}_0 \phi = E \phi \quad -(1)$$

where:

$$\hat{H}_0 = \frac{\hbar^2 R}{m} + V + \frac{mc^2}{2\hbar^2 R}\hat{p}^2 - \frac{m^2c^2}{4\hbar^4 R}\underline{\sigma}\cdot\underline{\hat{p}}\left(\frac{(\epsilon - V)}{R}\underline{\sigma}\cdot\underline{\hat{p}}\right) \quad -(2)$$

where

$$\epsilon = E - \frac{\hbar^2 R}{m} \quad -(3)$$

is the generally covariant equivalent of the kinetic energy.

So eq. (1) can be written as:

$$\boxed{\hat{H}\phi = \epsilon\phi} \quad -(4)$$

where

$$\hat{H} = V + \frac{mc^2}{2\hbar^2 R}\hat{p}^2 - \frac{m^2c^2}{4\hbar^4 R}\underline{\sigma}\cdot\underline{\hat{p}}\left(\frac{(\epsilon - V)}{R}\underline{\sigma}\cdot\underline{\hat{p}}\right) \quad -(5)$$

i.e.

$$\hat{H}\phi = \left(V + \frac{mc^2}{2\hbar^2 R}\right)\phi - \frac{m^2c^2}{4\hbar^4 R}\underline{\sigma}\cdot\underline{\hat{p}}\left(\frac{(\epsilon - V)}{R}\underline{\sigma}\cdot\underline{\hat{p}}\phi\right) \quad -(6)$$

In the limit:

$$R = \left(\frac{mc}{\hbar}\right)^2 \quad -(7)$$

eq. (6) becomes:

$$\hat{H}\phi = \left(V + \frac{\hat{p}^2}{2m}\right)\phi - \frac{\underline{\sigma}\cdot\underline{\hat{p}}}{4m^2c^2}\left((\epsilon - V)\underline{\sigma}\cdot\underline{\hat{p}}\phi\right) \quad -(8)$$

2) which is the equation developed in UFT 178, in which

$$\epsilon = E - mc^2 = (\gamma - 1) mc^2 \quad -(9)$$

where

$$\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \quad -(10)$$

and

$$E = \gamma mc^2. \quad -(11)$$

Eq. (11) originates in:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(12)$$

the Einstein energy equation of special relativity. In eq. (13)

$$\mathbf{p} = \gamma m \mathbf{v} \quad -(13)$$

is the relativistic momentum. So:

$$E^2 = c^2 \gamma^2 m^2 v^2 + m^2 c^4$$

$$= m^2 c^2 (\gamma^2 v^2 + c^2)$$

$$= m^2 c^2 \left(\left(1 - \frac{v^2}{c^2}\right)^{-1} v^2 + c^2 \right)$$

$$= m^2 c^2 \left(\frac{v^2}{1 - \frac{v^2}{c^2}} + c^2 \right) = m^2 c^2 \left(\frac{v^2 + c^2\left(1 - \frac{v^2}{c^2}\right)}{1 - \frac{v^2}{c^2}} \right)$$

$$= m^2 c^4 \left(\frac{v^2 + c^2 - v^2}{c^2 - v^2} \right)$$

$$= m^2 c^4 \left(1 - \frac{v^2}{c^2}\right)^{-1} = \gamma^2 m^2 c^4 \quad -(14)$$

So

$$E = \gamma mc^2 \quad -(15)$$

Q.E.D.

3) In note 179(1), the non-relativistic limit of eq. (19) was taken:

$$E = (\gamma - 1) mc^2 = \left(\left(1 - \frac{v^2}{c^2} \right)^{-1/2} - 1 \right) mc^2$$

$$\rightarrow \left(1 + \frac{1}{2} \frac{v^2}{c^2} - 1 \right) mc^2$$

$$= \frac{1}{2} mv^2 = \frac{p^2}{2m} \quad -(16)$$

These procedures are repeated with the starting equation:

$$E^2 = p^2 c^2 + \left(\frac{R \hbar^2}{m} \right)^2 \quad -(17)$$

in which the covariant mass is defined by:

$$R = \left(\frac{mc}{\hbar} \right)^2 = q^{\nu}_{a} \partial^{\mu} \left(\omega^{a}_{\mu\nu} - \Gamma^{a}_{\mu\nu} \right) \quad -(18)$$

i.e.

$$\boxed{m^2 = \frac{\hbar^2}{c^2} q^{\nu}_{a} \partial^{\mu} \left(\omega^{a}_{\mu\nu} - \Gamma^{a}_{\mu\nu} \right)} \quad -(19)$$

In general:

$$(\square + R) q^{a}_{\mu} = 0 \quad -(20)$$

which is the ECE wave equation. In the limit:

$$R \rightarrow \left(\frac{m_0 c}{\hbar} \right)^2 \quad -(21)$$

eq. (20) becomes the fermion equation:

$$\left(\Box + \left(\frac{m_0 c}{\hbar} \right)^2 \right) q^a_\mu = 0 \quad -(22)$$

in which m_0 is the measured mass of the fermion, a constant. Using the Schroedinger postulate:

$$p^\mu = i\hbar \partial^\mu \quad -(23)$$

in relativistic format, eq. (22) becomes:

$$E_0^2 = p_0^2 c^2 + m_0^2 c^4 \quad -(24)$$

where

$$\underline{p}_0 = \gamma m_0 \underline{v} \quad -(25)$$

Using eqs. (20) and (23) gives the more general equation:

$$E^2 = p^2 c^2 + \hbar^2 R c^2 \quad -(26)$$

$$\boxed{E^2 = c^2 \left(p^2 + \hbar^2 R \right)} \quad -(27)$$

in which

$$\underline{p} = \gamma m \underline{v} \quad -(28)$$

$$m c^2 = \frac{\hbar^2 R}{m} \quad -(29)$$

and where m is the covariant mass. Eq. (26) is the generally covariant energy equation of classical physics. Eq. (28) is the generally covariant momentum.

Eq. (27) can be developed as:

$$E^2 = m^2 c^2 \left(\gamma^2 v^2 + \frac{\hbar^2 R}{m^2} \right) \quad -(30)$$

5)

i.e. as:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(31)$$

where m is the covariant mass. Eqs. (24) and (31) have the same structure, but in eq. (31) m is the quantity defined by eq. (19), the quantity named the covariant mass. Unlike m_0, m is not constant and is given by Cartan geometry. Therefore:

$$E = mc^2 \quad -(32)$$

is the generally covariant rest energy. The generally covariant total energy is:

$$E = \gamma mc^2 \quad -(33)$$

The rest energy and total energy of Einstein are:

$$E_{00} = m_0 c^2 \text{ and } E_0 = \gamma m_0 c^2 \quad -(34)$$

In the limit:

$$v \ll c \quad -(35)$$

$$\epsilon = (\gamma - 1) mc^2 \rightarrow \frac{p^2}{2m} \quad -(36)$$

where ϵ is the generally covariant kinetic energy.

Therefore the powerful result is obtained that the hamiltonian (6) can be written as:

$$\hat{H} = V - \frac{\hbar^2}{2m}\left(1 + \frac{V}{2m^2c^2}\right)\nabla^2 + \frac{\hbar^4}{8m^3c^2}\nabla^4$$

$$+ \frac{e^2}{8\pi\epsilon_0 r^3 m^2 c^2}\,\underline{\hat{S}}\cdot\underline{\hat{L}} - \left(\frac{\hbar^2}{4m^2c^2}\,\underline{\nabla} V\right)\underline{\nabla} \quad -(37)$$

in which n is defined by eq. (19). Eq. (37) has the same structure as the hamiltonian of UFT 178, but m_0 is replaced by m. This is a law of physics, the equations of special relativity become equations of general relativity when:

$$\boxed{m_0 \longrightarrow m} \quad -(38)$$